

Fast and Robust Detection of Epilepsy in Noisy EEG Signals Using Permutation Entropy

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Abstract—Permutation entropy (PE) is a new complexity measure which can extract important information from long, complex and high-dimensional time series. The advantages of this measure such as its fast calculation and robustness with respect to additive noise make it suitable for biomedical signal analysis. In this paper the ability of PE for characterizing the normal and epileptic EEG signals is investigated. Classification is performed using discriminant analysis. The effect of additive Gaussian noise on the discrimination performance is also studied and some parameters derived from PE are suggested to improve the classification accuracy when the signal is contaminated with noise. The results indicate that the proposed measures can distinguish normal and epileptic EEG signals with an accuracy of more than 97% for clean EEG and more than 85% for highly noised EEG signals.

Keywords—EEG; epilepsy; ordinal pattern; permutation entropy

I. INTRODUCTION

About 1% of the world's population suffers from epilepsy. 75% of the patients can be cured by anti-epileptic drugs or epilepsy surgery. The remaining 25% do not respond to available medical or surgical treatments [1]. Developing effective and reliable methods for detecting and predicting the onset of epileptic seizures would help to understand the mechanisms underlying epilepsy and can improve treatment strategies or allow preventive actions.

Electroencephalogram (EEG) is a recording of brain electrical activity from the scalp using an array of electrodes. EEG signal is the most widely used instrument for clinical evaluation of the brain activity. Visual analysis of the EEG by a trained and skilled electroencephalographer has been the most commonly used method to detect epilepsy.

Visual inspection and diagnosis of EEGs is a very time-consuming and tedious task. In the past two decades numerous approaches have been made for automatic detection of epileptic seizures.

The traditional techniques to characterize EEG signals are based on linear methods such as Fourier transforms and spectral analysis. However as EEG is a non-stationary signal and stems from a highly nonlinear system, such methods must be used with care and caution [2]. Recently, there has been an increasing interest in applying techniques developed in chaos

theory and nonlinear dynamical systems for EEG signal analysis.

Several methods have been developed based on nonlinear theory which include approaches based on correlation dimension [3], Lyapunov exponent [4], entropy [5], dissimilarity measures [6] and recurrence quantification analysis [7].

Chaos-based measures hypothesize that signal is stationary and originates from a low dimensional nonlinear system. The effectiveness of applying these measures for analysis of EEG which is a time varying signal and is contaminated with noise and so can be regarded as a high dimensional signal is still under question. Moreover most of these methods are computationally costly and efficient algorithms must be used to speed up the computation.

In 2002 Bandt and Pompe introduced the concept of ordinal patterns which describe the order relations between the values of a time series [8]. They proposed a new complexity measure named permutation entropy (PE) to quantify the diversity of these patterns. It has been shown that this measure can be used to detect changes in the dynamics underlying nonlinear time series data [9].

Permutation entropy is computationally very fast and seems to be robust with respect to noise as reported in [8].

This paper aims to study the ability of permutation entropy for characterizing the normal and epileptic EEG signals. The discrimination performance of this measure is tested using linear discriminant analysis. The effect of additive Gaussian noise on the performance is also investigated and some parameters derived from PE are proposed to improve the classification accuracy in the presence of high noise.

II. METHOD AND MATERIALS

A. The concept of ordinal patterns and permutation entropy

Consider a scalar time series $(x_t)_{t \in 1, 2, \dots, N}$. As is well known, the first step in nonlinear data analysis is phase space reconstruction. The most common method to do this is using delay time embedding theorem [10].

In this approach values of the time series are transformed into a delay vector

$$x_i \rightarrow (x_{i-(d-1)\tau}, x_{i-(d-2)\tau}, \dots, x_{i-\tau}, x_i)$$

Which assign to each time, i , the d -dimensional vector of values at time $i, i - \tau, i - 2\tau, \dots, i - (d - 1)\tau$.

d is called the embedding dimension and τ is the embedding lag. This converts the N scalars into $N - \tau(d - 1)$ vectors with overlapping entries.

We can arrange the values in a d -dimensional delay vector in an increasing order to achieve an ordinal pattern: $[x_{i-r_d\tau} \leq x_{i-r_{d-1}\tau} \leq \dots \leq x_{i-r_1\tau} \leq x_{i-r_0\tau}]$

By the ordinal pattern we mean the permutation $(r_0, r_1, \dots, r_{d-1})$ of $(0, 1, \dots, d - 1)$ satisfying $x_{i-r_d\tau} \leq x_{i-r_{d-1}\tau} \leq \dots \leq x_{i-r_1\tau} \leq x_{i-r_0\tau}$

When equality occurs we set $r_l \leq r_{l-1}$. Thus, any of the delay vectors is uniquely mapped onto an ordinal pattern.

It is clear that there are $d!$ ordinal patterns for d -dimensional delay vectors, so we can identify each of the ordinal patterns with exactly one of the symbols $j = 1, 2, \dots, n = d!$.

In this way the reconstructed trajectory in the d -dimensional space is represented by a symbol sequence.

Now, we calculate the probability distribution for every symbol

$$p(j) = \frac{\#\{i | 1 \leq i \leq N - (d - 1)\tau, \text{ where } i \text{ has type } j\}}{N - (d - 1)\tau}$$

$$j = 1, 2, \dots, n = d!$$

Then the permutation entropy is defined as

$$H(d, \tau) = - \sum_{j=1}^{d!} p(j) \log p(j)$$

Clearly, $0 \leq H(d, \tau) \leq \log d!$ where the lower bound is attained for an increasing or decreasing sequence of values.

When $p(j) = \frac{1}{d!}$ then $H(d, \tau)$ attains the maximum value, $\log(d!)$, so for convenience we normalize $H(d, \tau)$:

$$0 \leq \frac{H(d, \tau)}{\log(d!)} \leq 1$$

Permutation entropy is a complexity measure for time series. The ordinal patterns can be computed in a very fast and easy way. As we deal with the order relations between values instead of the values themselves, the permutation entropy is robust with respect to noise corrupting the data.

B. Discrimination parameters

In addition to the permutation entropy described above, two novel parameters are proposed to improve the classification

accuracy when the signal is contaminated with noise. As will be shown in further details in next section, PE-Mean and PE-MD which are defined as the mean value and the mean deviation of the permutation entropies calculated for a range of embedding lags respectively, are much more robust with respect to additive Gaussian noise.

C. EEG data description

The EEG data were obtained from the EEG database made available online by Dr. Ralph Andrzejak [11]. Two sets containing healthy and epileptic subjects were composed for this research. Each set consisted of 100 single channel EEG segments of 23.6 sec duration sampled at 173.61 Hz. Healthy EEG dataset contains segments taken from surface EEG recordings of five healthy volunteers with eyes open. Epileptic EEG dataset includes data recorded during seizure activity. The type of epilepsy was diagnosed as temporal lobe epilepsy.

III. EXPERIMENTS AND RESULTS

The permutation entropy was applied to the described EEG signals. To reconstruct the phase space firstly we have to suggest suitable values for embedding lag and dimension. Different methods have been suggested to obtain the embedding lag. Here τ is selected as the time the autocorrelation function drops to $1/e$ of its initial value. Consequently, τ is estimated to be 3.

As mentioned before, the algorithm of the permutation entropy calculation is very fast but large values for embedding dimension, d , may cause some memory limitation on a PC. The length of the time series imposes an upper bound on the embedding dimension. Actually for an accurate calculation the length must be significantly larger than $d!$. Furthermore the value of the embedding dimension must be at least 3. Bandt and Pompe recommend $d=3, 4, \dots, 7$ for real-world time series [8]. Here we determine the embedding dimension $d = 3$. Selecting larger embedding dimensions will have little impact on the results.

Fig. 1 shows permutation entropy calculated for 100 healthy and 100 epileptic EEG segments.

The box plot shows how this measure might discriminate between these two groups. Each box has lines at the lower quartile, median, and upper quartile values. The whiskers show overall data range and outliers are data with values beyond the ends of the whiskers.

A t-test was used to investigate whether the mean values of permutation entropies calculated for epileptic and normal subjects are statistically different.

The mean value and the standard deviation of healthy and epileptic groups and also the p-value of the t-test are shown in table I.

These statistical analyses indicate that a simple threshold is sufficient to distinguish between healthy and epileptic subjects.

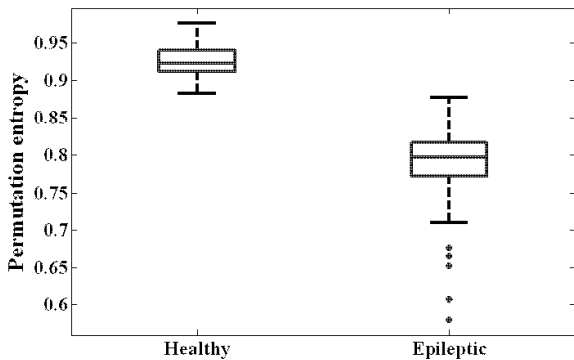
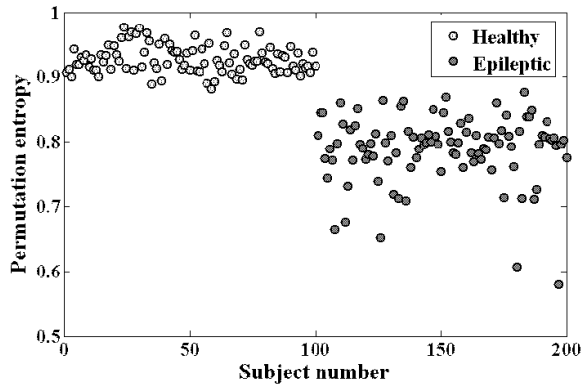


Figure 1. Permutation entropy for 100 healthy and 100 epileptic EEG signals (Top) and the corresponding box plot (Bottom).

To investigate the robustness, Gaussian noise was added and then the above measure was applied to detect the epileptic waveform patterns. Linear discriminant analysis is used for classification.

Fig. 2 shows percentage of correctly classified cases in a validating dataset (60% of the data) using a training set (40% of the data) for different signal-to-noise ratio (SNR) values. SNR represents the signal power (before addition of the noise) to the noise power.

As can be seen good discrimination between healthy and epileptic subjects can be obtained for $SNR \geq 20$. In the presence of high noise the ability of permutation entropy to distinguish between the two datasets decreases.

TABLE I

	Mean	Standard deviation	p-value
Healthy group	0.92	0.021	0.0000
Epileptic group	0.78	0.051	

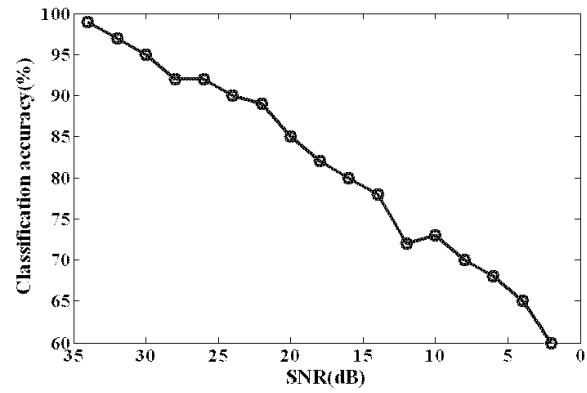


Figure 2. Classification accuracy versus signal-to-noise ratio (SNR) using permutation entropy

Fig. 3 shows the permutation entropy calculated in a range of embedding lags varying from $\tau = 1$ to $\tau = 30$ for healthy and epileptic subjects using clean and noisy EEG segments.

It can be seen that for clean data the permutation entropy is significantly smaller for epileptic EEG especially for small embedding lags but the permutation entropies calculated for highly noised EEG signals in an epileptic subject may exceed the entropies calculated for healthy subjects in some time lags.

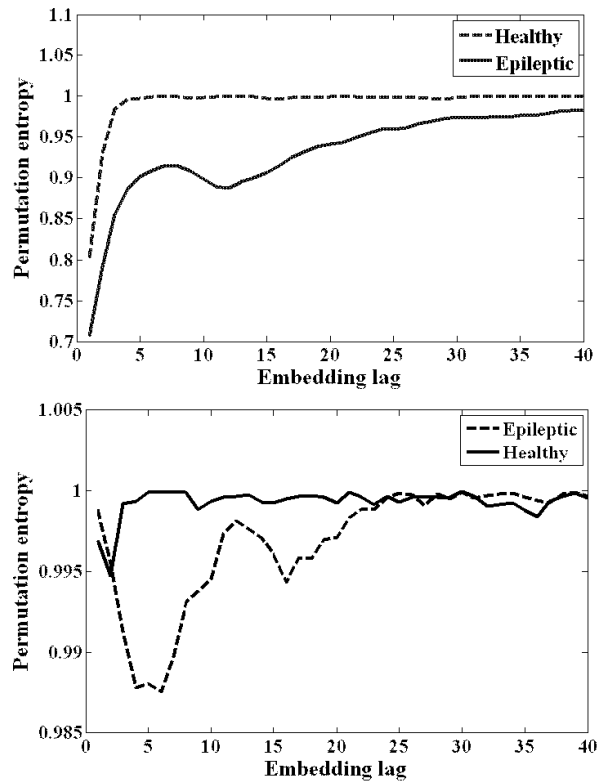


Figure 3. Variation of permutation entropy for different embedding lags for healthy and epileptic subjects using clean (top) and noisy EEG segments (bottom)

However the mean value of the entropies calculated for a range of embedding lags seems to be smaller for epileptic EEG signals.

Furthermore the permutation entropies estimated for an epileptic EEG segment shows more variations as the time delay increases. In line with these considerations, the average and the mean deviation of the permutation entropies over a range of embedding lags are used to distinguish between normal and epileptic subjects when the signals are buried in noise.

Fig. 4 shows the discrimination ability of these new measures. It can be seen that the classification accuracy for lower signal-to-noise ratios is significantly improved.

Fig. 5 shows that a multivariate discriminant analysis using all three parameters (permutation entropy, its mean and mean deviation over a range of embedding delays) has a better classification performance. The minimum classification accuracy reaches to 85% for SNR=2dB.

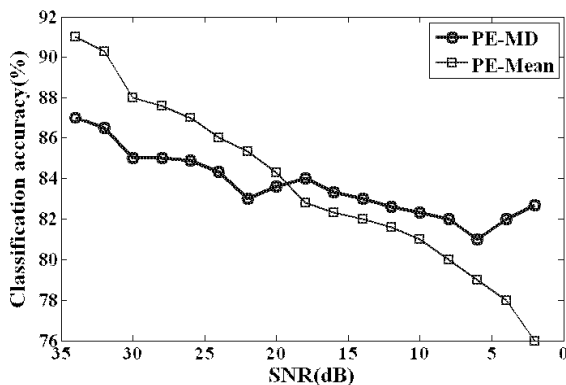


Figure 4. Classification accuracy versus SNR using the average (PE-Mean) and the mean deviation (PE-MD) of permutation entropies over a range of embedding lags

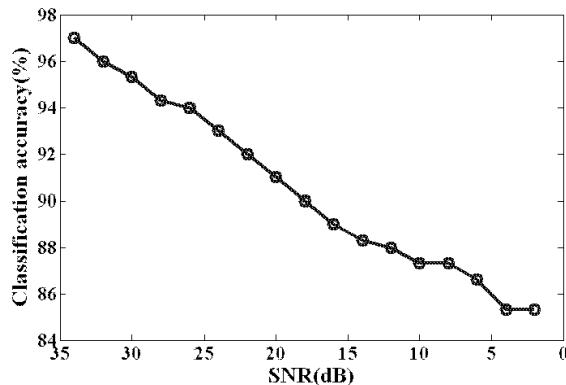


Figure 5. Classification accuracy versus SNR using a multivariate discriminant analysis combining all three proposed parameters

IV. CONCLUSION

Permutation entropy was used to characterize and discriminate normal and epileptic EEG signals. The results show that the entropy estimated for epileptic subject is

significantly less, compared to that of normal subject. This demonstrates the less complicated nature of Epileptic EEG signals. The discrimination performance of this measure in the presence of noise was also investigated using discriminant analysis. Although the discrimination power of PE is almost satisfactory in the presence of moderate noise, the ability of it to distinguish normal and epileptic subjects decreases for highly noised EEG segments.

The average and the mean deviation of the permutation entropies over a range of embedding lags were defined as two quantitative parameters to improve the classification accuracy. Results indicate that these new parameters are much more robust with respect to additive Gaussian noise. Best classification results could be reached by means of a multivariate discriminant analysis combining the permutation entropy with these proposed parameters.

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