A Systematic and Simple Approach for Designing Takagi-Sugeno Fuzzy Controller with Reduced Data

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***Abstract*—This paper introduces a simple, systematic and effective method for designing Takagi–Sugeno (T–S) fuzzy controller utilizing a significantly small training data. Creating proper training data is not an easy task and requires spending considerable time and resources. The proposed method first uses the three-level factorial design to partition the input space. Next the Response Surface Methodology (RSM) is used to estimate the output spaces. The membership functions are introduced with only three variables (min, max and number of membership functions). Fuzzy rules are generated with respect to the output surfaces and membership functions. The proposed method is applied for controlling an inverted pendulum. Simulation results demonstrate significant improvement for controlling the inverted pendulum.**

***Keywords*—TAKAGI–SUGENO controller, RSM, data reduction, inverted pendulum.**

I. INTRODUCTION

TAKAGI–SUGENO system has recently become a powerful practical engineering tool for controlling complex systems. It also has been applied to a variety of industrial applications [1] as well as complex robotics applications such as biped [2], snake [3] and fish robots [4]. The underlying T–S fuzzy control is an interpolation method which partitions the input space into fuzzy areas. Each area is approximated by a simple local model (often a linear model). The global model is obtained by interpolation between the different local models. This model permits the approximation of a strongly nonlinear function by a simple structure and a limited number of rules. The consequents of the fuzzy rules are expressed as analytic functions. The choice of the function depends on its practical applications.

Despite the many advantages of T-S controller, its design significantly hinders its application [5]. Carrying out design of T-S controller is difficult because the explicit structure of T-S system is generally unknown, and also due to their inherent nonlinear nature. Many efforts have been made to enhance systematic and simple design of T-S. In [6] the premise and consequent identification are separately performed using fuzzy c-means and the orthogonal least squares method, respectively. Reference [7] considers the T-S models as fuzzy-neural networks and neural-type algorithms are used for model learning. Reference [8] developed several approaches that attempt to reduce the number of fuzzy rules by assessing their degrees of importance using singular value decomposition (SVD). They start with an oversized rule base and then remove redundant or less important fuzzy rules. Recently, considerable number of methods uses genetic Algorithms to build fuzzy Sugeno models [9,10]. For instance [11] proposed a genetic-based algorithm for generating simple and well-defined T-S models.

An important step in designing the T-S controller is creating the training data. Most of the above methods require a large number of training data for designing. However, data generation is not always an easy task. It can require excessive time and resources. It is clear that a method which requires less number of data and is simpler to apply while producing acceptable results, is more valuable. With respect to the author's knowledge only a few works have dealt with reducing the number of training data. Most researchers are focused on simplifying the T-S system by reducing the number of rules with similarity measure [12,13]. They assume that sufficient data is available and attempt to simplify the system after design is completed.

The primary objective of this research is to develop an efficient and simple method for designing T-S controller with reduced number of training data. To do this, RSM (Response Surface Methodology) is used for modeling the output space. RSM is one of the most powerful methods used in DOE (Design of Experiments) [14]. It is used for modeling the behavior of an unknown system with a reduced number of data and experiment. RSM is widely used in modeling and analysis of various complex processes [15,16,17]. In this paper, the marriage of RSM and T-S controller has made a simpler and efficient method which is then applied to controlling a complex system.

The rest of this paper is organized as follows. A section 2 provides the necessary background information on the Takagi-Sugeno fuzzy system as well as Response Surface Methodology. The main contribution of the paper is presented in Sections 3. It describes the proposed systematic design of the T-S controller. Section 4 provides an example which demonstrates the applicability of the method. Finally,

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concluding remarks relating the overall study will be drawn in the last section

II. PRELIMINARIES

A. Takagi-Sugeno fuzzy controller

A T-S fuzzy controller is described by a set of fuzzy "IF … THEN" rules. A generic T-S rule can be written as follows:

R_i : IF x_1 is A_{i1} AND x_2 is A_{i2} … AND x_r is A_{ir} , THEN

$$y_i = f_i(x_1, x_2, \ldots, x_r), \quad i=1,2,\ldots,n_R \tag{1}$$

where $A_{i1}, A_{i2},\ldots, A_{ir}$ are fuzzy sets in the antecedent, while y_i is a crisp function in the consequent. y_i is usually a polynomial function of input variables. However, it can be any function as long as it can appropriately describe the output of the model within the fuzzy region specified by the antecedent of the rule. When y_i is a first-order polynomial, as in this paper, the resulting fuzzy inference system is called a first-order Sugeno fuzzy model [18].

$$y_i = a_{i1}\, x_1 + a_{i2}\, x_2 + \ldots + a_{ir}\, x_r + b_i, \qquad i=1,2,\ldots,n_R. \tag{2}$$

where $a_{i1}, a_{i2},\ldots, a_{ir}$ and b_i are parameters which should be identified. The consequents of the T-S controller are hyperplanes (r-dimensional linear subspaces) in $\Re^{r+1}$, whereas the if-part of the rule partitions the input space and determines the validity of the n_R locally linear model for different regions of the antecedent space. Since each rule has a crisp output, the overall output of the T-S system could be obtained via weighted average formula (3).

$$y=\frac{\sum_{i=1}^{n_R} y_i \,.\, w_i}{\sum_{i=1}^{n_R} w_i} \quad , \quad w_i = \prod_{i=1}^{r} \mu_{A_i}(x_i) \tag{3}$$

n_R is equal to the numberof rules

What remains to complete the description of T-S controller is a method to estimate parameters $a_{i1}, a_{i2},\ldots, a_{ir}$ and b_i of the model shown in (2). In the next section Response Surface Methodology [19] is utilized in order to provide estimates for the model parameters.

B. Response surface methodology

Response surface methodology (RSM) was invented by Box and Wilson in 1951 [20] and has been applied in a wide variety of industrial setting and such as, chemical, semiconductor and electronic manufacturing, machining and metal cutting processes. RSM is a collection of mathematical and statistical techniques that are used to model and analyze engineering applications. RSM provides an output response surface that describes the overall behavior of the input variables.

In this study, the standard procedure for RSM has been modified in a way to be used with T-S system. It consists of the following steps:

Step 1. Design and conduct a series of experiments to get adequate and reliable measurements of the response output (e.g. orthogonal array experiment).

Step 2. Develop best fittings mathematical model for the first order response surface.

The relationship between the dependent y (response output) and input variables ($x_1, x_2, x_3, \ldots$) may be known exactly of the form

$$y = f(x_1, x_2, x_3, \ldots) + \varepsilon \tag{4}$$

Where ε represents the model error, measurement error and other variations. f is a first or second order polynomial which is the empirical or response surface model. The successful application of RSM relies on the identification of a suitable approximation for f . This will generally be a first order model of the form

$$f = \beta_0 + \beta_1\, x_1 + \ldots + \beta_k\, x_k \tag{5}$$

The response surface methodology is intimately connected to regression analysis. For example when considering the first order model, the β terms comprise the unknown parameter set which can be estimated by collecting experimental system data. This data can either be sourced from physical experiments or from previously designed dynamic computer models. The parameter set can be estimated by regression analysis based upon the experimental data. The method of least squares is typically used to estimate, β**'s,** the regression coefficients.

III. PROPOSED METHOD

This section investigates the general procedure for controlling the systems using the proposed method. In order to better describe the method, first a simple model having two inputs and one output is selected. It is worth noting that the proposed method can be generalized to higher dimensional systems. The general steps are defined as following.

A. Establish the upper and lower margins for the inputs

The assignment process to define the upper and lower margins can be intuitive or it can be based on some algorithmic or logical operation. It is however, usually derived through understanding and prior knowledge about the system. For example, if temperature is used as an input variable to define the range of human comfort we get one range, and if temperature is used to define the range of safe operating temperature for a steam turbine we get another range. If prior knowledge about the system is not available the designer may need to conduct a series of baseline experiments to help establish these margins [14]. This may be viewed as an algorithmic or logical approach.

B. Defining fuzzy membership functions

The choice of membership functions will help define output surface which itself is made of combining multiple surfaces. In

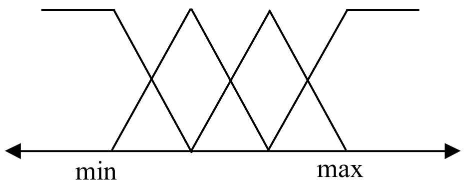

Figure 1. Membership functions

order to insure smooth transition among these surfaces, the number of membership function must be even, same type and input domain must be equally divided (Fig. 1).

C. *Training data*

Having an appropriate set of input-output training data is one of the most important factors in designing T-S system. This data should explain the behavior of unknown system. However, creating data is not an easy task and requires spending excessive time and resources. The main goal of the proposed method is to design a suitable system which uses the least number of training data. If *n* indicates the number of inputs, the total number of required data is computed by (6).

$$\text{Total number of data} = 3^n \tag{6}$$

The required number of data in this method is the same as a three-level factorial design [14]. Each input has three levels (low, medium and high). For a system with two inputs, the input space is divided as shown in Fig. 2. The star points indicate the location in input space where experiments are conducted.

D. *Consequents part of the fuzzy rules*

RSM is used to construct the consequents part of the fuzzy rules in T-S fuzzy system. The input domain is divided into four sections. Each section is represented by a first order surface (RSM_1- RSM_4). Surfaces are formed through four data

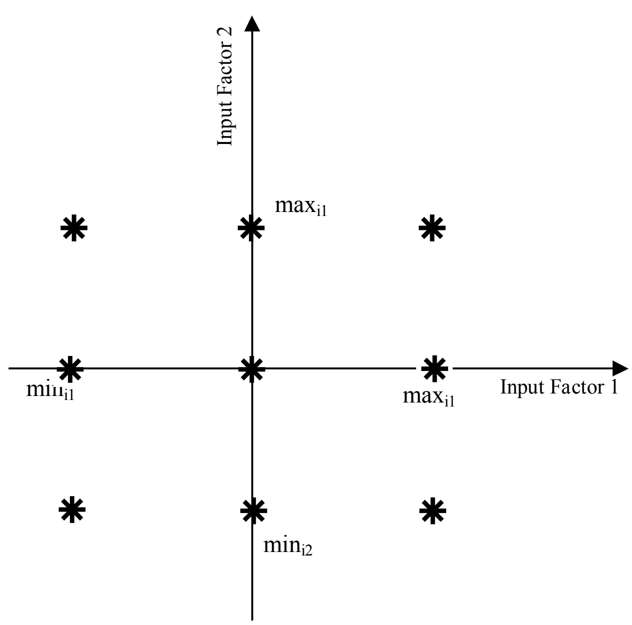

Figure 2. Treatment combination in a two input (3^2) system

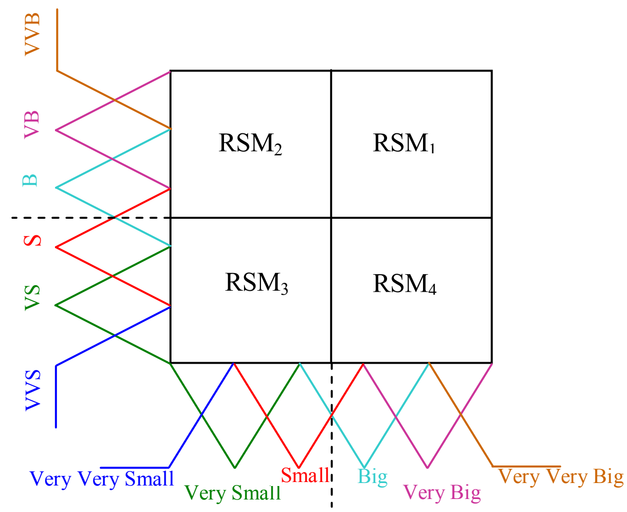

Figure 3. Relationship between inputs and output

with the aid of RSM. If x_1 and x_2 are first and second input variables and a_{ij} are constant parameters, then the four surfaces are defined by (7).

$$\begin{aligned}
RSM_1 &= y_1 = a_{11}\,x_1 + a_{12}\,x_2 + a_{13} \\
RSM_2 &= y_2 = a_{21}\,x_1 + a_{22}\,x_2 + a_{23} \\
RSM_3 &= y_3 = a_{31}\,x_1 + a_{32}\,x_2 + a_{33} \\
RSM_4 &= y_4 = a_{41}\,x_1 + a_{42}\,x_2 + a_{43}
\end{aligned} \tag{7}$$

Fig. 3 will clearly illustrate the relationship between inputs and output.

E. *Fuzzy rules and deffuzification*

Assuming six membership functions for each input, as in Fig. 3, the fuzzy rules for this system are given in Table I. Finally, the weighted average method is employed to defuzzify the output variable (3).

IV. Numerical Example Using the Template

In this section, we demonstrate the effectiveness of our approach by showing results of controlling the inverted pendulum around its unstable equilibrium point.

A. *Inverted pendulum*

The inverted pendulum is a highly nonlinear and unstable system. It is therefore often used as a benchmark for verifying the performance and effectiveness of new control methods. The system consists of an inverted pole hinged on a cart which is

TABLE I. Fuzzy Rules

x2 \ x1	**VVS**	**VS**	**S**	**B**	**VB**	**VVB**
VVS	RSM_3	RSM_3	RSM_3	RSM_4	RSM_4	RSM_4
VS	RSM_3	RSM_3	RSM_3	RSM_4	RSM_4	RSM_4
S	RSM_3	RSM_3	RSM_3	RSM_4	RSM_4	RSM_4
B	RSM_2	RSM_2	RSM_2	RSM_1	RSM_1	RSM_1
VB	RSM_2	RSM_2	RSM_2	RSM_1	RSM_1	RSM_1
VVB	RSM_2	RSM_2	RSM_2	RSM_1	RSM_1	RSM_1

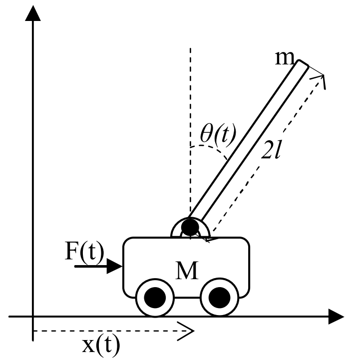

Figure 4. Inverted pendulum

free to move in the horizontal direction. The inverted pendulum inherently has two equilibria, one of which is stable while the other is unstable. The stable equilibrium corresponds to a state in which the pendulum is pointing downwards. In the absence of any control force, the system will naturally return to this state. The unstable equilibrium corresponds to a state in which the pendulum points strictly upwards and, thus, requires a control force to maintain this position. The basic control objective of the inverted pendulum problem is to maintain the unstable equilibrium position when the pendulum initially starts with some nonzero angle from the vertical position. An inverted pendulum is shown in Fig. 4. The nonlinear dynamical equations are given by (8) [21].

$$\begin{aligned} &(M+m)\ddot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F \\ &ml\ddot{x}\cos\theta+\tfrac{4}{3}ml^2\ddot{\theta}-mgl\sin\theta=0 \end{aligned} \tag{8}$$

where θ and $\dot{\theta}$ are the angular displacement and angular velocity of the pole, g (acceleration due to the gravity) is 9.8 m/s^2, M (mass of the cart) is 1 kg , m (mass of the pole) is 0.1 kg , l (half length of the pole) is 0.5m and F is the application force in Newton. F is determined by the controller to bring the pole into equilibrium position. $\theta(deg)$ and $\dot{\theta}$ *(deg/s)* are the controller inputs and are varied within [-20, 20] and [-70, 70] respectively. The boundary conditions can be chosen by the designer according to feasible domains of input variables [22]. Six membership functions are assumed for each input variable and are divided equally between the two limits (Fig. 5).

The controller has two inputs, therefore, a three-level factorial design requires having nine training data set (6). With respect to system dynamics and expert knowledge, forces for specific inputs conditions may be selected as in Fig. 6.

Four linear surfaces defined in (7) can now be formed through the use of RSM. Results are shown in (9). The order of numbering is inspired by Fig. 3.

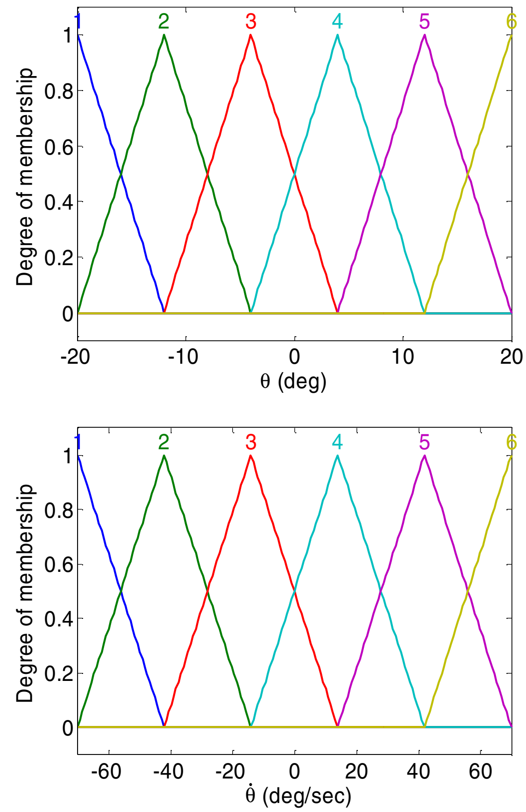

Figure 5. Membership functions for inverted pendulum problem

$$\begin{aligned} &RSm1=17.1887\theta+4.9111\dot{\theta}+1.5 \\ &RSm2=34.3775\theta+4.9111\dot{\theta}+1.5 \\ &RSm3=17.1887\theta+4.9111\dot{\theta}-1.5 \\ &RSm4=34.3775\theta+4.9111\dot{\theta}-1.5 \end{aligned} \tag{9}$$

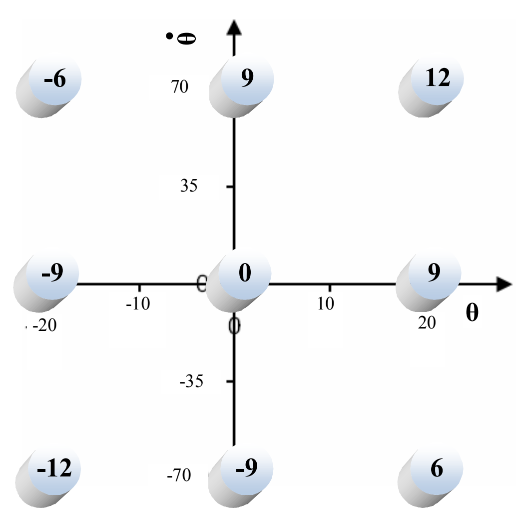

Figure 6. Input-output training data

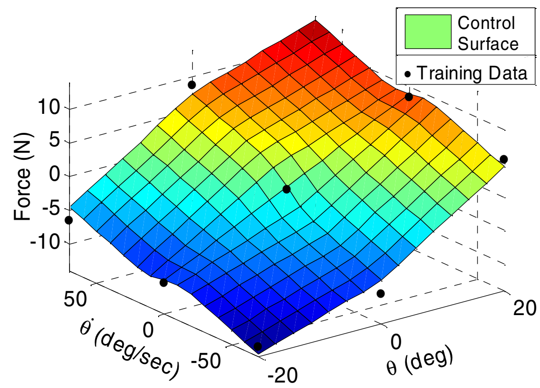

Figure 7. Control surface

The fuzzy rules are listed in Table I. The overall response of the system is shown in Fig. 7. Performance of the controller under various initial conditions is evaluated. The output responses are plotted in Fig. 8. This Figure indicates that the controller performs well even with a large deviation ($-71.4^o \le \theta \le 67.9^o$) from the equilibrium point. It should be noted that these large deviations do not fall into the region of the training data set ($-20^o \le \theta \le 20^o$). This demonstrates the robustness of the developed fuzzy control system with respect to unseen initial conditions [22]. Next, a comparison is performed with the fuzzy controller proposed by Y. L. Sun et al. [23].

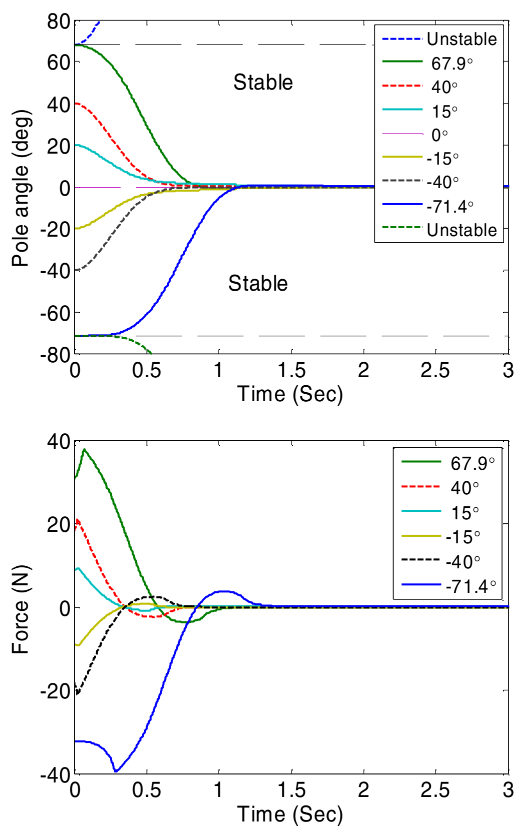

Figure 8. The response of the system

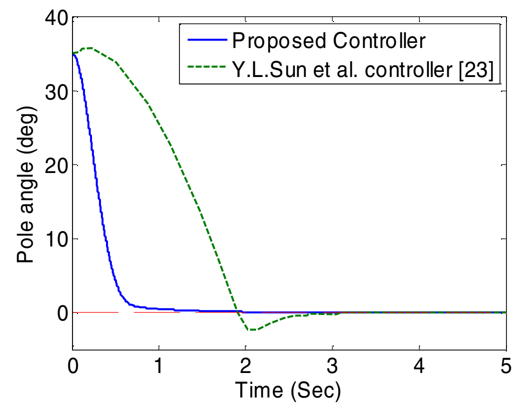

Figure 9. Comparison between two controller

Results indicate that our proposed controller significantly performances better than [23] (Fig. 9). Furthermore, the controller proposed by [23] becomes unstable after 41.83^o. Our proposed controller not only can stabilize the system faster than the [23] controller, but also as shown in Fig. 8, remains stable for a significantly larger deviations from vertical, up to 67.9^o.

V. CONCLUSIONS

We proposed a simple systematic procedure for design of T-S fuzzy controller, based on Response Surface Methodology. Membership functions and fuzzy rules were defined in a straightforward manner. The significance contribution of the study is the reduction of training data required for designing the T-S fuzzy controller while obtaining good system performance. The systematic approach facilitates conducting the T-S design in comparison with other methods.

To demonstrate the effectiveness, the proposed method was applied to control of an inverted pendulum. Results demonstrated a significant performance improvement.

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