Proceedings of PVP2008
2008 ASME Pressure Vessels and Piping Division Conference
July 27-31, 2008, Chicago, Illinois, USA

PVP2008 61606

MULTI-OBJECTIVE OPTIMAL DESIGN OF SANDWICH COMPOSITE LAMINATES USING SIMULATED ANNEALING AND FEM

A. Sarhadi / Department of Mechanical Engineering, Ferdowsi University, Mashhad, Iran

M. Tahani / Department of Mechanical Engineering, Ferdowsi University, Mashhad, Iran

F. Kolahan / Department of Mechanical Engineering, Ferdowsi University, Mashhad, Iran

ABSTRACT
Multi-objective optimal design of sandwich composite laminates consisting of high stiffness and expensive surface layers and low-stiffness and inexpensive core layer is addressed in this paper. The object is to determine ply angles and number of surface layers and core thickness in such way that natural frequency is maximized with minimal material cost and weight. A simulated annealing algorithm with finite element method is used for simultaneous cost and weight minimization and frequency maximization. The proposed procedure is applied to Graphite-Epoxy/Glass-Epoxy and Graphite-epoxy/Aluminum sandwich laminates and results are obtained for various boundary conditions and aspect ratios. Results show that this technique is useful in designing of effective, competitive and light composite structures.

INTRODUCTION
In recent years, application of composite laminates has been increased because of their high stiffness-to-weight or strength-to-weight ratios and their specific design capabilities. Extensive applications and numerous design parameters of composite laminates such as fiber orientations, thickness and material of layers, indicate utilization of suitable optimization techniques in design of composite structures. Composite laminates are usually employed in aerospace, defense, marine and automotive industries. Most composite structures have components that may be modeled as rectangular plates. To reduce the cost and enhance the mechanical properties of these structures, sandwich design is usually used. Such designs frequently employ high-stiffness and expensive materials as surface layers and a low-stiffness and inexpensive material as core layer. This idea combines the advantages of two materials. In general, there are various boundary conditions for composite laminated plates that only a small number of these boundary conditions can be solved by analytical solutions. Therefore utilization of numerical solution methods such as finite element method seems to be necessary. The present study, aims at multi-objective optimal design of sandwich laminated plates with general boundary conditions subjected to free vibrations.

Several integer programming techniques were investigated by Haftka and Wash [1], Nagendra et al. [2] to determine the optimal stacking sequences of simply supported laminated plates. Buckling load maximization of simply supported laminated plates was studied by Haftka and Leriche [3,4] using genetic algorithm. Liu et al. [5] applied genetic algorithm (GA) to find the optimal stacking sequence of a simply supported laminated plate for maximum buckling load. In the design of laminates, maximum frequency problems are of practical importance. Adali et al. [6] used an integer programming approach with Boolean variables for frequency maximization of simply supported hybrid laminates undergoing free vibrations. Weight minimization of simply supported composite laminated panels subjected to strength and buckling load constraints was investigated by Gantovnik et al. [7]. Spallino et al. [8] studied thermal buckling load maximization of a simply supported laminated plate subjected to thermal strain constraint. Lin and Lee [9] applied a GA procedure with local improvement for optimum stacking sequence of a composite plate. Erdal and Snomez [10] proposed a simulated annealing algorithm (SA) to maximize buckling load of a composite laminated plate with simply supported boundary conditions.

Research studies on optimal design of composite laminates are extensive. However, most investigations have been devoted to single objective laminate optimal design problems with simply supported boundary conditions. Further, some important design aspects such as material cost have been disregarded.

The present study aims at design optimization of sandwich laminated plates with general boundary conditions for maximum frequency with minimum cost and weight. The paper has been organized as follows: first, a brief description about SA algorithm and its components is given. Then, the optimization problem is described. After that, optimization procedure of the problem is illustrated. Finally, some numerical results are provided and discussed to show the efficiency of the proposed solution technique in optimal design.

SIMULATED ANNEALING ALGORITHM

As proposed by Kirkpatrick et al. [11] for the first time, simulated annealing (SA) is a powerful stochastic search technique. The method name induces the physical process whereby the temperature of a solid is raised to a melting point, where the atoms can move freely and then slowly cooled. This method models the behavior of solid material atoms during annealing in forming arrangements. There is an analogy between an optimization process and the physical annealing process. Different configurations of the problem correspond to different arrangements of the atoms. The cost of a configuration corresponds to the energy of the system. Optimal solution corresponds to the lowest energy state. In annealing process the atoms find their way to build a perfect crystal structure through random movements; similarly the global optimum is reached through a search within randomly generated configurations.

In the SA (Simulating Annealing) algorithm, a random initial point is selected and systematically updated until a stopping criterion is satisfied. Updating is an iterative procedure. A random point is generated in the neighborhood of the current configuration, iteratively. The new point is accepted, if the point has a smaller value of cost function compared to that of the current record. This point replaces the old one. On the other hand, if the new cost function has a larger value, the acceptability of the point is decided according to the probability of Boltzman distribution. The probability of accepting a new solution is given as follows:

$$p=\begin{cases}1 & if \quad \Delta<0 \\ e^{-\Delta/t} & if \quad \Delta\geq 0\end{cases} \tag{1}$$

The calculation of this probability relies on a temperature parameter, T, which is referred to as temperature, since it plays a similar role as the temperature in the physical annealing process. The initial temperature parameter is determined in such way that at first, almost every solution is accepted by algorithm. The temperature parameter is kept constant for a number of trials and then reduced. To avoid getting trapped at a local minimum point, the rate of reduction should be slow. For instance, the following method can be proposed to reduce the temperature:

$$\begin{aligned} &T_{i+1}=cT_i \qquad i=0,1,\dots \\ &0.9\leq c<1 \end{aligned} \tag{2}$$

At initial stages of the algorithm (at high temperatures), the probability of accepting worse designs is higher but at low temperatures, this probability becomes smaller and smaller so that in the end the designs having higher cost are almost never accepted.

There are different stopping criteria in SA algorithm. The number of iterations in which the solution doesn't improve or the final temperature value may be taken as stopping criteria.

PROBLEM DESCRIPTION

A laminated plate of length a, width b and thickness h is considered in the x, y and z directions respectively. The problem is investigated in two different cases. At case (1) optimal design of a simply supported hybrid laminates solvable with analytical solutions is investigated. At case (2) optimal design of sandwich laminates with general boundary conditions which are not solvable with analytical solution is done by FEM. Detailed descriptions of these cases are as follows:

Case 1. At first case a simply supported hybrid laminate consisting of high stiffness outer layers (Graphite/Epoxy) and low stiffness inner layers (Glass/Epoxy) is considered. Each layer has a constant thickness t so that $h=N\times t$ where N is the total number of layers. The hybrid laminates are made of N_i inner plies and N_o outer plies such that $N=N_i+N_o$. The equation governing the free vibrations of this laminate by neglecting the rotatory inertia effects and bending-twisting coupling stiffness D_{16} and D_{26} is given by:

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}=\rho h\frac{\partial^2 w}{\partial t^2} \tag{3}$$

In equation (3), w denotes the deflection in the *z* direction, ρ is the mass density, and *h* is the total thickness of the laminate. The bending stiffness D_{ij} is computed by:

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{N}(\bar{Q}_{ij})_k\,(z_k^3-z_{k-1}^3) \tag{4}$$

Where z_k is the distance from the middle plane of the laminate to the top of the *k*th layer and $\bar{Q}_{ij}$ is the plane stress reduced stiffness component of the *k*th layer which can be calculated as a function of fiber orientations and material properties using standard transformation relations.

The mass density of a hybrid laminate is computed as a thickness weighted average given by:

$$\rho=h^{-1}\int_{-h/2}^{h/2}\rho^{(k)}dz \tag{5}$$

Where $\rho^{(k)}$ indicates the mass density of the material in the *k*th layer.

The boundary conditions for the simply supported plate are calculated as:

$$\begin{aligned} &w=0,\ M_x=0 \qquad at\ x=0,a \\ &w=0,\ M_y=0 \qquad at\ y=0,b \end{aligned} \tag{6}$$

Where M_x and M_y represent the bending moments about x and y axes, respectively. The influence of the bending-twisting coupling stiffness coefficients D_{16} and D_{26} is assumed insignificant and hence will be omitted in the analysis. The error induced by this assumption is negligible if the following non-dimensional ratios γ and δ are satisfied by constraints (8).

$$\gamma = D_{16}(D_{11}^3 D_{22})^{-1/4}, \quad \delta = D_{26}(D_{11} D_{22}^3)^{-1/4} \tag{7}$$

$$\gamma \le 0.2, \quad \delta \le 0.2 \tag{8}$$

A detail discussion of this condition and its implications is given in Nemeth [12], where it is shown that for buckling problems constraints (8) are effective in reducing bending-twisting coupling to a negligible level. Due to similarity of expressions for buckling load and frequencies, the same constraints are used to reduce the error introduced by neglecting D_{16} and D_{26}. Due to simply supported boundary conditions of the hybrid laminate, Navier's solution is used to solve the eigenvalue problem (3) subjected to the boundary conditions (6). In this approach the deflection w of the vibration mode *(m, n)* is considered as:

$$w(x, y, t) = W(x, y)\ e^{i\Omega t} \tag{9}$$

$$W(x, y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{10}$$

By substituting equation (10) into (3), we compute the eigen-frequency Ω_{mn} as:

$$\Omega_{mn}^2 = \frac{\pi^4}{\rho h}\left\{D_{11}\left(\frac{m}{a}\right)^4 + 2(D_{12} + 2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4\right\} \tag{11}$$

Where the various frequencies Ω_{mn} correspond to different mode shapes (different values of *m* and *n* in equation (11)). The fundamental frequency is obtained when *m* and *n* are both one.

The design object is to find the best sequence of ply angles and number of low stiffness and less expensive layers (Glass/Epoxy) to maximize fundamental frequency and minimize material cost of the hybrid laminate simultaneously.

From mathematical standpoint this design optimization problem can be defined as follows:

Find the solution vector $X = [\theta_1, \theta_2, ..., \theta_n, N_l]$ subjected to constraints $\begin{matrix} \gamma - 0.2 \le 0 \\ \delta - 0.2 \le 0 \end{matrix}$ in order to minimize $F(X) = [\Omega(X), Cost(X)]$ where θ_i is the fiber angle of the i th layer, N_l is the number of low stiffness layers, $\Omega(X)$ and $Cost(X)$ represent fundamental frequency and cost of the hybrid laminate material respectively.

Case 2. At this case a sandwich laminate made of Aluminum core and Graphite/Epoxy as outer layers is considered. There is no limitation on boundary conditions and any combination of simply, clamped and free boundary conditions can be selected for sandwich laminated plate.

From mathematical standpoint, this optimization problem can be defined as follows:

Find the solution vector $X = [\theta_1, \theta_2, ..., \theta_n, t]$ to minimize $F(X) = [\Omega(X), Cost(X), Weight(X)]$ where θ_i is the fiber angle of the i th outer layer and may be vary between -90 to 90 degrees with increments of 15 degree. t is the core thickness of the sandwich laminate , $\Omega(X)$, $Cost(X)$ and $Weight(X)$ represent fundamental frequency , cost and weight of the sandwich laminate respectively.

Because of that calculating of $\Omega(X)$ by analytical solutions is possible only for a few numbers of boundary conditions, FEM is used to calculate $\Omega(X)$ in any combination of simply, clamped and free boundary conditions.

OPTIMIZATION PROCEDURE

Optimization procedure of the above cases is described at this section.

Case 1. A sequence of ply angles and number of low stiffness layers are considered as an initial solution in SA algorithm. This initial solution consists of a sequence of integer numbers to represent the ply angles ranging from -90 to 90 with 15° increment and number of low stiffness layers (Glass/Epoxy). The insertion method is used for neighborhood generation in SA algorithm.

Since the design space of a laminate stiffness optimal design problem is not convex [13], the weighted sum method is not an appropriate technique and it is possible not to result in optimum solution. Therefore to find the optimal solution, the general objective function of the current multi-objective optimization problem is defined as follows:

$$F = \begin{cases} \dfrac{1}{f_1^2 + f_2^2} & if\ (3.5)\ and\ (4.5)\ satisfied \\ \dfrac{1}{f_1^2 + f_2^2 + c_1 g_1^2 + c_2 g_2^2} & otherwise \end{cases} \tag{12}$$

Where g_1, g_2 and f_1, f_2 are as follows:

$$g_1 = (\gamma - 0.2) \tag{13}$$

$$g_2 = (\delta - 0.2) \tag{14}$$

$$f_1 = \left(\frac{\Omega^* - \Omega}{\Omega^*}\right) \tag{15}$$

$$f_2 = \left(\frac{Cost - Cost^*}{Cost}\right) \tag{16}$$

Coefficients c_1 and c_2 should be chosen so that any violation in the constraints imposes a considerable penalty in the fitness function. Therefore, there is low probability of accepting a solution if the constraints are not satisfied. Ω^* represents the maximum frequency of the hybrid laminate in the case that all layers are made of high stiffness material (Graphite/Epoxy) and $Cost^*$ is the material cost of the hybrid laminate in case that all layers are made of low stiffness material (Glass/Epoxy). In order to ensure that this method results in optimal solution, Ω^* and $Cost^*$ should be calculated relatively accurate. To calculate Ω^* a single objective optimal design problem should be solved. Ω is calculated by equation (11) and the material cost function is defined as:

$$\mathrm{cost} = ab \frac{h}{N_t} g \left(\alpha_o \rho_o N_o + \alpha_i \rho_i N_i \right) \tag{17}$$

Where h is the total thickness of the laminate, N_t is total number of layers, ρ_o is the density of high-stiffness layer material (Graphite/Epoxy), N_o is the number of high-stiffness layers, α_o is the material cost factor of high-stiffness layer, ρ_i is the density of low-stiffness layer material (Glass/Epoxy), N_t is number of low-stiffness layers, α_i is the material cost factor of low-stiffness layer, a is the length of the plate and b is the width of the plate.

Case 2. A sequence of ply angles and the core (Aluminum) thickness of the sandwich laminate are considered as an initial solution in SA algorithm. Like previous case, the insertion method is used for neighborhood generation in SA algorithm. The general objective function of the current multi-objective optimization problem is defined as follows:

$$F = \frac{1}{f_1^2 + f_2^2 + f_3^2} \tag{18}$$

$$f_1 = \left(\frac{\Omega^* - \Omega}{\Omega^*}\right) \tag{19}$$

$$f_2 = \left(\frac{Cost - Cost^*}{Cost}\right) \tag{20}$$

$$f_3 = \left(\frac{Weight - Weight^*}{Weight}\right) \tag{21}$$

Ω^*, $Cost^*$ and $Weight^*$ represent the optimal value of frequency, cost and weight respectively in case each of these objective functions are optimized singly. To calculate Ω^* a single objective optimal design problem in which angle plies are design parameters should be solved.

Ω indicates the fundamental frequency and is calculated by FEM software (ANSYS).

$Cost$ represents the material cost function and is calculated by following equation:

$$Cost = ab\,(\alpha_g t_g \rho_g N_g + \alpha_{Al} t_{Al} \rho_{Al}) \tag{22}$$

α_g, t_g, ρ_g and N_g represent cost per unit weight, thickness of layers, number of layers and density of the Graphite/Epoxy layers respectively. α_{Al}, t_{Al} and ρ_{Al} represent cost per unit weight, thickness and density of the Aluminum core, respectively. a and b indicate dimensions of the sandwich laminate.

$Weight$ is the weight of the sandwich laminate and is calculated from the following equation:

$$Weight = ab\,(t_g \rho_g N_g + t_{Al} \rho_{Al}) \tag{23}$$

Description of the used parameters in this equation is as previous.

Optimization process is done by linkage between optimization computer codes and FEM software thus optimization computer codes generate the input data for ANSYS software, then ANSYS software calculates the fundamental frequency to calculate the general objective function. After that, input data is varied by SA algorithm to generate new input data for ANSYS software. Output of ANSYS software is used to calculate the general objective function. This process continues until convergence happens and optimal design solution is found. Figure (1) shows the schematic presentation of the optimization procedure.

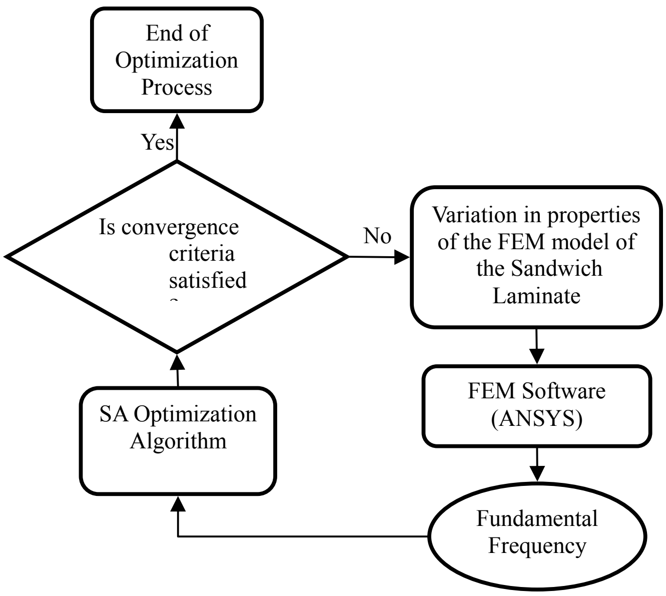

Figure 1. Schematic presentation of the optimization procedure

NUMERICAL RESULTS

In order to evaluate the performance of the proposed optimal design procedures, numerical examples of both of the above cases are presented and solved in this section.

Case 1. A hybrid laminate made of Glass/Epoxy in inner layers and Graphite/Epoxy in outer layers with geometrical dimensions of *b=0.25m*, *h=0.002m* is considered. Forgoing hybrid laminate is shown in figure (2).

Graphite/Epoxy
Graphite/Epoxy
⋮
Glass/Epoxy
Glass/Epoxy
Glass/Epoxy
Glass/Epoxy
⋮
Graphite/Epoxy
Graphite/Epoxy

Figure 2. General form of the hybrid laminate

Material properties of layers are shown in table (1).

Table 1. Material properties of the hybrid laminate

Properties \ Material	Graphite/Epoxy	Glass/Epoxy
$E_1 (GPa)$	181	38.6
$E_2 (GPa)$	10.3	8.27
$G_{12} (GPa)$	7.17	4.14
υ_{12}	0.28	0.26
$\rho \left({Kg}/{m^3} \right)$	1600	1800
α (Cost Factor)	8	1

As seen from table (1) the material cost factor (α) of the Graphite/Epoxy layer is eight times more than that of the Glass/Epoxy while the stiffness to weight ratio (E/ρ) of the Graphite/Epoxy is about 5 times more than that of the Glass/Epoxy. The idea of locating expensive material in the outer layers and inexpensive material in the inner layers can reduce the material costs while satisfies the design specifications. It is known that the more graphite layers, the higher frequency and material cost. Therefore, it is necessary to find the best trade off between frequency and material cost.

A computer code has been developed in order to find the best sequence of ply orientations and number of glass layers for various aspect ratios. In every case, the computer code was run to get the optimum angle sequence and layer numbers of each material.

Table (2) contains the best sequence of ply angles (θ_{best}) and number of Glass/Epoxy layers ($n_{glass/epoxy}$) for N=8 and different aspect ratios varying from 0.2 to 2. This table shows that the best sequence of ply angles changes from 0 to 90 if the aspect ratio varies from 0.2 to 2. The last two columns of the table present the percent of material cost and frequency reduction with respect to the situation which all of layers are made of Graphite/Epoxy. As illustrated by Table 2, it is clear that a small decrease in the natural frequency leads to a considerable decrease in the material costs. The table also shows that use of Glass/Epoxy layers in the composite laminate made of Graphite/Epoxy can decrease material costs considerably while there is a small reduction in the fundamental frequency. Table 2 shows that for an average decrease of 22.8% in the fundamental frequency, there is a 64.5% reduction in the material costs.

As seen from table (2) the best sequence of ply angles are mostly composed of 0 for an aspect ratio between 0.2 and 0.4, ± 45 for aspect ratios between 0.8 and 1.2, ± 60 for an aspect ratio between 1.4 and 1.6 and 90 for aspect ratios between 1.8 and 2.

Adali et al. [6] designed laminates composed of piles with 0º, ±45º and 90º orientations only and showed that optimal stacking sequences for all layers are 0º for aspect ratios between 0.2 and 0.6, ±45º for aspect ratios between 0.7 and 1.4, and 90º for aspect ratios between 1.5 and 2. Due to restriction of ply orientations in ref [6], it is not possible to make a complete comparison but it is seen that for aspect ratios from 0.2 to 0.4, 1 and 1.8 to 2, both of results are the same. In other aspect ratios due to more extensiveness of the ply orientations in our design, the obtained results are more accurate in comparison with the results in ref [6] for this special case

Figure (3) shows the convergence graph of SA algorithm during optimization process. Horizontal axis represents the number of iterations and the vertical axis corresponds to the general objective function values (F).

As figure (3) represents at initial stages of the optimization process by SA algorithm due to high temperature conditions some oscillations occur and some worse solutions are accepted but gradual decrease in temperature parameter causes gradual convergence of the SA algorithm so that in low temperature area, optimal solution is found after about 1500 iterations.

Table 2. Optimal design results of simply supported hybrid laminate (N=8)

a/b	b (m)	h (m)	θ_{best}	$\omega_{\max}$ $(rad\ /s)$	$\cos t_{\min}$	$n_{glass\ /epoxy}$	Cost reduction (%)	Frequency reduction (%)
0.2	0.25	0.002	$[0/0/0/0]_s$	19093	1.1375	6	64.5	20
0.4	0.25	0.002	$[0/0/0/0]_s$	4844.3	2.275	6	64.5	21.4
0.6	0.25	0.002	$[0/15/-30/-15]_s$	2232.5	3.4125	6	64.5	20.3
0.8	0.25	0.002	$[0/30/-45/-45]_s$	1334.2	4.55	6	64.5	25.8
1	0.25	0.002	$[-45/45/45/45]_s$	1154.5	5.6875	6	64.5	25.9
1.2	0.25	0.002	$[90/-45/45/45]_s$	855.64	6.825	6	64.5	27.7
1.4	0.25	0.002	$[90/60/60/-60]_s$	820.4	7.9625	6	64.5	23.9
1.6	0.25	0.002	$[90/75/75/60]_s$	799.8	9.1	6	64.5	21.3
1.8	0.25	0.002	$[90/90/90/90]_s$	790.07	10.238	6	64.5	21.2
2	0.25	0.002	$[90/90/90/90]_s$	784.04	11.3750	6	64.5	21.3

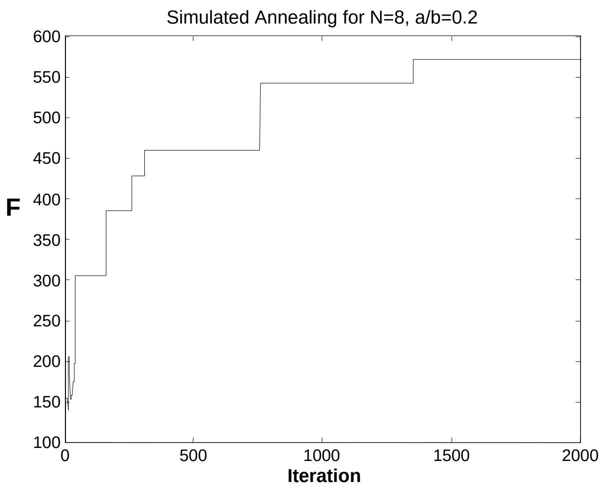

Figure 3. convergence graph of SA algorithm

Case 2. A sandwich laminate composed of ***AL-3003-H12*** as its core and ***Graphite/Epoxy*** layers as its outer layers with geometrical dimensions *b=0.25m, h=0.002m* is considered. Each layer of Graphite/Epoxy has a thickness of *t=0.125 mm*. General form of this sandwich laminate is shown in figure (4).

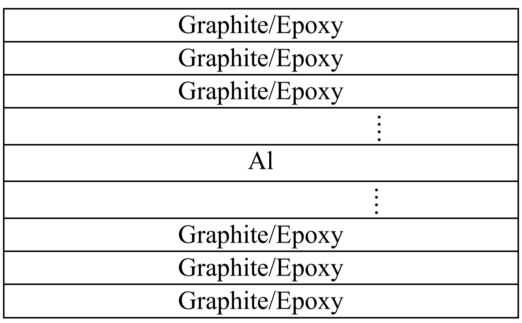

Figure 4. General form of the considered sandwich laminate.

Material properties and costs of the sandwich laminate layers are shown in table (3).

Table3. Material properties of the sandwich laminate

Material / Properties	Graphite/Epoxy	AL-3003-H12
$E_1(GPa)$	181	68.9
$E_2(GPa)$	10.3	68.9
$G_{12}(GPa)$	7.17	25
υ_{12}	0.28	0.33
$\rho\left(Kg/m^3\right)$	1600	2730
α $(\$/Kg)$	26.64	8.88

As table (3) represents the stiffness to weight ratio $(E\ /\ \rho)$ of the Graphite/Epoxy is about 4.5 times more than that of the Aluminum while cost per weight value (α) of the Graphite/Epoxy is about 3 times more than that of the Aluminum. Therefore the objective of the design optimization is to find the best sequence of ply angles of the Graphite/Epoxy layers, The Aluminum core thickness and number of the Graphite/Epoxy layers in order to maximize the fundamental frequency and minimize the cost and the weight of the sandwich laminate.

In order to generalize the optimization design procedure for any combination of boundary conditions, Finite Element Method (FEM) is used. Shell 99 element is used to model the sandwich laminate. Shell 99 is an eight-node element which every node has six degrees of freedom, three rotational and three translational degrees of freedom toward x, y and z directions. Input data of this element includes thickness of each layer, ply angles and orthotropic properties of layers.

At any iteration, computer optimization programs change the sequence of ply angles, The Aluminum core thickness and number of the Graphite/Epoxy layers as input data of the finite element software (ANSYS) according to the SA algorithm. Then finite element package calculates fundamental frequency of the sandwich laminate on the basis of new FEM model. After that general objective function is calculated and this procedure continues according to the described process in figure (1) until the optimal solution is found.

Numerical results of running computer programs for sandwich laminates with various boundary conditions and aspect ratios are shown in tables from (4) to (7). The parameters θ_{best} , $n_{graphite/epoxy}$, t_{Al} , $\omega_{\max}$, $\cos t_{\min}$ and $weight_{\min}$ in the following tables indicate the best sequence of ply angles, number of the Graphite/Epoxy layers, thickness of the Aluminum core, maximum fundamental frequency and minimum cost of the designed laminate, respectively. Characters **S**, **C** and **F** present simply supported, clamped and free boundary conditions, respectively. The order of denomination is on the basis of the opposite edges.

Table (4) represents optimal design results for simply supported sandwich laminates. As results present the sequence of ply angles for similar aspect ratios are approximately like obtained results for simply supported hybrid laminates in case (1) so that for aspect ratios equal to 1 and 2 the most ply angles are ±45º and 90º, respectively. This issue approves the validity of the explained approach in optimal design of sandwich laminates.

Numerical results of multi-objective optimal design of sandwich composite laminates with arbitrary boundary conditions are shown in tables (5) to (7). In these tables the best sequence of ply angles, the Aluminum core thickness and number of the Graphite/Epoxy layers are calculated so that the fundamental frequency is maximized while cost and weight of the sandwich laminate are minimized, simultaneously. As the following tables show the best sequence of ply angles and other design parameters change on the basis of the boundary conditions and aspect ratios. Table (5) shows that optimal design results of sandwich laminates with SSCS boundary conditions for aspect ratios 0.5 and 2 are like the corresponding results from table (4). Optimal design results for sandwich laminates with CCCC boundary conditions are shown in table (6). Since there are no analytical solutions for sandwich laminates with CCCC boundary conditions, utilization of FEM solution is necessary to find the optimal design of the laminate. In table (7) optimal design results for sandwich laminates with CFFF boundary conditions are shown.

As final results approve, utilization of FEM package (ANSYS) with SA algorithm provides the possibility of design optimization of composite laminates with complicated boundary conditions. Figure (5) displays the convergence graph of the SA algorithm during the optimization process. As figure shows after initial oscillations, optimal solution is found after about 500 iterations, finally.

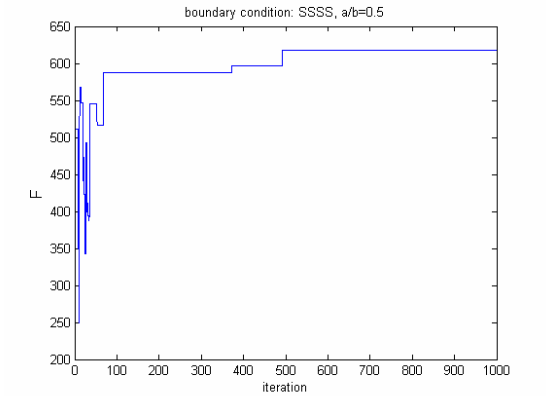

Figure 5. convergence graph of SA algorithm

CONCLUSION

In this paper, multi-objective optimal design of the composite laminates was investigated in two different instances. At first case, multi-objective optimal design of hybrid laminates with simply supported boundary conditions was performed. In order to calculate the fundamental frequency of the simply supported hybrid laminates, Navier's solution was employed. Numerical results approved the proficiency of SA algorithm in the effective design of the hybrid laminates. Comparison of the obtained results from the proposed approach with the results from ref [6] for special cases validated our approach in the design optimization. At case 2, the design optimization of the sandwich composite laminates with various boundary conditions was investigated. At case 2, due to absence of analytical solutions for arbitrary boundary conditions, Finite Element Method was employed to calculate the fundamental frequency of the sandwich laminate. Final results approved the capability of SA algorithm in the optimal design of the composite laminates. As computational results represent, combination of SA algorithm as a flexible optimization algorithm and FEM as an efficient numerical solver provides a powerful means in optimal design of the sandwich composite laminates with complicated boundary conditions and geometry. This approach can be extended to optimal design of composite plates and shells including pressure vessels, pipes and so on.

REFERENCES

[1] Haftka R, Walsh J. ”Stacking-sequence optimization for buckling of laminated plates by integer programming”. AIAA J 1992.

[2] Nagendra S, Haftka R, Gflrdal Z. ”Stacking sequence optimization of simply supported laminates with stability and strain con straints”. ALA.A J 1992;30:2132 7.

[3] Le Riche R, Haftka R. ”Optimization of laminate stacking sequence for buckling load maximization by genetic algorithm.” AIAA J 1993;31:951-6.

[4] Le Riche R, Haftka R.T. Improved genetic algorithm for minimum thickness composite laminate design. Composite Engineering, Vol. 5, No. 2,143–161, 1995.

[5] Liu B, Haftka R.T, Akgün M.A, Todoroki A. Permutation genetic algorithm for stacking sequence design of composite laminates. Computer Methods in Applied Mechanics and Engineering, Vol. 186, pp. 357–372, 2000

[6] Adali S, Verijenko V. Optimal Stacking Sequence Design of Symmetric Hybrid Laminates Undergoing Free Vibrations. *Composite Structures*, Vol. 54, pp. 131-138, 2001.

[7] Gantovnik V.B, Gurdal Z, Watson L.T. A genetic algorithm with memory for optimal design of laminated sandwich composite panels. *Composite Structures,* Vol. 58, pp. 513–520, 2002.

[8] Spallino R, Thierauf G. Thermal buckling optimization of composite laminates by evolution strategies. *Computers & Structures*, Vol. 78, pp. 691-697, 2000.

[9] C.C Lin and Y.J Lee, Stacking sequence optimization of laminated composite structures using genetic algorithm with local improvement, *Composite Structures* Vol. 63, pp. 339–345, 2004.

[10] Erdal Ö, Sönmez F.Ö. Optimum Design of Composite Laminates for Maximum Buckling Load Capacity Using Simulated Annealing. *Composite Structures,* Vol. 71, Issue 1, pp. 45-52, 2005

[11] Kirkpatrik S, Gelatt C.D and Vecchi M.P. Optimization by Simulated Annealing. *IBM Research Report RC 9355*, 1982.

[12] Nemeth MP. "Importance of anisotropy on buckling of compression-loaded symmetric composite plates." AIAA J 1986;24:1831-5.

[13] Marler R.T, Arora J.S. Review of Multi-Objective Optimization Concepts and Algorithms For Engineering. *Technical Report*, *Optimal Design Laboratory*, College of Engineering, the University of Iowa, USA, 2004.

APPENDIX A

NUMERICAL RESULTS

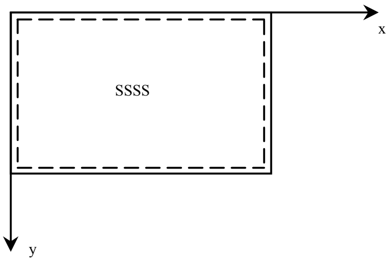

Table 4. Optimal design results of simply supported sandwich laminate

a/b	b (m)	Boundary conditions	θ_{best}	$n_{graphite/epoxy}$	t_{Al} (mm)	$\omega_{\max}$ (rad/s)	$cost_{\min}$ ($)	$weight_{\min}$ (gr)
0.5	0.25	SSSS	$[0/0/0/0/0/0/Al]_s$	12	0.5	3651.9	2.377	117.7
1	0.25	SSSS	$[-45/45/-60/45/-45/Al]_s$	10	0.75	1215.8	4.466	253
1.5	0.25	SSSS	$[75/-75/75/-60/-60/Al]_s$	10	0.75	909.1	6.7	379.5
2	0.25	SSSS	$[90/90/90/90/90/90/Al]_s$	12	0.5	914.2	9.51	470.6

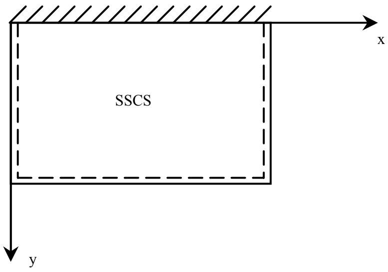

Table 5. Optimal design results of SSCS boundary conditions sandwich laminate

a/b	b (m)	Boundary conditions	θ_{best}	$n_{graphite/epoxy}$	t_{Al} (mm)	$\omega_{\max}$ (rad/s)	$cost_{\min}$ ($)	$weight_{\min}$ (gr)
0.5	0.25	SSCS	$[0/0/0/0/0/0/Al]_s$	12	0.5	3671.6	2.377	117.7
1	0.25	SSCS	$[60/-60/75/-60/-60/Al]_s$	10	0.75	1459.8	4.466	253
1.5	0.25	SSCS	$[90/90/90/-60/90/90/Al]_s$	12	0.5	1404.9	7.13	353
2	0.25	SSCS	$[90/90/90/90/90/90/Al]_s$	12	0.5	1405.6	9.51	470.6

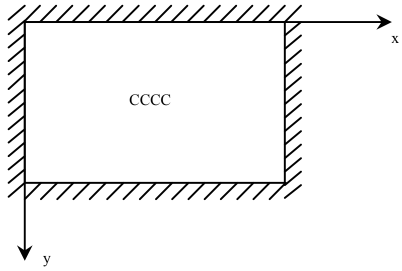

Table 6. Optimal design results of CCCC boundary conditions sandwich laminate

a/b	b (m)	Boundary conditions	θ_{best}	$n_{graphite/epoxy}$	t_{Al} (mm)	$\omega_{\max}$ (rad/s)	$\cos t_{\min}$ ($)	$weight_{\min}$ (gr)
0.5	0.25	CCCC	$[0/0/15/0/0/0/Al]_s$	12	0.5	7991.4	2.377	117.7
1	0.25	CCCC	$[0/90/0/-15/0/Al]_s$	10	0.75	2060.4	4.466	253
1.5	0.25	CCCC	$[90/90/60/90/90/90/Al]_s$	12	0.5	2000.8	7.13	353
2	0.25	CCCC	$[90/90/90/90/90/90/Al]_s$	12	0.5	2026.9	9.51	470.6

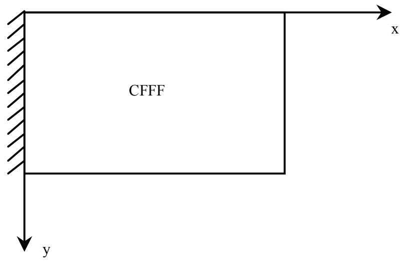

Table 7. Optimal design results of CFFF boundary conditions sandwich laminate

a/b	b (m)	Boundary conditions	θ_{best}	$n_{graphite/epoxy}$	t_{Al} (mm)	$\omega_{\max}$ (rad/s)	$\cos t_{\min}$ ($)	$weight_{\min}$ (gr)
0.5	0.25	CFFF	$[0/0/15/0/0/0/Al]_s$	12	0.5	1250.6	2.377	117.7
1	0.25	CFFF	$[0/0/0/-15/0/0/Al]_s$	12	0.5	314.5	4.753	235.3
1.5	0.25	CFFF	$[15/-15/0/0/15/0/Al]_s$	12	0.5	134.5	7.13	353
2	0.25	CFFF	$[0/0/15/0/15/-15/Al]_s$	12	0.5	77.3	9.51	470.6