Effects of Friction Factor and Inlet Stagnation Conditions on the Self-Condensation of Steam in a Supersonic Nozzle

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This paper describes a theoretical study of the self-condensation of non-equilibrium wet steam during supersonic expansion in a de Laval nozzle. Nucleation and droplet growth theories have been combined with the equations of one-dimensional gas dynamics and the system integrated numerically on a computer. The method of solution has been applied to predict the self-condensation zone in the supersonic region of a converging-diverging nozzle and it has been shown that the condensation of vapour in divergent part of the nozzle causes a pressure rise in flow. The results have been confirmed experimentally and, on the strength of this agreement, the treatment has been extended to predict the effects of friction factor, stagnation pressure and temperature on the location of the condensation shock. The work can be extended to predict the formation and growth of liquid droplets in steam turbines.

INTRODUCTION

The importance of supersaturation in the separation of moisture from a vapor undergoing expansion has been recognized since the early days of the steam turbine. Although the basic physical phenomena were imperfectly understood, the Wilson line and later the Wilson zone were formulated to describe the limiting conditions of the process.

The first analytical investigation of condensing flow in which the nucleation theory was combined with gas dynamics equations was carried out by Oswatitsch [1].

In the development of supersonic wind tunnels, the condensation of water vapor present in the air caused some problems and there have been a number of contributions to the study of phase change in this context, e.g. [2-4].

In parallel with the above investigations, the development of steam turbines was progressing with remarkable speed and problems associated with the presence of moisture were being experienced. An important contribution to the subject is contained in the work of Gyarmathy who places emphasis on the rate of expansion [5]. Gyarmathy and Meyer [6], Pouring [7], Campbell and Bakhtar [8], Wegener [9] and Bakhtar et al. [10-13] are among many researchers who have reported investigations in this field.

With the advent of nuclear reactors, studies of nucleation and spontaneous condensation at high-pressure steam were reported by Gyarmathy et al. [14] and Ryley and Tubman [15]. Bakhtar et al. [16,17] have published some investigations in connection with limiting supersaturation in high pressure steam. Moisture separation in turbines of nuclear power stations has also been studied by a number of investigators, including Filippov and Povarov [18].

Since 1980, theoretical and experimental investigations have been extended to two- and three-dimensional fields. Nevertheless, because of the complexity of these flows and the methods of treatment, one-dimensional treatment is still a useful tool for the study of nucleation because of its simplicity.

The treatment described in this paper is one that was developed at Birmingham University [8]. Because of the inclusion of higher Viral terms in the equation of state for the vapor and other refinements, it is applicable to one-dimensional condensing flows of steam over a wide range of conditions.

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Thermodynamic and kinetic aspects of phase change have been studied in the literature in other connections, however, before considering the main equations of flow, basic results relevant to the present problem will be briefly summarized.

THERMODYNAMIC ASPECTS

The state of vapor is specified by its pressure P and temperature T_G. To take a concrete example say that $P = 1$ bar and $T_G = 75°C$. The saturation temperature corresponding to P is denoted by $T_s(P)$, which, in the present case, will be 99.6°C and the saturation pressure, corresponding to T_G as $P_s(T_G)$, which will be 0.3855 bar.

The comparison of either P with $P_s(T_G)$ or $T_s(P)$ with T_G indicates that the vapor is not in an equilibrium state. Its condition is said to be supercooled or supersaturated.

The departure from equilibrium can be quantified either by the ratio of $P/P_s(T_G) = 2.594$, which is called the supersaturation ratio, denoted by S as:

$$S = \frac{P}{P_s(T_G)}, \tag{1}$$

or $T_s(P) - T_G = 24.6°C$, which gives the degrees of supercooling and which is defined as:

$$\Delta T = T_s(P) - T_G. \tag{2}$$

When vapor is cooled to its nominal saturation point, the answer to the question of whether phase transition will or will not immediately occur is dependent upon whether or not nucleation of the new phase can take place. In the absence of all foreign surfaces or particles (homogeneous case), phase transition does not occur until some finite supersaturation or degree of supercooling has developed.

For a mass of fluid to condense from vapor at P and temperature T_G to a droplet of radius r and density ρ_L, there is an increase in Gibbs free energy of:

$$\Delta G = 4\pi r^2 \sigma_r - \frac{4}{3}\pi r^3 \rho_L (\Delta G_1), \tag{3}$$

where σ_r is the surface tension in the case of a liquid droplet [19]. The influence of curvature on the surface tension of a droplet has been the subject of much controversy. In the early work of Benson & Shuttleworth [20], it was predicted that the surface tension of small droplets was lower than the surface tension of a flat surface. Benson & Shuttleworth adopted a molecular interaction method to give a relation between the surface tension of a small droplet, δ_r, and that of a flat surface, δ_∞, and suggested that $\delta_r < \delta_\infty$. Further work by Plummer and Hale [21], suggested that the surface tension of small clusters is

close to the bulk value, δ_∞. In this work, the variations of δ with radius has been neglected and the surface tension of a small cluster is taken to be that of a flat surface. This gives a good all-round agreement with experimental results.

The term ΔG_1 in Equation 3 is affected by the form of the equation of state. By using a truncated Viral series, it becomes :

$$\Delta G_1 = RT_G\{\ln \frac{\rho}{\rho_s(T_G)} + 2B_1[\rho - \rho_s(T_G)]$$
$$+ \frac{3}{2}B_2[\rho^2 - \rho_s^2(T_G)] + \frac{4}{3}B_3[\rho^3 - \rho_s^3(T_G)] + \cdots\}, \tag{4}$$

$\rho_s(T_G)$ is the density of saturated vapor at T_G. With reference to Figure 1, for any supersaturation ratio, $S > 1, \Delta G$ passes through a maximum:

$$\Delta G^* = \frac{16\pi\sigma_r^3}{3\rho_L^2(\Delta G_1)^2}. \tag{5}$$

The maximum ΔG occurs at some embryo radius r^*, defined to be the critical radius:

$$r^* = \frac{2\sigma_r}{\rho_L\Delta G_1}. \tag{6}$$

This equation is known as the Kelvin-Helmholtz equation and for a given supersaturation ratio gives the size of a critical droplet, which will be in a meta-stable equilibrium.

It is pointed out that only droplets having a radius denoted by r^* and free energy change denoted by ΔG^* will be in meta-stable equilibrium. Droplets larger than this will act as centers of condensation, since their growth is accompanied by a reduction in free energy. The reverse is true for droplets of a smaller size, which can achieve a decrease in free energy by losing molecules and have an inherent tendency to evaporate.

A thermodynamic consideration of Equation 3 and Figure 1 poses this dilemma.

Although supersaturated vapor is in a meta-stable state, because its specific Gibbs free energy is more
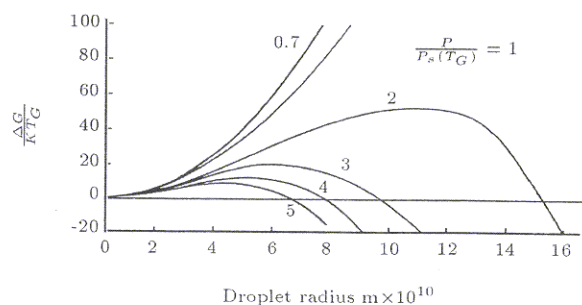

Figure 1. Variations of the change in Gibbs free energy with radius of droplet.

than that of liquid at the same temperature, it cannot jump to the preferred lower free-energy state. That jump can only proceed via formation of tiny drops beginning with clusters of two, three or four molecules, etc. The growth of such embryos of the new phase carries the system up to the state where the total free-energy of the system has, according to Equation 3, risen more by creation of the new surface-free-energy of the embryonic drops than it has fallen by virtue of the bulk-free energy change accompanying the phase transition that produced those drops. That is, phase transition is always, in the homogeneous case, blocked by an activation energy barrier.

The way out of the dilemma is a fortuitous formation of clusters due to statistical fluctuations within the body of the vapor. Inspection of Equations 5 and 6 and Figure 1 shows that ΔG^* decreases with increases in supersaturation. Thus, increases in supersaturation lower the activation barrier to the formation of critical droplets.

Theory and observation both suggest that a sufficient increase of S, to around 5 or 6, finally serves to raise the molecular bombardment rate and to reduce r^* and the attendant height of the activation barrier ΔG^* to such an extent that fluctuations can send some favored embryos over the top of the free energy barrier. In other words, at this condition, the probability of some subcritical embryo fortuitously growing to supercritical size in a short time approaches unity. At that S value, homogeneous nucleation becomes an effective process and phase transition can finally begin.

KINETIC ASPECTS

The controlling factor in the condensation of liquid from a supercooled vapor is the rate at which critical droplets can be formed.

In the present investigation, the nucleation theory adopted is the classical result, subject to the refinements made by Courtney [22] and Kantrowitz [23].

The kinetic aspects of homogeneous nucleation and the rate of formation of critical clusters within the body of a supercooled vapor were originally studied by Zeldovich [24] and Frenkel [25].

A full derivation of the equation for the rate of formation of critical embryos, with a lucid discussion of the procedure, is given by McDonald [19]. The classical expression for the rate of formation of critical droplets per unit volume per second, is:

$$J = q \sqrt{\frac{2\sigma_r}{\pi m^3}} \cdot \frac{\rho_G^2}{\rho_L} \cdot \exp\left\{-\frac{\Delta G^*}{kT_G}\right\}, \qquad (7)$$

where m is the mass of a water molecule, k is the Boltzman constant and q is the condensation coefficient representing the fraction of molecules striking a cluster and which condense on it.

The condensation coefficient for water has been measured and discussed by a number of investigators, including Alty, Machay and Jamieson [8,26]. Jamieson obtained a value of 0.04 in the instance of steam condensing on a flat liquid surface at rest. This is so little different from the direct measurements of Alty and Mackay that it appears reasonable to accept their value of 0.036 for the condensation coefficient.

With the corrections due to Courtney and Kantrowitz included, the expression for the steady state nucleation rate, J, used in the calculations, becomes:

$$J = \frac{q}{1+\nu} \sqrt{\frac{2\sigma_r}{\pi m^3}} \cdot \frac{\rho_G \rho_s(T_G)}{\rho_L} \cdot \exp\left\{-\frac{\Delta G^*}{kT_G}\right\}, \qquad (8)$$

where:

$$\nu = \frac{q\rho_G}{\alpha_r}\left(\frac{RT_G}{2\pi}\right)^{\frac{1}{2}}\left(\frac{L^2}{RT_G^2} - \frac{L}{2T_G}\right), \qquad (9)$$

α_r, is the heat transfer coefficient and is given later by Equation 13.

CONDENSATION OF STEAM DURING SUPERSONIC EXPANSION IN A LAVAL NOZZLE

If the enthalpy drop from the inlet stagnation, required to precipitate condensation, is greater than the critical enthalpy drop giving the sonic flow, the nucleation zone occurs in the supersonic region of the nozzle.

A typical condensing flow of steam in a convergent/divergent nozzle is shown schematically in Figure 2. The path of the expansion is shown on the Mollier diagram (Figure 3). Steam expands from, initially, a dry-superheated stagnation state (point 1), to a sonic condition at the throat (point 2). At point 3,
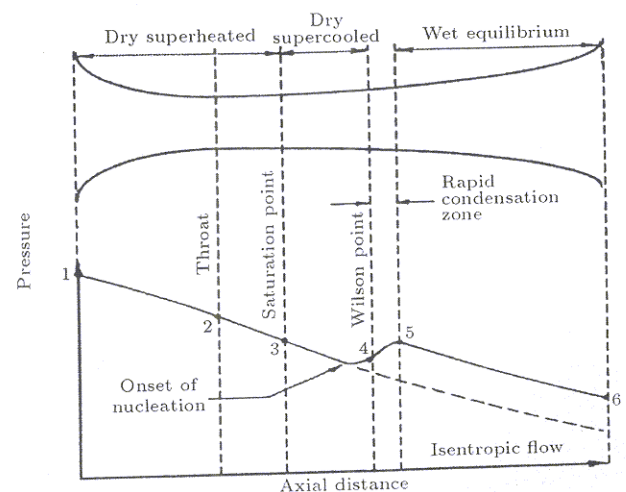

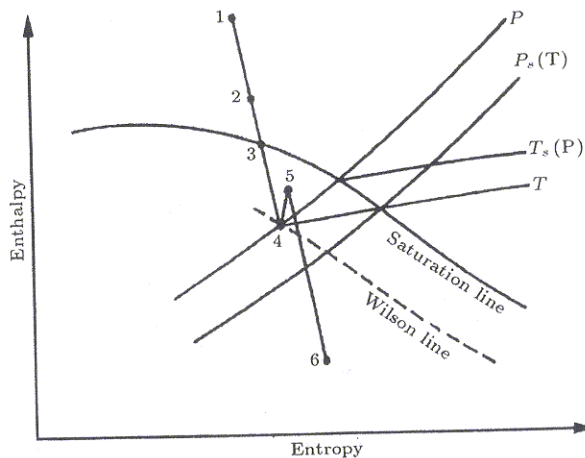**Figure 2.** Axial pressure distribution in a nozzle with spontaneous condensation.

Figure 3. State line for expanding steam with spontaneous condensation.

the saturation line is crossed and droplet embryos begin to form.

The nucleation rates associated with these early embryos are so low that the steam continues to expand as a dry single-phase vapor in a meta-stable supercooled or supersaturated state (when steam is expanded isentropically, its temperature drops faster than the saturation temperature corresponding to its pressure).

Depending on the local conditions and rate of expansion, the nucleation rate increases dramatically and reaches its maximum at point 4, where breakdown of supersaturation occurs.

The region just upstream of point 4 is termed the nucleation zone and is terminated by the Wilson point at 4, which is the point of maximum supercooling.

The nuclei grow rapidly between points 4 and 5 by exchanging heat and mass with the surrounding vapor and restore the system to thermodynamic equilibrium. The conduction of latent heat, which is released at the droplet surfaces to parent vapor, gives rise to a gradual increase in pressure. This phenomenon, between points 4 to 5, is referred to as condensation shock by some authors and decelerates the already supersonic flow.

The term condensation shock is, however, misleading because the changes in flow properties between these points are continuous.

Since heat transfer takes place through a finite temperature difference between phases, the process is essentially irreversible and has been associated with a net rise in entropy from points 4 to 5.

This is referred to as "Thermodynamic Nucleation Loss".

Because of supercooling forced on the vapor by the curvature of the droplets, a small amount of supersaturation persists during the expansion of points 5 to 6 of the nozzle exit. Further expansion

of the flow between these points takes place close to equilibrium.

DROPLET GROWTH THEORY

The supercritical droplets will grow and exchange mass, momentum and heat with the surrounding vapor. These exchange rates depend on the relative magnitude of the droplet and the mean free path. A Knudsen number is defined as the ratio of the mean free path of the vapor molecules, $\bar{l}$ to the droplet diameter, $2r$, i.e.:

$$K_n = \frac{\bar{l}}{2r}. \tag{10}$$

From kinetic theory of gases:

$$\bar{l} = \frac{1.86\mu}{\rho_G \sqrt{RT_G}}, \tag{11}$$

where μ denotes the viscosity of the vapor. The exchange processes are arbitrarily divided into four regimes:

$$
\begin{aligned}
&& K_n &< 0.01 && \text{continuum,} \\
0.01 &< & K_n &< 0.18 && \text{slip,} \\
0.18 &< & K_n &< 4.60 && \text{transition,} \\
4.50 &< & K_n &&& \text{free molecular.}
\end{aligned}
$$

Freshly created droplets have a Knudsen number much greater than unity and, for large droplets, the Knudsen number is very much less than unity. Thus, the nucleated droplets pass through all regimes mentioned above during the expansion process.

In this work, the slip between the vapor phase and the droplets has been neglected but, for calculation of droplet growth rates, heat and mass transfer rates must be simultaneously considered.

An energy balance is established between liquid droplets and vapor.

The latent heat of condensation is released to the liquid when a vapor molecule condenses onto a liquid droplet. Part of this heat raises the temperature of the droplet and the remainder is transferred to the vapor. The energy balance gives:

$$L\frac{dm_r}{dt} = 4\pi r^2 \alpha_r (T_L - T_G) + m_r c_L \frac{dT_L}{dt}, \tag{12}$$

where c_L is the specific heat of the liquid and α_r is the heat transfer coefficient between the droplet surface and the vapor. The last term is small compared with the others and can be neglected.

The heat transfer coefficient in various flow regimes for a sphere relative to a surrounding medium, as a function of K_n, has been studied amongst others by Schaff and Chamber [27].

Gyarmathy [5] showed that the results can be fitted by the following single expression over the whole range of Knudsen numbers:

$$\alpha_r = \frac{\lambda}{r(1 + 3.18K_n)}.$$ (13)

The droplet mass can be expressed as:

$$m_r = \frac{4}{3}\pi r^3 \rho_L,$$ (14)

and:

$$\frac{dm_r}{dt} = 4\pi r^2 \rho_L \frac{dr}{dt}.$$ (15)

Substituting Equations 13 and 15 into Equation 12 and neglecting the thermal inertia of the droplet yields:

$$\frac{dr}{dt} = \frac{\lambda}{(r + 1.59\bar{l})\rho_L} \frac{T_L - T_G}{L}.$$ (16)

In the free molecular range, the rate of change of mass can be calculated as the difference between the rate at which droplets capture molecules and the rate at which they lose them. On the other hand, in a continuum regime, the growth of a droplet can be calculated from the diffusion of the vapor to its surface.

Bakhtar and Zidi's general expression for the net mass transfer rate valid in all the regimes is [28] :

$$\frac{dr}{dt} = \left(\frac{1}{1 + 0.375qS_n/K_n}\right)\frac{q}{\rho_L}\left(\frac{R}{2\pi}\right)^{\frac{1}{2}}$$
$$\times \left(\rho_G T_G^{\frac{1}{2}} - \rho_s(T_L, r)T_L^{\frac{1}{2}}\right),$$ (17)

where S_c is the Schmidt number [$S_c = \mu/(\rho_G D)$] and D is the self-diffusion coefficient developed by Bakhtar and Zidi.

By eliminating dr/dt between Equations 16 and 17, a relationship between the droplet radius and its temperature is established. The equation obtained is numerically differentiated at each stage to obtain dT_L/dr. The rate of change of the droplet temperature is, then, derived from the relation:

$$\frac{dT_L}{dt} = \frac{dT_L}{dr}\frac{dr}{dt}.$$ (18)

This equation is integrated using either Equations 16 or 17 to obtain the variations of droplet radius and temperature with time.

FUNDAMENTAL EQUATIONS

General Outline

Before presenting into mathematical details, the main features of the problem will be considered briefly.

A condensing flow of steam can be regarded as the special case of a compressible fluid in which heat addition is the result of latent heat being released into the flow by condensation, which also affects the mass flow rate of vapor phase. The mixture, as a whole, must obey the conservation laws of mass, momentum and energy, which can also be expressed as mathematical equations. The set of equations, thus obtained, describes the system completely and is capable of solution by numerical methods. It is notable that at each given instant, a test must be carried out concerning the state of equilibrium within the system. If the vapor temperature is below that necessary for thermodynamic equilibrium, the vapor will condense, which it can do by the growth of any existing droplets, as well as the formation of new ones.

Furthermore, in problems of wet steam flows, generally, the combination of the prevailing dryness fraction and the much smaller specific volume of the liquid, in comparison with that of the vapor, is such that the fraction of the mixture volume occupied by the liquid phase can be regarded as small. Under these circumstances, the mixture can be taken as the summation of a large number of liquid droplets of specified size, which are distributed uniformly in the vapor of given pressure and temperature.

The governing equations may be derived as follows.

Equation of State for Steam

Consistent properties of steam can be calculated by assuming an equation of state, an equation for the saturation line and one for the specific heat at zero pressure.

A number of very accurate equations of state have been developed in the literature for steam, but they all refer to the equilibrium states. The properties of supercooled vapor have to be calculated by extrapolation from these equations into the meta-stable regions.

The equation of state used is the Viral equation developed by Vukalovich [29], which, as shown in [30], can be extrapolated to the meta-stable state satisfactorily:

$$P = \rho_G R T_G (1 + B_1\rho_G + B_2\rho_G^2 + B_3\rho_G^3).$$ (19)

Vukalovich used the association theory of gases as a basis for obtaining expressions for the Viral coefficients. In this way, he included the effect of clusters of up to five molecules. The higher terms correspond with the interactions between the larger clusters of molecules. This form of equation is very convenient in cases where the equation has to be manipulated. By differentiating, the equation of state gives:

$$\frac{dP}{P} - X\frac{d\rho_G}{\rho_G} - Y\frac{dT_G}{T_G} = 0,$$ (20)

where:

$$X = \frac{\rho_G}{P}\left(\frac{\partial P}{\partial \rho_G}\right)_{T_G}$$

$$= \frac{1 + 2B_1\rho_G + 3B_2\rho_G^2 + 4B_3\rho_G^3}{1 + B_1\rho_G + B_2\rho_G^2 + B_3\rho_G^3}, \qquad (21)$$

$$Y = \frac{T_G}{P}\left(\frac{\partial P}{\partial T_G}\right)_{\rho_G}$$

$$= 1 + \frac{\rho_G T_G}{1 + B_1\rho_G + B_2\rho_G^2 + B_3\rho_G^3}$$

$$\times \left(\frac{dB_1}{dT_G} + \rho_G\frac{dB_2}{dT_G} + \rho_G^2\frac{dB_3}{dT_G}\right). \qquad (22)$$

Considering the problem as one-dimensional, Figure 4 shows the flow over an incremental distance, dx, along the duct axis, in which the pressure, temperature, density, flow area, velocity and mass flow rate, change from P, T, ρ, A, U, W to $P+dP, T+dT, \rho+d\rho, A+dA, U+dU$ and $W+dW$, respectively. Using suffixes G and L to denote the vapor and liquid phases, respectively, the fundamental equations of motion for the steady state flow may be derived as follows.

Mass Continuity

The equation of mass continuity is expressed as:

$$W = W_L + \rho_G A U_G = \text{Constant}, \qquad (23)$$

where the total mass flow rate, W, is the sum of the liquid mass flow rate, W_L, and vapor mass flow rate, $\rho_G A U_G$.

It is assumed that the volume occupied by the liquid phase is negligible in comparison with the volume
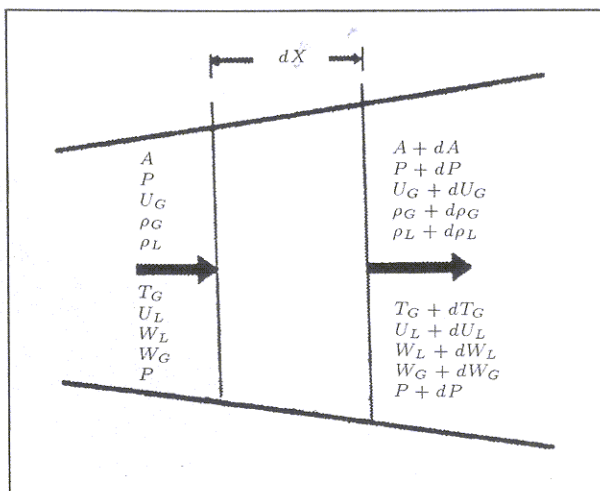

Figure 4. Flow element.

of the vapor. Thus, in differential form:

$$\frac{d\rho_G}{\rho_G} + \frac{dA}{A} + \frac{dU_G}{U_G} + \frac{dW_L}{W - W_L} = 0. \qquad (24)$$

Momentum Equation

The change in momentum of the system across the element is equal to the sum of the pressure and frictional forces and can be written as:

$$d(W_G U_G + W_L U_L) = -AdP - \frac{fA\rho_G U_G^2 dx}{2D_e}, \qquad (25)$$

f denotes the friction factor and D_e the duct equivalent diameter.

Expanding this equation, regarding $(U_G - U_L)dW_L$ as small, because there is little or no slip between the phases, $(U_G \approx U_L = U)$, results in the following:

$$\frac{dP}{P} = -\frac{f\rho_G U_G^2}{2PD_e}dx - \frac{(W - W_L)U_G}{AP}\frac{dU_G}{U_G}$$

$$- \frac{W_L U_L}{AP}\frac{dU_L}{U_L}. \qquad (26)$$

Energy Equation

Assuming no interphase slip, which is a justifiable assumption, as the droplets nucleated in wet steam are of small diameter, the steady state energy equation for adiabatic flow through the control volume may be written as:

$$d\left[(W - W_L)\left(h_G + \frac{U^2}{2}\right) + W_L\left(h_L + \frac{U^2}{2}\right)\right] = 0. \qquad (27)$$

Writing $(h_G - h_L)$ as L and dividing throughout by Wc_PT_G results in:

$$\frac{dh_G}{c_PT_G} - \frac{d(W_L L)}{Wc_PT_G} + \frac{U^2}{c_PT_G}\frac{dU}{U} = 0. \qquad (28)$$

The change of enthalpy of the vapor phase can be expressed by:

$$dh_G = c_PdT_G + \left[V_G - T_G\left(\frac{\partial V_G}{\partial T_G}\right)_P\right]dP. \qquad (29)$$

With the equation of state adopted, the following may be obtained:

$$\left[V_G - T_G\left(\frac{\partial V_G}{\partial T_G}\right)_P\right] = \frac{1}{\rho_G}\left(1 - \frac{Y}{X}\right). \qquad (30)$$

Substituting the value of dh_G, together with Equation 30 in Equation 28, after rearrangement and ignoring the result from $(W_L/W)dL$, the final form of the energy equation will result as:

$$\frac{dT_G}{T_G} + \frac{P}{\rho_G c_P T_G}\left(1 - \frac{Y}{X}\right)\frac{dP}{P} + \frac{U^2}{c_P T_G}\frac{dU}{U}$$

$$- \frac{L}{c_P T_G}\frac{dW_L}{W} = 0. \qquad (31)$$

Vapor Phase Mach Number

For the development of equations of flow, it is algebraically convenient to work in terms of a Mach number, which is defined in terms of the frozen speed of sound but without implying physical significance:

$$Z = M^2 = \frac{U_G^2 \rho_G}{\gamma P}. \qquad (32)$$

Upon differentiation this yields:

$$\frac{dZ}{Z} = 2\frac{dU_G}{U_G} + \frac{d\rho_G}{\rho_G} - \frac{dP}{P}. \qquad (33)$$

The change in the isentropic exponent, γ, is imperceptible and has been neglected.

REARRANGEMENT OF THE GOVERNING EQUATIONS

Taking $dA/A, f.dx/2D_e, dW_L/(W - W_L), dW_L/W$ and $dL/c_P T_G$ as independent variables and assuming no phase slip, the four Equations 24, 26, 31, 32 and the Equation of state 20 are sufficient to solve the five unknowns, $dP/P, dU/U, d\rho_G/\rho_G, dT_G/T_G$ and dZ/Z.

By introducing a dryness fraction:

$$\xi = \frac{W - W_L}{W}, \qquad (34)$$

the five equations may be combined, eliminating dU/U and dP/P in the process, to give the following three flow equations:

$$\frac{dZ}{Z} = \frac{1}{1 - (Z/\xi)[1 + (1 - \xi)(\gamma - X)/Y]}$$

$$\times \left\{ \left[\left(\frac{\gamma - X}{Y}\right)(\gamma Z + 1) + \gamma + 1\right]\frac{f.Z.dx}{2D_e} \right.$$

$$+ \left(1 + \frac{\gamma Z}{\xi}\right)\frac{Y}{X}\frac{L}{c_P T_G}\frac{dW_L}{W}$$

$$\left. - \left\{2 + \frac{Z}{\xi}\left[(\gamma - 1) - (1 - \xi)\left(\frac{\gamma - X}{Y}\right)\right]\right\} \right.$$

$$\times \left. \left[\frac{dA}{A} + \frac{dW_L}{W - W_L}\right]\right\}, \qquad (35)$$

$$\frac{dT_G}{T_G} = \frac{1}{1 - (Z/\xi)[1 + (1 - \xi)(\gamma - X)/Y]}$$

$$\times \left\{ \left(\frac{\gamma - X}{Y^2}\right)(X - Y - \gamma Z)\frac{f.Z.dx}{2D_e} \right.$$

$$+ \left(1 - \frac{\gamma Z}{\xi X}\right)\frac{L}{c_P T_G}\frac{dW_L}{W}$$

$$+ Z\left(\frac{\gamma - X}{Y^2}\right)\left(X + \frac{Y - X}{\xi}\right)$$

$$\left. \times \left[\frac{dA}{A} + \frac{dW_L}{W - W_L}\right]\right\}, \qquad (36)$$

$$\frac{d\rho_G}{\rho_G} = \frac{1}{1 - (Z/\xi)[1 + (1 - \xi)(\gamma - X)/Y]}$$

$$\times \left\{ -\frac{Z}{Y}(\gamma - X + Y)\frac{f.dx}{2D_e} \right.$$

$$- \frac{Y}{X}\frac{L}{c_P T_G}\frac{dW_L}{W} + \frac{Z}{\xi}\left[1 + (1 - \xi)(\frac{\gamma - X}{Y})\right]$$

$$\left. \times \left[\frac{dA}{A} + \frac{dW_L}{W - W_L}\right]\right\}. \qquad (37)$$

The above equations have to be integrated numerically and, for this purpose, the terms dA/A and $f.dx/2D_e$ have to be calculated from the shape and aerodynamic behavior of the flow through the duct. Also, the terms dW_L and W_L have to be regarded as variable coefficients and re-evaluated at each stage of the process. This is because equations describing droplet formation and growth are stiff and have to be evaluated over smaller incremental steps than those used for the integration of main equations.

ANALYSIS PROCEDURE

As already described, if the enthalpy drop from the inlet stagnation condition, required to precipitate condensation, is greater that the critical enthalpy drop giving sonic flow, the nucleation zone occurs in the supersonic region of the nozzle. This implies that the water content of the steam is negligible in the subsonic region and the wetness terms in the flow Equations 35 to 37 may be ignored when dealing with the converging section.

Furthermore, if the heat released by condensation is insufficient to choke the flow (subcritical heat addition), it will remain steady, continuous and supersonic. A large number of tests have been conducted on the wet steam test-rig at the University of Birmingham. These tests fall into the above categories and the

present work is based on these conditions. In the calculation procedure in the converging section, the mass flow rate is specified by the choking condition at the throat. Frictional effects have been included and tend to lower the mass flow rate and throat pressure marginally. Thus, the value of the mass flow rate is estimated and the dry flow equations integrated by the standard Runge Kutta technique, until the sonic condition is reached. The mass flow rate can then be adjusted until the sonic point corresponds with the actual throat.

The complete iteration procedure takes only a few seconds on a PC computer. With the mass flow rate known, the complete flow equations can be integrated step by step in divergent sections, a test being carried out at each point in this section to determine whether the local pressure and supersaturation ratio induce appreciable nucleation. If so, a complete droplet formation and growth sequence is employed to predict condensation shock and the following near equilibrium.

Within each elemental step, the behavior of two series of droplets is of interest. The first set is formed within an element and, depending on the exact location of their birth, the droplets will have grown by varying amounts on leaving it. The second group is formed upstream of the element and simply grows on traversing it.

Assuming the variations in the flow pressure and temperature over the element to be small, the rate of formation of new particles and their critical radius can be calculated from Equations 8 and 6. Thus, knowing the initial values of droplet radius and temperature for both the new and existing particles and using the calculation procedure already described, one may next calculate their subsequent variations with time. On the basis of the above explanation, in the nucleation zone, where nucleation rate and vapor properties vary rapidly, it may be necessary to divide the intervals into a number of subdivisions. With respect to the present computers, in this investigation, up to 25 sub-divisions were used. With these calculations completed, the results are expressed as:

a) Values of dW_L and W_L for introduction into the main flow equations.

b) Weighted average values of droplet parameters at exit from the element for subsequent use as entry conditions to the succeeding element.

SUMMARY OF RESULTS

The theoretical calculations have been carried out for a curved throat convergent-divergent nozzle, shown in Figure 5.

The uncertainty in the nucleation theory, the surface tension and the condensation coefficient mean
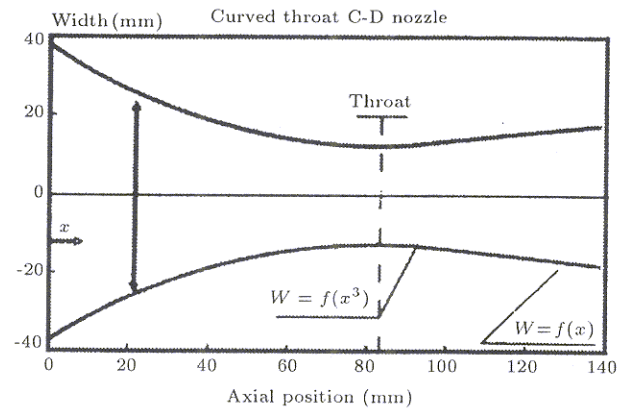

Figure 5. Shape of duct.

that many different combinations can be used to bring the calculations into close agreement with the experimental results. During the current investigation, it was found that the best all round agreement was obtained by using the classical nucleation theory, subject to refinements by Courtney and by Kantrowitz, with a condensation coefficient of 0.036 and by taking the surface tension of small droplets to be that of bulk water.

The experimental results presented consist of axial pressure distributions for flows with various stagnation pressures and temperatures at the inlet [31]. Comparisons with theoretical solutions are given in Figure 6. It is clear that the agreement between theoretical treatment and experimental observations is good.

The variations in the rate of expansion and other important characteristics of the flow with distance along the duct are shown in Figures 7 and 8 ($f = 0.018$, $P_0 = 147$ KPa, $T_0 = 399.2$ K and $q = 0.036$).

The pressure rise in the divergent part of the nozzle is caused by steam condensation. The latent heat of the phase change given up, initially, to the liquid, can be transferred to the vapor and decelerated from supersonic to nearly sonic flow. In this case, the velocity and Mach number decrease and pressure increases suddenly.

It can be seen that the maximum supercooling occurs just before condensation. The latent heat released reduces the rate of increase of supercooling and the subsequent value of the nucleation current.

It should be understood that nucleation occurs immediately on crossing the saturation line, but, at low values of the supersaturation ratio, the quantity of heat released is minute and has no effect on the expansion curve. It is only in a very small zone immediately prior to the shock that the heat produced affects the pressure distribution. This is caused almost entirely by growth of existing droplets rather than by the heat released in their formation. The function of nucleation is to produce a large surface area, by virtue of the large value
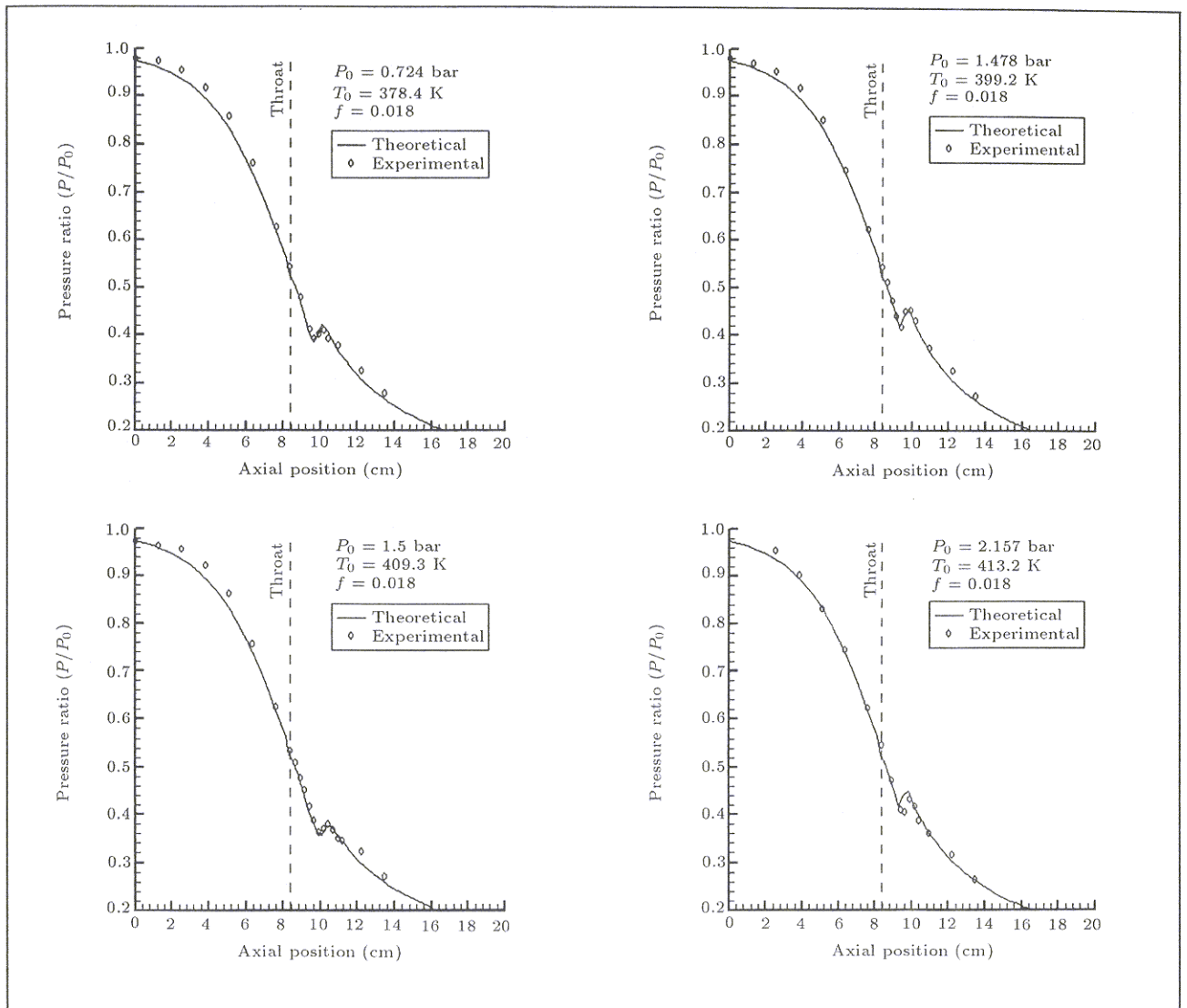
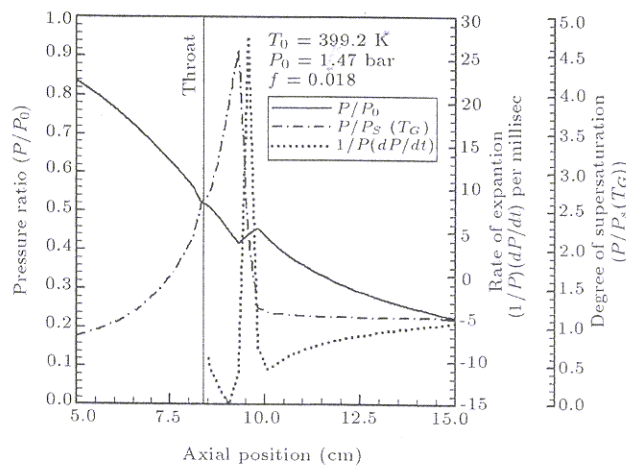**Figure 6.** Comparison between theoretical curves and experimental results.

Figure 7. Variation of pressure, rate of expansion and degree of supersaturation along the duct axis.
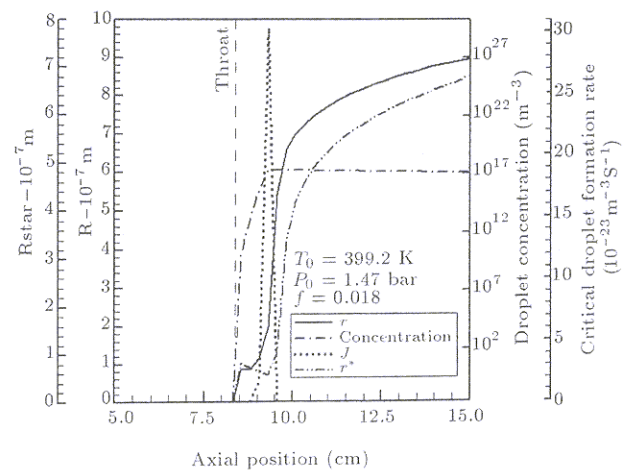

Figure 8. Variation of droplet radius, critical radius, concentration and rate of nucleation along the duct axis.
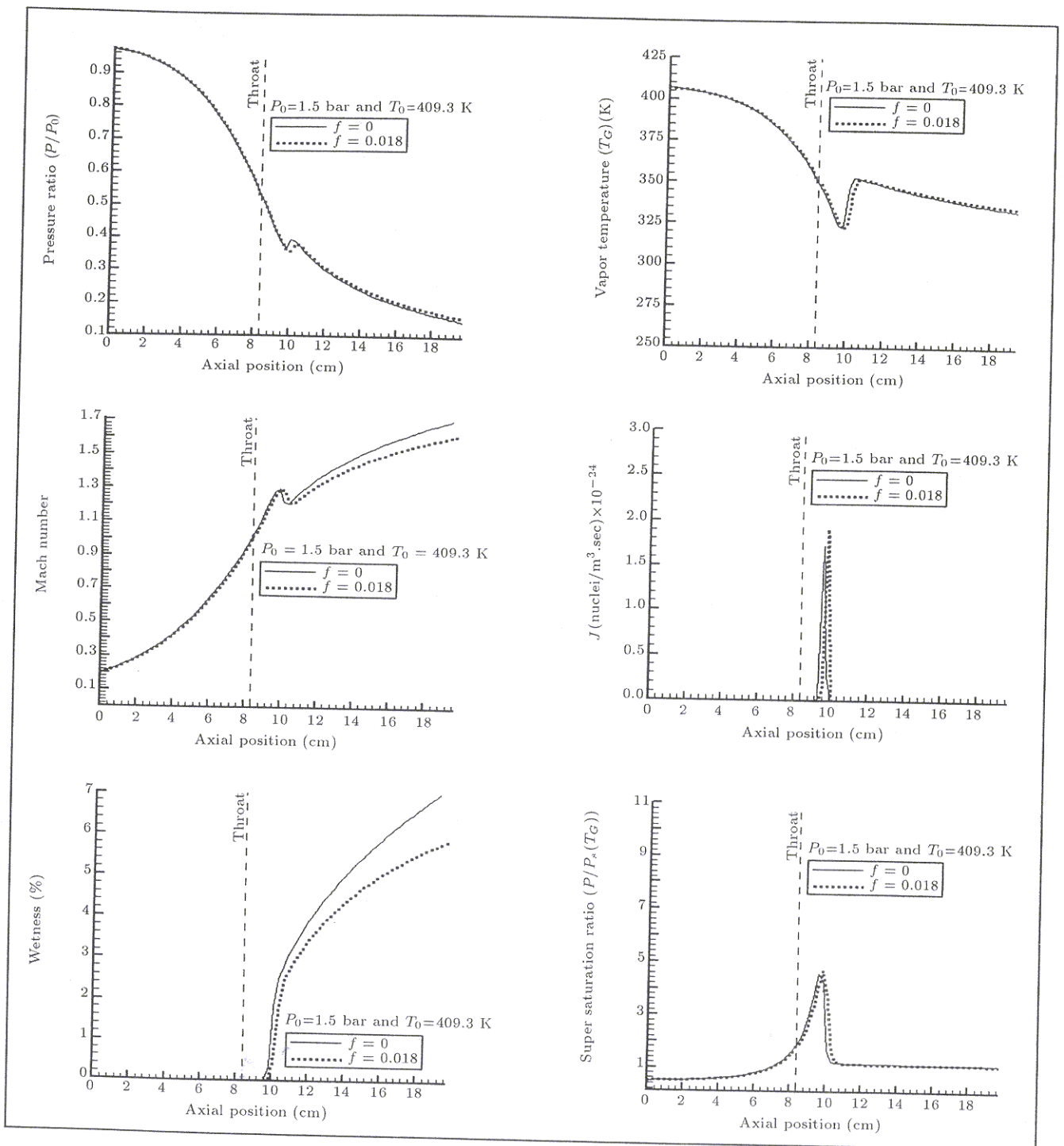
Figure 9. Effect of friction factor on the important axial characteristics of the flow.

of the nucleation current, J, but the actual mass of water is very small.

Increase of wetness or weighted average radius of droplets in divergent parts of the nozzle is because of nucleation and growth processes. In the nucleation zone, the number of droplets and their radius increase, however, in wet zone concentration, they will be constant and only the size of droplets

increases. The position of the condensation shock can be influenced by friction factor, inlet stagnation pressure and temperature.

The influence of friction factor on the results can be seen, with reference to distributions of important characteristics of flow along the duct axis, shown in Figure 9, and the effect of changes in inlet P_0 and T_0, which can be seen in Figures 10 and 11.
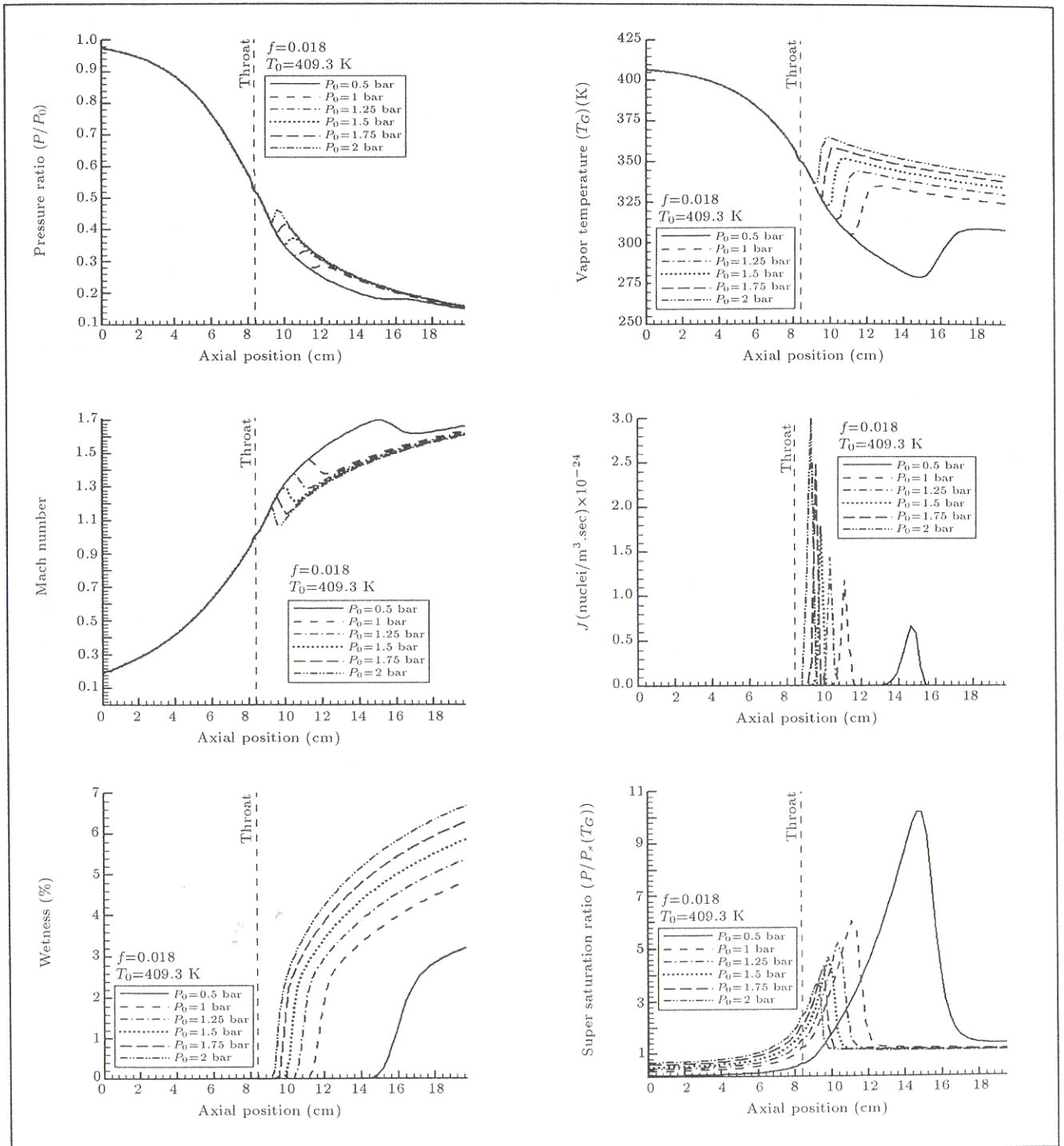
Figure 10. Effect of changes in inlet stagnation pressure on the important axial characteristics of the flow.

CONCLUDING REMARKS

In this paper, it has been shown that there are some differences between dry and condensing steam flow. It has also been illustrated that the theoretical prediction of the reversion point or the location of the condensation shock for supersaturated steam at high

speed flow is sensitive to friction factor, inlet stagnation pressure and stagnation temperature.

Because of the reheat caused by friction, this factor has a great influence on the condensation process. A lower friction factor reduces the amount of frictional reheat and, therefore, the limiting supersaturation conditions are reached earlier during steam expansion
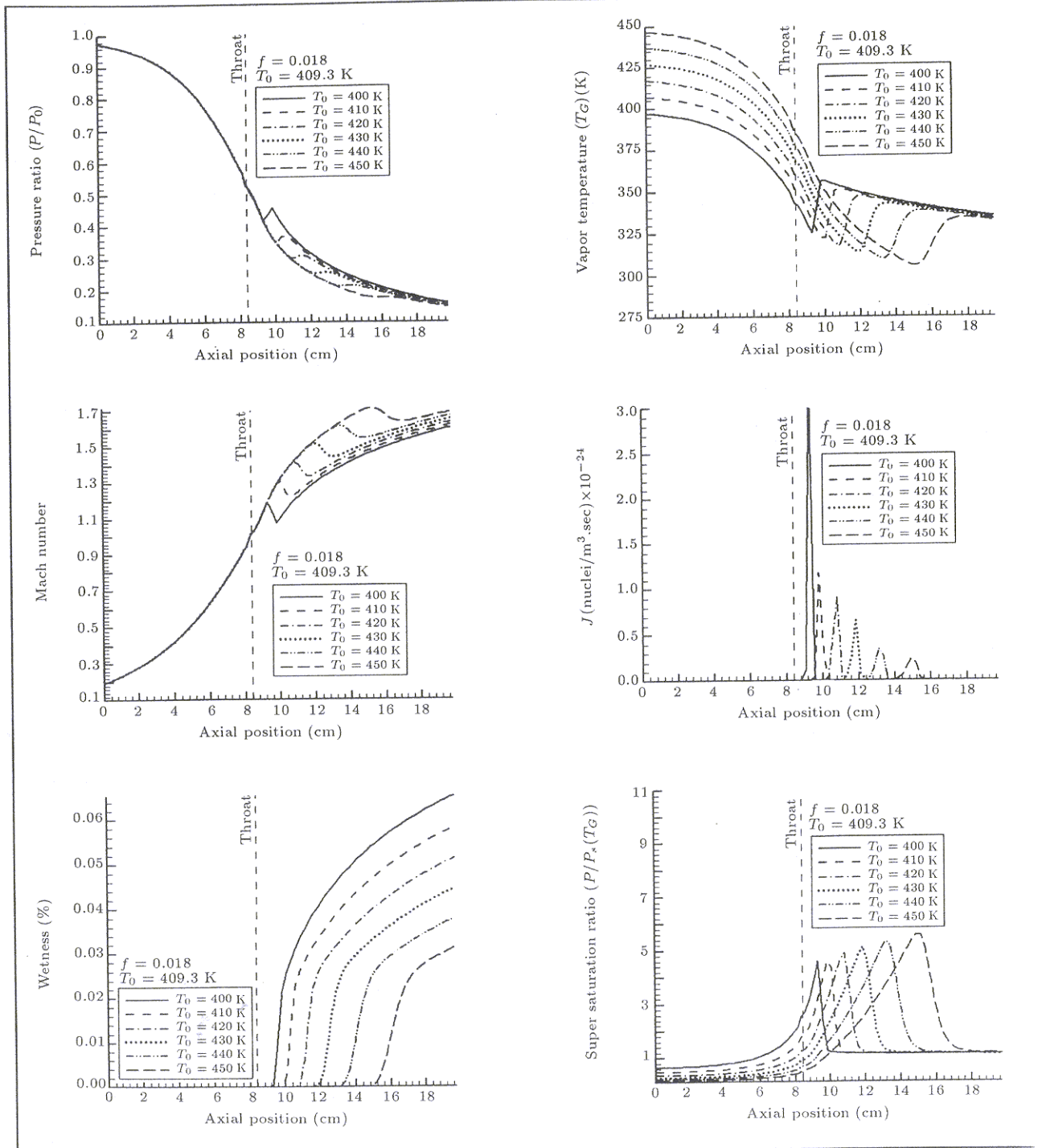
Figure 11. Effect of changes in inlet stagnation temperature on the important axial characteristics of the flow.

Increasing the inlet stagnation temperature shifts the condensation zone to downstream and the wetness ratio decreases effectively.

A decrease of P_0 displaces the location of the zone of rapid condensation and pushes the reversion further downstream the divergent section where higher Mach numbers prevail.

The main consequence of the small rate of cha of pressure is to allow the very first centers of densation generated to grow large enough so the latent heat released reduces the rate of incr of supercooling and the subsequent value of the cleation current. In general, when reversion oc at relatively low Mach numbers, within a relati

short distance downstream the throat, the pressure rise associated with homogeneous condensation is easier to identify.

At small rates of expansion, steam supercools and reverts to equilibrium at a slower rate over a large distance, making the detection of the onset of condensation by simple comparison of dry and wet pressure distributions more difficult and uncertain.

NOMENCLATURE

A	area
c_P	specific heat at constant pressure
D_e	equivalent diameter
f	friction factor
ΔG	change in Gibbs free energy
J	rate of formation of critical droplets per unit volume and time
K_n	Knudsen number
L	latent heat
M	Mach number
m_r	mass of a droplet
P	vapor pressure
$P_s(T_G)$	saturation pressure at T_G
$\dot{P}$	rate of expansion $(1/P)(dP/dt)$
q	condensation coefficient
R	gas constant for water vapor
r	radius of a droplet
S	supersaturation ratio $(P/P_s(T_G))$
T	temperature
$T_s(P)$	saturation temperature at P
ΔT	degree of supercooling $(T_s(P) - T_G)$
t	time
U	velocity
V	specific volume
W	total mass flow rate
x	distance along duct axis
X, Y	functions of temperature and density in equation of state
α_r	coefficient of heat transfer
γ	ratio of specific heats
μ_G	dynamic viscosity of vapor
ζ	dryness fraction
ρ	density
λ	coefficient of thermal conductivity
σ	surface tension
$\rho_s(T_L, r)$	density corresponding to saturation pressure at temperature T_L over a surface of curvature r

S_c	Schmidt number

Subscripts

G	vapor phase
L	liquid phase
0	stagnation condition
r	a droplet of radius r
s	saturation

Superscripts

$*$	critical droplet

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