

Disc. 102-S54/From the July-Aug. 2005 *ACI Structural Journal*, p. 535

### Punching Shear Strength of Reinforced Concrete Slabs Strengthened with Glass Fiber-Reinforced Polymer Laminates. Paper by Cheng-Chih Chen and Chung-Yan Li

#### Discussion by M. Reza Esfahani and Vahid Moradi

Associate professor, Ferdowsi University, Mashhad, Iran; and MSc student, Ferdowsi University.

The authors have presented valuable experimental results in punching shear strength of reinforced concrete slabs strengthened with glass fiber-reinforced polymer (GFRP) laminates.

In an attempt to apply the proposed equations on a new, similar test series conducted by the discussers, some problems were observed and are listed as follows:

1. For the authors' test series, the discussers have calculated  $d_{eqv}$  and  $\rho_{eqv}$  using Eq. (6) to (17), and ultimate shear strength of slabs using Eq. (3a) to (5). The results of these calculations are given in Table A. For the control specimens, strength values similar to those calculated by the authors were obtained. There are some differences, however, between the values calculated by the authors and the discussers. It is appreciated if the authors would confirm their calculations;

2. According to Fig. 12(b), the measured value of  $\epsilon_s$  in Specimen SR1-C2-F1a is approximately 0.0025 while, based on Eq. (6), the calculation shows that the steel strain is approximately four times greater than the aforementioned value (Table B). This large difference between measured and calculated values cannot be justified. Also, the calculated values of  $\epsilon_s$  in Table B indicate yielding of reinforcing bars in most of the specimens (with the exception of Specimens SR2-C1-F1 and SR2-C1-F2). These values that were calculated by the proposed equations are not in agreement with the test results; and

3. According to authors, test results indicated that the strains of the GFRP laminates, at ultimate loads, ranged from 0.0033 to 0.0069. Calculations based on the proposed equations (Table B) give values of  $\epsilon_f$  that are very different than those of the test results. How can this be justified?

Table A—Comparison of test results with calculated values

Specimen	$V_{u,test}$ , kN	$V_{u,predict}$ , kN					
		Authors' (ACI)	Discussers' (ACI)	Authors' (BS)	Discussers' (BS)	Authors' (JSCE)	Discussers' (JSCE)
SR1-C1-F0	103.9	84.8	84.87	104.4	104.3	74.4	74.3
SR1-C1-F1a,b	148.0	92.0	101.68	135.7	137.9	97.3	99.6
SR1-C1-F2a,b	202.1	96.2	102.27	147.7	139.1	106.3	100.5
SR1-C2-F0	123.8	121.0	121.1	122.1	124.9	106.1	106.0
SR1-C2-F1a,b	180.0	131.3	155.21	175.7	187.9	153.7	162.4
SR1-C2-F2a,b	218.8	137.3	160.7	193.6	205.0	169.9	177.7
SR2-C1-F0	146.1	84.8	84.87	135.9	136.0	96.8	96.9
SR2-C1-F1a,b	189.6	88.4	90.28	154.7	148.3	110.6	106.2
SR2-C1-F2a,b	224.2	90.7	93.64	164.9	156.8	118.1	112.6
SR2-C2-F0	225.7	121.0	121.1	158.9	162.9	138.2	138.3
SR2-C2-F1	263.9	126.1	135.0	201.4	191.3	175.7	163.7
SR2-C2-F2	289.4	129.4	140.8	217.8	204.5	190.3	175.5

Table B—Calculated values

Specimen	$\epsilon_s$	$\epsilon_f$	$d_{eqv}$ , mm	$\rho_{eqv}$
SR1-C1-F0	0.0098	—	70.5	0.0059
SR1-C1-F1a,b	0.0054	0.0089	80.7	0.0078
SR1-C1-F2a,b	0.0052	0.0087	81.0	0.0079
SR1-C2-F0	0.0222	—	70.5	0.0059
SR1-C2-F1a,b	0.0100	0.0155	84.8	0.0094
SR1-C2-F2a,b	0.0079	0.0125	87.0	0.0110
SR2-C1-F0	0.0028	—	70.5	0.0131
SR2-C1-F1a,b	0.0023	0.0045	73.8	0.0140
SR2-C1-F2a,b	0.0021	0.0043	75.9	0.0148
SR2-C2-F0	0.0083	—	70.5	0.0131
SR2-C2-F1	0.0060	0.0098	76.5	0.0152
SR2-C2-F2	0.0051	0.0085	78.9	0.0163

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### Punching Shear Strength of Reinforced Concrete Slabs Strengthened with Glass Fiber-Reinforced Polymer Laminates. Paper by Cheng-Chih Chen and Chung-Yan Li

#### Discussion by Himat Solanki and Chandra Khoe

Professional Engineer, Building Dept., Sarasota County Government, Sarasota, Fla.; Engineer, MMXF Steel Corp., Tampa, Fla.

It is true that the strengthening with GFRP would increase the punching shear capacity of slabs. The discussers, however, would like to offer the following comments:

1. The test results appear to be inconclusive because when the concrete strength and GFRP maintain the same level in the slabs and the GFRP reinforcement ratio varies, the strength increase is significantly higher than the reinforcement ratio allows. This

can be explained using the test specimens. For example, for test Specimen SR1-C2-F1, the average ultimate shear strength is 180.0 kN. When the shear strength is compared with Specimen SR2-C2-F1's shear strength, the shear strength increases by approximately 46.6%. In test Specimen SR1-C2-F2, when comparing shear strength with test Specimen SR2-C2-F2, the shear strength increases by approximately 32.2%. When



**Table C—Comparison of test results and predictions**

Specimen designation	$d_{eqv}$ , mm	$\rho_{eqv}$ , %	$V_{u,test}$ , kN	$V_{u,predicted}$ , kN				$V_{u,test}/V_{u,predicted}$			
				ACI	BS	JSCE(1)	JSCE(2)	ACI	BS	JSCE(1)	JSCE(2)
SR1-C1-F0	70.47	0.59	103.9	84.8	104.3	96.2	74.4	1.22	1.00	1.08	1.40
SR1-C1-F1	81.70	0.82	148.0	103.3	142.3	128.3	102.9	1.43	1.04	1.15	1.44
SR1-C1-F2	84.22	0.93	202.1	107.7	154.3	138.5	111.9	1.88	1.31	1.46	1.81
SR1-C2-F0	70.47	0.59	123.8	121.0	125.0	137.3	106.1	1.02	0.99	0.90	1.17
SR1-C2-F1	85.57	1.00	180.0	157.0	193.7	206.5	167.6	1.15	0.93	0.87	1.07
SR1-C2-F2	87.98	1.17	218.8	163.1	212.0	225.1	183.9	1.34	1.03	0.97	1.19
SR2-C1-F0	70.47	1.31	146.1	84.8	135.9	125.3	96.9	1.72	1.08	1.17	1.51
SR2-C1-F1	74.57	1.43	189.6	91.4	151.1	138.2	108.3	2.07	1.25	1.37	1.75
SR2-C1-F2	77.05	1.54	224.2	95.5	161.6	147.1	116.2	2.35	1.39	1.52	1.93
SR2-C2-F0	70.47	1.31	225.7	121.0	162.8	178.8	138.2	1.87	1.39	1.26	1.63
SR2-C2-F1	77.42	1.56	263.9	137.1	195.8	212.0	167.7	1.92	1.35	1.24	1.57
SR2-C2-F2	79.79	1.69	289.4	142.8	209.6	225.9	180.1	2.03	1.38	1.28	1.61
Mean value								1.67	1.18	1.19	1.51
Coefficient of variation								0.25	0.15	0.18	0.18

the reinforcement ratio (0.59% versus 1.31%) is considered in the test specimens, the shear strength increases approximately 23.0%;

2. The load displacement characteristic in Fig. 6 and 7 appears to be inconsistent when it compares with Fig. 8 and 9;

3. Based on Fig. 10, both flexural and punching shear strength would be equal at a reinforcement ratio of approximately 0.65. Why was the higher reinforcement ratio considered if specimens considered fail in punching shear?

4. From Fig. 12(a) and (b), it appears that the load-versus-strain relationship for a No. 3 bar and two layers of GFRP does not depart significantly, but the test shows higher shear strength.

5. The yield strength of GFRP fabric is lower than No. 3 reinforcing bar yield strength. This means the GFRP would fail prior to yielding of a No. 3 reinforcing bar. Therefore, it is very difficult to predict the correct value of  $T_s$ ;

6. The discussers believe Eq. (16) should be limited to  $d_{eqv} \leq h$ , and Eq. (17) should be limited to  $\rho_{eqv} \leq 2.0\%$ ; and

7. There are typographical errors in Eq. (4) and (5) and Table 3,  $V_{u,predicted}$ , JSCE and  $V_{u,test}/V_{u,predicted}$ , JSCE for all specimens.

#### AUTHORS' CLOSURE

The authors would like to thank the discussers for their valuable comments and interest in the paper. In regards to the discussion by Esfahani and Moradi, the authors are especially grateful for their detailed comments regarding the proposed method to predict the ultimate punching shear strength of reinforced concrete slabs strengthened with GFRP laminates. Due to a typing error, the predicted punching shear strengths tabulated in Table 3 should be corrected as shown in Table C; however, the ratios of tested values to predictions in Table 3 were fairly correct. Equivalent depth  $d_{eqv}$  and equivalent reinforcement ratio  $\rho_{eqv}$  calculated by the authors are also included in Table C. Regarding the questions posed, Question 1 is related to the differences between the predictions calculated by the authors and the discussers. The inconsistency is now minor except for Specimen SR1-C1-F2, in which the discussers may have incorrect equivalent depth. Table C also presents the punching shear strengths predicted using the JSCE code. The JSCE(1) column indicates the punching shear strengths calculated by neglecting the limitation for  $\beta_d$  to investigate the effect of the depth on the punching shear strength. The JSCE(2) column considers the limitation for  $\beta_d$

that is limited to 1.5. The  $\beta_d$  is related to the effective depth of the slab. As indicated in the ratios of tested-to-predicted strength, JSCE(2) gives a much more conservative prediction than JSCE(1) does. It should be noted that both the BS and JSCE codes use  $(1/d)^{1/4}$  to calculate the punching shear strength, and there is no upper limit for that in the BS code. Questions 2 and 3 are related to the measured and calculated strains of reinforcing bars and GFRP laminates. The authors would like to point out that the proposed method is based on the hypothesis of Moe<sup>15</sup> that the punching shear strength of a slab is calculated from its flexural strength. Equation (6) to (17) can be adopted to calculate the flexural strength. Therefore, the calculated strains shown in Table B provided by the discussers represent the ultimate strains of the reinforcing bars and GFRP laminates corresponding to the flexural strength. Punching shear failure occurs, however, rather than flexural failure for all specimens except Specimens SR1-C1-F0 and SR1-C2-F0. The strains measured at punching shear failure would be less than those corresponding to the flexural failure.

In regards to the discussion by H. Solanki and C. Khoe, the following presents a closure for each comment. With regards to Question 1, it is believed that GFRP laminates function as the tensile steel reinforcement. The punching shear strength of reinforced concrete slabs strengthened with GFRP laminates influenced by the laminates in addition to the steel reinforcement. Therefore, the authors proposed an equivalent reinforcement ratio to account for the contribution from both materials. The comparison presented by the discussers is not appropriate. Moreover, the relation between punching shear strength and reinforcement ratio is not linearly proportional, as seen from the BS and JSCE codes. Regarding Question 2, the load-displacement curves in Fig. 6 and 7 are different from those in Fig. 8 and 9, and that is contributed to the different amount of reinforcement. Specimens in Fig. 6 and 7 are lightly reinforced slabs and demonstrated a flexural punching or shear punching with higher post-peak strength than specimens in Fig. 8 and 9, which have more reinforcement. For Question 3, the tensile steel reinforcement is usually determined based on the requirement for flexural strength. The slabs subjected to a heavy load lead to a higher amount of the tensile steel reinforcement, which may fail in punching shear. Regarding Question 4, the authors fail to understand the discussers' comment and would like to point out that Fig. 12(a) presents the strains of reinforcing bar



for SR1-C2-F0, while Fig. 12(b) shows strains of reinforcing bar and GFRP laminates for SR1-C2-F1a, which has only a single-layer, not a double-layer, laminate. It is clear that, with the GFRP laminates, SR1-C2-F1a reached a higher shear strength than SR1-C2-F0. Regarding Question 5, the ultimate tensile strength is usually termed for the GFRP laminates instead of yield strength used by the discussers, because the GFRP laminates behave linearly elastic up to failure. Although the ultimate tensile strength of the GFRP laminates is lower than the yield strength of a No. 3 reinforcing bar, the ultimate tensile strains (0.018 and 0.021 for the single- and double-layer, respectively) of GFRP laminates are much higher than the yield strain (0.0024) of the reinforcing bar. Clearly, the GFRP laminates will not fracture before yielding of the reinforcing bar.

Therefore, there is no problem in calculating any internal force, and the proposed method has been followed by the other discussers. For Question 6, the authors agree that the equivalent depth  $d_{eqv}$  will be definitely less than the slab thickness  $h$  and there is no need to specify. However, the authors disagree to limit the equivalent reinforcement ratio  $\rho_{eqv}$  to be less than 2.0%. It is important to recognize that the BS code limits the reinforcement ratio to 3%, while the JSCE code limits implicitly to 3.375%. For Question 7, Eq. (4) and (5) are both correct. For Eq. (4), a factor of  $(f_{cu}/25)^{1/3}$  considering for concrete compressive strength greater than 25 MPa has been mentioned in the paragraph following Eq. (4). Comments related to Table 3 are addressed in the response to the other discussers.

Disc. 102-S56/From the July-Aug. 2005 ACI Structural Journal, p. 550

**Analytical Model to Predict Nonlinear Flexural Behavior of Corroded Reinforced Concrete Beams.** Paper by Tamer El Maaddawy, Khaled Soudki, and Timothy Topper

**Discussion by Peter H. Bischoff**

ACI member, Professor, Department of Civil Engineering, University of New Brunswick, Fredericton, New Brunswick, Canada.

The authors have undertaken an ambitious task of developing a rational model to predict the flexural response of corroded beams. Their model uses bond to control deflection of the computed beam response, and deflection is calculated from elongation of the reinforcement between cracks. The approach used to model bond is somewhat similar to the tension chord model developed by Kaufmann and Marti.<sup>26</sup> This discussion provides another perspective to the problem of beam deflection by relating the author's method of computing deflection with the concept of tension stiffening. A comparison is made in the linear elastic range before yielding of the steel reinforcement, but can be extended to include nonlinear behavior.

Combining the author's equations for curvature and elongation of the reinforcement gives

$$\phi_i = \frac{\epsilon_{max}}{d - c} \left( 1 - \frac{\tau s_m}{d_b f_{max}} \right) = \phi_{i,max} \left( 1 - \frac{\tau s_m}{d_b f_{max}} \right) \quad (41)$$

Hence, the average curvature of an element is related to the maximum curvature  $\phi_{i,max}$  at the crack in the middle of the element, and this value is reduced by the factor to account for tension stiffening of the reinforcing steel. Figure A shows the variation of stress  $f_s$  in the reinforcing steel for a stabilized crack pattern assumed by the authors, whereas Eq. (11) defines the reduction in steel stress at the end of an element as  $\Delta f_{max} = 4\tau s_m / 2d_b$  for a constant value of bond stress. Hence, the maximum change in bar stress at first cracking  $\Delta f_{max,cr} = 4\tau_o s_m / 2d_b$  occurs when the element cracks under a cracking moment  $M_{cr}$ . The expression  $\tau_o = \Delta f_{max,cr} d_b / 2s_m$  then represents an upper limit on bond stress ( $\tau \leq \tau_o$ ) because the concrete is not able to carry a tensile stress greater than  $f_r$ . The latter part of this discussion shows an anomaly in the authors' approach where a value of  $\tau > \tau_o$  is used to calculate deflection of their control beam. The maximum change in bar stress at first cracking is also defined by  $\Delta f_{max,cr} = f_{max,cr} - f_{min,cr} =$

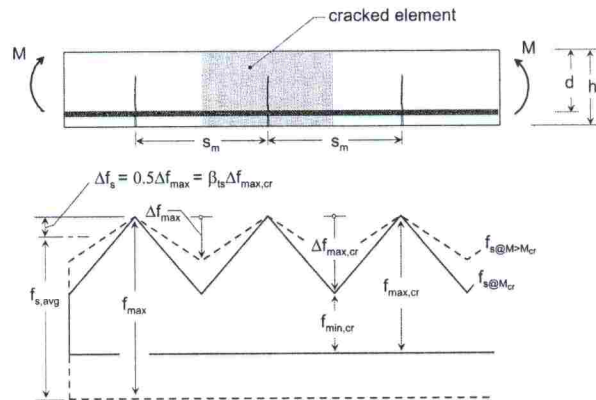


Fig. A—Assumed variation of stress in embedded reinforcement for stabilized crack pattern.

$\frac{n_r M_{cr} (d - c_{cr})}{I_{cr}} - n_r f_r \frac{d - c_g}{h - c_g} = \eta' f_{max,cr}$  where  $\eta' = 1 - (I_{cr}/I_g)$   $(d - c_g)/(d - c_{cr})$ . The variables  $c_{cr}$  and  $c_g$  represent the neutral axis depths of the cracked and uncracked sections, respectively. Substituting this information into Eq. (41) then gives a curvature linked with the bond stress-slip model.

$$\begin{aligned} \phi_i &= \phi_{i,max} \left( 1 - 0.5 \frac{\tau}{\tau_o} \frac{\Delta f_{max,cr}}{f_{max}} \right) \\ &= \phi_{i,max} \left( 1 - 0.5 \frac{\tau}{\tau_o} \eta' \frac{M_{cr}}{M_{ext}} \right) \end{aligned} \quad (42)$$

Alternatively, the tensile response of reinforcement embedded in concrete can be modeled with a tension stiffening factor  $\beta_{ts} = \Delta \epsilon_s / \Delta \epsilon_{max,cr}$  related to strain in the reinforcement.<sup>27</sup> This gives an average strain in the reinforcing bar (refer to