SOME EQUIVALENT MULTIRESOLUTION CONDITIONS ON LOCALLY COMPACT ABELIAN GROUPS

R. RAISI TOUSI *, R.A. KAMYABI GOL

ABSTRACT. Conditions under which a function generates a multiresolution analysis are investigated. The definition of the spectral function of a shift invariant space is generalized from $\mathbb{R}^n$ to a locally compact abelian group and the union density and intersection triviality properties of a multiresolution analysis are characterized in terms of the spectral functions.

REFERENCES

1. A.A. Arefijamal, R.A. Kamyabi Gol, *A characterization of square integrable representations associatd with CWT*, Journal of Sciences of Islamic Republic of Iran, **18**, (2), (2007), 159–166.

2. L.W. Baggett, H.A. Medina, K.D. Merrill, *Generalized multi-resolution analyses and a construction procedure for all wavelet sets in* $\mathbb{R}^n$, J. Fourier Anal. Appl., **3**, (1999), 451–456.

3. J.J. Benedetto, S. Li, *The theory of multiresolution analysis frames and applications to filter banks*, Appl. Comput. Harmon. Anal. **5** (1998), 389–427.

4. C. de Boor, R.A. DeVore, A. Ron, *On the construction of Multivariate (pre)wavelets*, Constr. Approx. **9**, (1993), 123–166.

5. M. Bownik, *The structure of shift invariant subspaces of* $L^2(\mathbb{R}^n)$, J. Funct. Anal. **177** (2000), no (2), 282–309.

6. M. Bownik and G. Garrigos, *Biorthogonal wavelets, MRA's and shift invariant spaces*, Studia Math. **160** (2004), 231-248.

7. M. Bownik and Z. Rzeszotnik, *On the existence of multiresolution analysis for framelets*, Math. Ann. **332** (2005), 705–720.

8. M. Bownik, Z. Rzeszotnik, *The specral function of shift invariant spaces on general lattices*, Contemp. Math. 345 (2004), 49-59.

9. S. Dahlke, *Multiresolution analysis and wavelets on locally compact abelian groups*, Wavelets, Images, and Surface Fittings, (1994), 141–156.

10. I. Daubechies, *Ten Lectures on Wavelets*, Society for Industrial and Applied Mathematics, (SIAM), Philadelphia, 1992.

11. G. B. Folland, *A Course in Abstract Harmonic Analysis*, CRC Press 1995.

12. F. Galindo and J. Sanz, Multiresolution analysis and Radon measures on a locally compact abelian group, Czechoslovak Math. J. **51** (2001), 859–871.

13. D. Han, D. R. Larson, M. Papadakis, T. Stavropoulos, *Multiresolution analyses of abstract Hilbert spaces and wandering subspaces*, The functional and harmonic analysis of wavelets and frames , Contemp. Math., **247**, (1999)259–284.

14. E. Hernandez, G. Weiss, *A First Course on Wavelets*, CRC Press, Boca Raton, (1996).

15. E. Hewitt, K. A. Ross, *Abstract harmonic analysis*, Vol 1, Springer-Verlag, 1963.

16. R.A. Kamyabi Gol, R. Raisi Tousi, *A range function approach to shift-invariant spaces on locally compact abelian groups*, Int. J. Wavelets Multiresolut. Inf. Process., **8**, (2010), 49-59.

17. R.A. Kamyabi Gol, R. Raisi Tousi, *The structure of shift invariant spaces on a locally compact abelian group*, J. Math. Anal. Appl. **340**, (2008), 219–225.

18. E. Kaniuth, G. Kutyniok, Zeros of the Zak transform on locally compact abelian groups, Proc. Amer. Math. Soc. 126 (1998), no. 12, 3561–3569.

19. G. Kutyniok, *Time Frequency Analysis on Locally Compact Groups*, Ph.D. thesis, Padeborn University, (2000).

20. W.R. Madych, Some elementary properties of multiresolution analyses of $L^2(\mathbb{R}^n)$, in Wavelets: A tutorial in Theory and Applications (C.K. Chui, Ed.), Academic Press, 1992, 259–294.

21. S.G. Mallat, *Multiresolution approximations and wavelet orthonormal bases of $L_2(\mathbb{R})$*, Trans. Amer. Math. Soc., **315**, (1989), 69–87.

22. Z. Rzeszotnik, *Characterization theorems in the theory of wavelets*, ph.D. thesis, Washingtone University, (2000).

² Department of Pure Mathematics, Ferdowsi University of Mashhad, P. O. Box 1159, Mashhad 91775, Iran.

E-mail address: raisi@um.ac.ir

[1] Department of Pure Mathematics, Ferdowsi University of Mashhad, and Center of Excellence in Analysis on Algebraic Structures (CEAAS), P.O.Box 1159-91775, Mashhad, Iran.

E-mail address: kamyabi@um.ac.ir