

Control Structure Design in Power Systems

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Abstract: Most available control theories assume that a control structure is given at the outset, so they do not use the provisions of control structure design. Control structure design tries to close the gap between theory and practice. In fact control structure design helps the designer to utilize the maximum capability of the system. Control structure design is clearly more important in large scale systems.

Power systems are large scale systems that are subject to low frequency oscillations. These oscillations degrade the system performance. Power system stabilizers (PSSs) used to improve the performance of power systems, determination of the suitable signal for applying to power system stabilizer and determination of their locations is an important issue in power systems.

Some of the control structure design measures are introduced. These measures are used to determine the inputs, outputs and control configuration based on a systematic procedure that reduces the reliance on experiment and the art of the designer. Applicability of these measures in power systems is shown. Selection of a suitable signal for applying to a power system stabilizer and determination of the PSS locations are the major contributions of this paper.

Keywords: Power system stabilizer, singular value decomposition, relative gain array and block relative gain.

1 Introduction

Most available control theories assume that a control structure is given at the outset. The first step for an engineer is to determine the variable that should be controlled, the variable that should be measured, and the input that should be manipulated. Control structure design leads to closing the gap between theory and application in this area. Singular value decomposition (SVD), relative gain arrays (RGA) and block relative gains (BRG) are used as some measures in control structure design [1].

Power systems as large complex multivariable systems are suitable systems to consider control structure design on them. With the increasing interconnection of power systems, electromechanical oscillations restrict the steady-state power transfer limits and affect operational system economics and security. As a consequence these oscillations have become one of the main problems requiring solution in power system stability. Power system stabilizers are used to overcome this problem.

With its important capability of predicting, from the open loop system model, effective controllers that should be installed in a system, modal analysis offers a special application in power system stabilizers to improve system oscillation stability. Singular value decomposition (SVD) has been proposed as a measure of the distance of a controllability and observability matrix from singularity in a state space model [2].

This distance is used as an index to compare the ability of inputs to control an oscillation mode.

Above mentioned analysis is based on the conclusions of eigenvalues and left and right eigenvectors of the full power system state matrix. For large power systems it can hardly be used because of the numerical difficulty in the eigen solution of their huge state matrix. It is also illustrated in [3] that if the system has oscillation modes close to each other, indices according to modal analysis do not work correctly.

This paper deals with these difficulties by using suitable control structure design measures. This method guarantees not only the applicability to apply on systems with modes close to each other but also the order of matrices involved in the calculation is not higher than the number of machines in a power system [4-6].

The paper is organized as follows: the suitable control structure design measures are briefly described in Section 2; application of control structure design indices on power systems are described in Section 3; and finally results of simulation studies on a multi machine power system are described in Section 4.

2 Indices in control structure design

Singular value decomposition, relative gain array and block relative gain are suitable measures in control structure design and their conjunction with power systems seems to be applicable.

• Singular value decomposition (SVD)

Consider M is a constant matrix in $C^{l \times m}$. Then M can be decomposed into its singular value decomposition according to the following theorem.

Theorem: Let $M \in C^{l \times m}$, then there exist $\Sigma \in R^{l \times m}$ and unitary matrices $U \in C^{l \times l}$ and $V \in C^{m \times m}$ such that:

$$M = U \Sigma V^H \quad (1)$$

where $\Sigma = \begin{bmatrix} S & 0 \\ 0 & 0 \end{bmatrix}$, $S = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_r\}$ with

$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r$ and $r \leq \min\{l, m\}$ and V^H is

complex conjugate transpose of V . The column vector elements of U , identified by u_i , are orthogonal and of unit length, and represent the output directions. The column vector elements of V , identified by v_i , are orthogonal and of unit length, and represent the input directions. These input and output directions are related through singular values by:

$$M v_i = \sigma_i u_i \quad (2)$$

It means that if an input is applied in the direction v_i , then the output is in the direction u_i and has a gain of σ_i . Input direction v_i for $i > r$ corresponds to inputs that do not have any influence on outputs and similarly output direction u_i for $i > r$ corresponds to the outputs that cannot be accessed by any input. Since the diagonal elements of S are arranged in a descending order, it can be shown that the largest gain for any input direction is equal to the maximum singular value σ_1 . So one can write:

$$\sigma_1 = \max_{d \neq 0} \frac{\|M d\|_2}{\|d\|_2} = \frac{\|M v_1\|_2}{\|v_1\|_2} \quad (3)$$

where d is any input signal and $\|\cdot\|_2$ is the Euclidian norm. Expansion of (1) leads to:

$$M = \sum_{k=1}^r \sigma_k u_k v_k^H \quad (4)$$

Singular value decomposition has the ability to find the most effective input and output direction. Controllability and observability of oscillation modes in power systems can be considered by input and output direction corresponding to maximum singular value at neighborhood of oscillation mode [6].

• Relative gain array (RGA)

The relative gain array (RGA) was introduced in [7]. It provides the designer with a quick assessment of interaction among the control loops of a multivariable system. The relative gain array of an $l \times m$ matrix G is defined as

$$RGA(G) = \Lambda(G) = G \times (G')^T \quad (5)$$

where $(\cdot)'$ is pseudo inverse, $(\cdot)^T$ means

transpose and \times denotes element by element multiplication. The RGA has a number of interesting control properties [8]. Relative gain array can exhibit the interaction of different input/output pair on the total system. This property can be used in the detection of convolving generators in each oscillation mode.

• Block relative gain (BRG)

The original definition of the relative gain as a scalar, has limited its applicability exclusively to SISO control loops. The relative gain array concept and its properties from a scalar to a matrix form were extended in [9] by defining it as block relative gain (BRG). It yields a more powerful synthesis framework that can address a broader class of control problems such as the synthesis of decentralized control structures that are not restricted to SISO control loops.

Consider a square $n \times n$ transfer function matrix $G(s)$ partitioned as below in (6):

$$G = \begin{array}{cc} \begin{array}{c} \longleftrightarrow m \\ \longleftrightarrow n-m \end{array} & \\ \begin{array}{c} \updownarrow m \\ \updownarrow n-m \end{array} & \\ \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} & \end{array} \quad (6)$$

Outputs y_1 are interconnected with the first m inputs u_1 and the last $n-m$ outputs y_2 are interconnected with the last $n-m$ inputs u_2 .

Relation between u_1 and y_1 when the last $n-m$ loops are open is:

$$y_1(s) = G_{11}(s)u_1(s) \quad (7)$$

If one considers perfect control on the last $n-m$ loops then relation between u_1 and y_1 is given as [9]:

$$y_1 = (BRG_l)^{-1} G_{11} u_1 \quad (8)$$

$$y_1 = G_{11} (BRG_r)^{-1} u_1 \quad (9)$$

where BRG_l and BRG_r left and right block relative gains correspondingly, are defined as:

$$BRG_l = \left[\left. \frac{\partial y_1(s)}{\partial u_1(s)} \right|_{u_2=0} \right] \left[\left. \frac{\partial y_1(s)}{\partial u_1(s)} \right|_{y_2=0} \right]^{-1} \quad (10)$$

$$= G_{11}(s) \cdot [G^{-1}(s)]_{11}$$

$$BRG_r = \left[\left. \frac{\partial y_1(s)}{\partial u_1(s)} \right|_{y_2=0} \right]^{-1} \left[\left. \frac{\partial y_1(s)}{\partial u_1(s)} \right|_{u_2=0} \right] \quad (11)$$

$$= [G^{-1}(s)]_{11} \cdot G_{11}(s)$$

When $BRG_l = I$ which implies $BRG_r = I$, the performance of the $m \times m$ block is isolated from the rest of the plant and operates under the influence of only its own control law. According to this property, closeness of BRG_l or BRG_r to identity matrix is a measure of interaction.

Above mentioned property of BRG can be used in the detection of convolving generators in each oscillation mode.

3 Systematic algorithm for determination of PSS locations

In a multi-machine power system it is important to determine the best location for the application of power system stabilizers. Following algorithm can be use to identify generators that contribute in each oscillation mode and the best location for the installation of the PSSs.

- 1- Consider set points of different exciters as input candidates and speed of different generators as output candidates.
- 2- Extract transfer function matrix for input and output candidates and do scaling [10].
- 3- Find the oscillation modes and perform the following steps for each oscillation mode [6].
- 4- Find convolving generators in each oscillation mode by one of the following indices.

a) Singular value decomposition index:

The proposed technique depends upon distinguishing the contribution of various generators to each oscillation mode by finding the minimum number of generators for which the corresponding reduced transfer function has the maximum singular value near to the largest singular value

of the whole system at that frequency. To do this, the row and column corresponding to each generator is omitted and its effect on the maximum singular value is examined. If the reduction in the maximum singular value is small, one understands that this generator does not contribute in this mode. Otherwise, this generator is contributing in this mode. Repeating this method for all generators and omitting the non-contributing ones, it is possible to find the minimum number of generators whose corresponding reduced transfer function has maximum singular value near to the largest singular value of the real system at that frequency [6].

b) Relative gain array index:

RGA elements are considered in the frequency of oscillation mode. The system has large sensitivity of element-by-element uncertainty for large elements in RGA matrix. Each mode of oscillation is due to contribution of many generators which have large interaction among them. Large RGA elements are a measure of interaction, so one can find generators that contribute in each oscillation mode through RGA elements [6].

c) Block relative gain index :

BRG is the extension of RGA. The contribution of various generators to each oscillation mode can be extracted by considering their BRG. Block relative gain of contributing generators in each oscillation mode must be close to identity matrix. This property is used to distinguish the contribution of various generators to each oscillation mode [1].

- 5- Delete from the transfer function matrix generators that do not contribute to the oscillation mode. The best location to apply PSS can be found by one of the following indices. The attention is focused on the use of local controllers. Because of practical

reasons, signals of distant generators are generally avoided for application to the PSS.

a) Use of reduction in maximum singular value:

By deleting the column and row corresponding to each generator in turn, and checking the maximum singular value of the remaining system, best PSS location for improving each oscillation mode can be found. A generator, the deletion of which leads to the smallest magnitude of the maximum singular value of the remaining system, is the most effective generator for that mode and is thus the most appropriate location for a PSS to damp this mode. This procedure combines the controllability and observability measures in one step [1].

b) Use of input and output direction from SVD and their product:

The input and output directions of transfer function matrix and their products at the neighborhood of oscillation mode are equivalent to controllability, observability, and residue correspondingly [6]. This property is used to select the suitable unit to which a PSS should be applied.

- 6- Repeat steps 2-5 for different operating points. Try to find the best location of PSS for each oscillation mode that is suitable for the whole operating points.

- 7- If one cannot find a local decentralized PSS for some oscillation mode that is viable for the whole operating point, the following steps are suggested.

a) Signals from other generators:

Use additional signals from distant generators as PSS input.

b) Other type of controller:

Use other types of control equipment such as FACTS, that change the effective parameters of the transmission line.

4 Application of control structure design measures in multi-machine power systems

Steps 4 and 5 of the proposed algorithm are illustrated in this Section to show the applicability of the proposed procedure. A 4-machine system is shown in Fig. 1. Parameters of the system are given in [3]. It is illustrated in [3] that if the oscillation modes are close to each other, the indices according to modal analysis do not work correctly and one must make minor variation in some system parameters (inertia of all machines of one area in [3]) to find suitable results for applying PSSs according to the eigenvector methods. The method proposed here is applicable to systems with oscillation modes that are close to each other.

Considering the exciters as inputs and generator speeds as outputs, transfer function of the system is extracted. To scale the variables, the maximum expected deviation from the normal value should be chosen. Dividing each variable by its maximum value scales all variables.

4.1) Identifying generators that contribute in each oscillation mode

According to step 4 of Section 3 there are three methods to distinguish the convolving generators in each oscillation mode.

a) Singular value decomposition:

Singular value plots of the system are shown in Fig. 2. It can be seen that there are two peaks in the singular values plot. The largest singular value obtained by deleting each generator in turn at 3.36 rad/s is shown in Table 1. For the peak at 3.36 rad/s the set of all generators has the maximum singular value comparable with the total system, so all generators contribute in this mode.

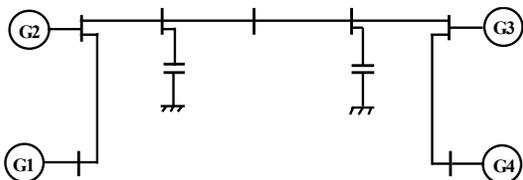


Figure 1. Example of a 4-machine power system

The largest singular value obtained by deleting each generator in turn at 6.4 rad/s is shown in Table 2. Table 2 shows that for the oscillation mode of frequency 6.4 rad/s both sets of {G1,G2} and {G3,G4} have the maximum singular value comparable with the total system. So there are two oscillation modes corresponding to 6.4 rad/s.

b) Relative gain array:

Elements of RGA versus frequency are shown in Fig. 3. It can be seen that there are two peaks in the RGA elements. Absolute values of RGA at 3.35 rad/s and 6.7 rad/s are shown by $\hat{\Lambda}$:

$$\hat{\Lambda}(j3.35) = \begin{bmatrix} 0.7169 & 1.0937 & 0.3551 & 0.4968 \\ 0.7529 & 0.4311 & 0.3633 & 0.5053 \\ 0.3709 & 0.4676 & 5.8485 & 6.1447 \\ 0.4002 & 0.5991 & 6.2690 & 5.6513 \end{bmatrix}$$

$$\hat{\Lambda}(j6.7) = \begin{bmatrix} 2.7723 & 2.4357 & 0.0146 & 0.0198 \\ 2.4054 & 2.7594 & 0.0098 & 0.0180 \\ 0.0351 & 0.0337 & 1.6323 & 1.6771 \\ 0.0370 & 0.0390 & 1.6918 & 1.5789 \end{bmatrix}$$

Relative values of RGA elements at 3.35 rad/s show that all generators contribute in this frequency. Relative values of RGA elements at 6.7 rad/s show that at this frequency there is interaction only between the sets {G₁,G₂} and {G₃,G₄}. Thus there are two local oscillation modes corresponding to each set at 6.7 rad/s.

c) Block relative gain:

Table 3 shows the absolute value of diagonal elements of right block relative gain at 3.36 rad/s. This table shows that just the 4 × 4 block has diagonal elements

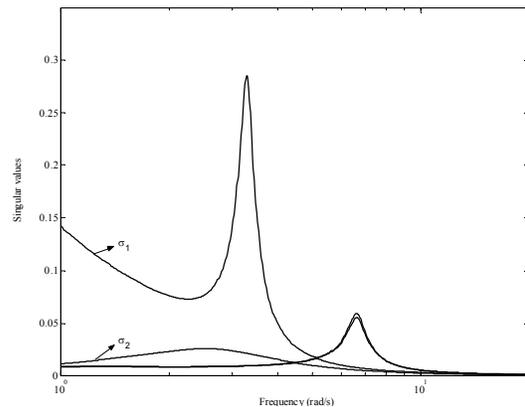


Figure 2. Singular values of 4-machines system

Table 1
3.36 rad/s oscillation mode

Number of omitted Generator	Max singular value
No omission	117.8816
1	102.5896
2	102.6965
3	68.8785
4	73.9812
1,2	84.7918

Table 2
6.4 rad/s oscillation mode

Number of omitted Generator	Max singular value
No omission	21.9891
1	19.6972
2	19.6950
3	21.8219
4	21.8342
1,2	19.6503
3,4	21.7861

near one. So they show that all generators contribute in this oscillation mode. Table 4 shows the absolute value of diagonal elements of right block relative gain at 6.4 rad/s. This table shows that at this frequency BRG of $\{G_1, G_2\}$ and $\{G_3, G_4\}$ is near to identity matrix. Thus there are two local oscillation modes corresponding to each set at 6.4 rad/s. All three above mentioned measures confirm that there is one inter-area mode around 3.36 rad/s and there are two local modes around 6.4 rad/s. Two local modes correspond to $\{G_1, G_2\}$ and $\{G_3, G_4\}$. Inter-area mode corresponds to oscillation of all generators. Now one must choose suitable PSS location for each mode.

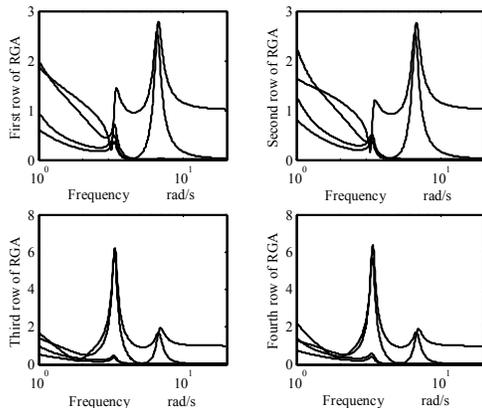


Figure 3. RGA elements of a 4-machine system

Table 3
Absolute value of diagonal elements of right block relative gain at 3.36 rad/s

1+2	<u>0.31</u>	<u>0.84</u>	0.71	0.98
1+3	<u>0.85</u>	1.07	<u>5.80</u>	6.24
1+4	<u>0.98</u>	0.86	6.40	<u>5.64</u>
2+3	0.99	<u>0.84</u>	<u>5.78</u>	6.28
2+4	0.87	<u>1.00</u>	6.43	<u>5.60</u>
3+4	0.75	1.03	<u>0.58</u>	<u>0.62</u>
1+2+3	<u>0.65</u>	<u>0.85</u>	<u>5.77</u>	6.34
1+2+4	<u>0.63</u>	<u>0.70</u>	6.50	<u>5.65</u>
1+3+4	<u>1.20</u>	0.97	<u>0.70</u>	<u>0.65</u>
2+3+4	1.13	<u>1.43</u>	<u>0.72</u>	<u>0.69</u>
1+2+3+4	<u>1.00</u>	<u>1.00</u>	<u>1.00</u>	<u>1.00</u>

4.2) Determination of the best PSS location for each oscillation mode

There are two methods to determine the PSS locations according to step5 of Section 3.

a) *Use of reduction in maximum singular value:*

By deleting the column and row corresponding to each generator in turn, check the maximum singular value of the remaining system. A generator that leads to the maximum reduction in the largest singular value is the most effective

Table 4
Absolute value of diagonal elements of right block relative gain at 6.4 rad/s

1+2	<u>0.97</u>	<u>1.00</u>	0.02	0.03
1+3	<u>2.63</u>	2.54	<u>1.37</u>	1.62
1+4	<u>2.69</u>	2.55	1.65	<u>1.34</u>
2+3	2.58	<u>2.68</u>	<u>1.36</u>	1.62
2+4	2.52	<u>2.65</u>	1.65	<u>1.33</u>
3+4	0.04	0.03	<u>0.99</u>	<u>0.98</u>
1+2+3	<u>0.97</u>	<u>1.03</u>	<u>1.35</u>	1.61
1+2+4	<u>1.00</u>	<u>0.97</u>	1.66	<u>1.35</u>
1+3+4	<u>2.65</u>	2.54	<u>1.00</u>	<u>0.99</u>
2+3+4	2.56	<u>2.68</u>	<u>0.99</u>	<u>0.99</u>
1+2+3+4	<u>1.00</u>	<u>1.00</u>	<u>1.00</u>	<u>1.00</u>

Table 5
Singular values of 4-machine system

Frequency rad/s	3.36	6.63	6.63
Convolving generators	#1,#2,#3,#4	#1,#2	#3,#4
Number of omitted generator	Maximum	singular	value
No omission	117.8816	21.7861	19.6503
#1	102.5896	12.0932	**
#2	102.6965	9.8403	**
#3	68.8785	**	11.1827
#4	73.9812	**	7.6023

generator for damping of that mode. The largest singular values obtained by deleting each generator in turn for all of the modes are shown in Table 5. According to Table 5 omitting G3 reduces the singular values corresponding to 3.36 rad/s the most, so it is the best position to apply PSS for improving the performance of the system, and G2 and G4 respectively are best positions for improving the local modes. A rank between generators for improving this oscillation mode is also shown in Table 5.

b) *Use of input and output direction from SVD and their product:*

For choosing the best input-output pair for improving the damping of inter-area mode p one must find the SVD of the system at $s = p + \varepsilon$. The SVD of the system at the inter-area mode is:

$$G_{all} \cong 41.6 \begin{bmatrix} 0.1578 - 0.0578i \\ 0.1117 - 0.0581i \\ -0.6039 - 0.3962i \\ -0.5487 - 0.3648i \end{bmatrix} \begin{bmatrix} 0.4794 \\ 0.4904 + 0.0239i \\ -0.5028 + 0.0004i \\ -0.5251 - 0.0242i \end{bmatrix}^H$$

Absolute values of u_1, v_1 and $u_1 v_1^H$ as residue of the system are shown by \hat{u}_1, \hat{v}_1 and \hat{R} :

$$\hat{u}_1 = \begin{bmatrix} 0.1680 \\ 0.1259 \\ 0.7223 \\ 0.6589 \end{bmatrix} \quad \hat{v}_1 = \begin{bmatrix} 0.4794 \\ 0.4910 \\ 0.5028 \\ 0.5256 \end{bmatrix}$$

$$\hat{R} = \begin{bmatrix} 0.0806 & 0.0825 & 0.0845 & 0.0883 \\ 0.0603 & 0.0618 & 0.0633 & 0.0662 \\ 0.3463 & 0.3547 & 0.3632 & 0.3797 \\ 0.3159 & 0.3236 & 0.3313 & 0.3464 \end{bmatrix}$$

Relative magnitude of \hat{v}_1 shows that exciter of generator four is the best input candidate, relative magnitude of \hat{u}_1 shows that speed of generator three is the best measurement signal. Residue matrix \hat{R} considers both controllability and observability and suggests generator four for the application of PSS and speed of generator three as a suitable signal to apply to the PSS. But the need for local controller suggests generator three for applying a local PSS.

For study on local oscillation modes, there are two local modes for sets $\{G_1, G_2\}$ and $\{G_3, G_4\}$. For local mode of set $\{G_1, G_2\}$ one must extract transfer function corresponding to these generators. The SVD of this transfer function at $s = p + \varepsilon$ is:

$$G_{all} \cong 22.51 \begin{bmatrix} 0.6213 - 0.0422i \\ -0.7807 + 0.0522i \end{bmatrix} \begin{bmatrix} 0.7143 \\ -0.6995 - 0.0236i \end{bmatrix}^H$$

Absolute value of u_1, v_1 and $u_1 v_1^H$ as residue of the system are shown by \hat{u}_1, \hat{v}_1 and \hat{R} :

$$\hat{u}_1 = \begin{bmatrix} 0.6227 \\ 0.7825 \end{bmatrix} \quad \hat{v}_1 = \begin{bmatrix} 0.7143 \\ 0.6999 \end{bmatrix}$$

$$\hat{R} = \begin{bmatrix} 0.4448 & 0.4358 \\ 0.5589 & 0.5476 \end{bmatrix}$$

Relative magnitude of \hat{v}_1 shows that exciter of generator one is the best input candidate, relative magnitude of \hat{u}_1 shows that speed of generator two is the best measurement signal. Residue matrix \hat{R} suggests generator two for applying a local PSS. The same procedure for the set $\{G_3, G_4\}$ suggests that exciter of generator four is the best input candidate and speed of generator four

is the best measurement signal. So generator four is the best candidate for applying a local PSS.

5 Conclusions

A systematic procedure based on control structure design measures is proposed to find the generators contributing in each oscillation mode and also the best location for applying PSSs is determined. The advantages of the proposed algorithm are:

- No computation involving the right (left) eigenvectors and eigen sensitivity analysis is required.
- This method can be used by information of simulation of a system or modeling the system.
- This method has the applicability to apply on systems with modes close to each other.
- The order of all matrices involved in the calculation is not higher than the number of machines in a power system.

Test results on 4-machine system show that the proposed method is reliable and straightforward.

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