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ANT COLONY OPTIMIZATION OF HYBRID LAMINATES FOR MINIMUM COST AND WEIGHT

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ABSTRACT

Ant colony optimization (ACO), a heuristic method formerly applied on combinatorial problems in the field of applied mathematics and industrial engineering, is described and employed for the multi-objective optimization of hybrid laminates for obtaining minimum weight and cost. The investigated laminate is made of glass-epoxy and graphite-epoxy plies to combine the lightness and economical attributes of the first with the high-stiffness property of the second using the weighting sum method in order to make the trade off with the cost and weight as the objective functions and the first natural frequency as the constraint possible. The results obtained by ACO including the Pareto set, optimum stacking sequences and number of plies made of either glass or graphite fibers are compared with those achieved by the GA reported in literature. The comparison, besides confirming the idea of hybridization, clearly shows that the ACO has outperformed GA or resulted in identical solutions. In the latter case, it seems that both GA and ACO have reached the global optimums.

1. INTRODUCTION

Laminated composites have been extensively used as structures in aerospace, defense, marine, automobile and many other industries. This is because of the fact that they are generally lighter and stiffer than the other structural materials. In contrast to isotropic materials, there are also some additional attributes in their design regarding the fiber orientations and stacking sequence which can be set in order to achieve the maximum efficiency. Optimization methods are the best and sometimes the unique means of utilizing these capacities.

In all applications, it is ideal to have the stiffest and meanwhile lightest and most economical structures. These three that normally act against each other, may come in compromise with the help of hybridization of composite laminates in which the high-stiffness material that is generally more expensive and heavier is used in the outer layers to provide enough rigidity and stiffness. The material which is used in the inner layers is low-stiffness, lighter and inexpensive.

Deflection, stress and natural frequencies are some supplementary aspects which have been investigated in hybrid laminates in a multi-objective optimization process. Maximizing natural frequencies especially the fundamental one is of critical importance in the design of laminates to decrease the risk of resonance caused by external excitations. Number, material and thickness of the surface and core layers as well as fiber orientations are the design variables in this process. However, in many engineering applications, it is reasonable to make use of standard layers with certain thicknesses and limited number of angles.

Single-objective maximization of the fundamental frequency for laminated plates was given by Bert [1, 2], Reiss and Ramachandran [3] and Grenestedt [4] using continuous design variables. The same design for cross-ply laminates was studied by Duffy and Adali [5] and for anisotropic laminates by Adali [6]. Minimum cost design of laminated plates undergoing free vibrations was investigated by Adali and Duffy [7]. Adali and Verijenco [8] discussed the optimum stacking sequence design of symmetric hybrid laminates undergoing free vibrations for fundamental frequency and frequency separation. Regarding multi-objective optimization, Spallino and Rizzo [9] presented the discrete optimization of laminated structures. Tahani et al. [10] optimized the fundamental frequency and cost in a multi-objective procedure using genetic algorithm (GA) and Kolahan et al. [11] also solved the same problem with the help of simulated annealing (SA).

In the present study, considering free vibrations of symmetric balanced hybrid laminates as the design constraint, weight and cost are optimized. The ant colony optimization (ACO) is the method employed. Ant colony optimization is a nature-inspired constructive based method which was first introduced by Dorigo in 1997 [12] and so far has been extensively applied on various types of combinatorial problems such as traveling salesperson problem (TSP) [12], quadratic assignment, vehicle routing [13] and job-shop scheduling (JSP) [14].

In the field of structural optimization, a few works being optimized by ACO have been reported. Camp et al. [15] studied the application of ACO for designing steel frames. Christodoulou [16] presented the optimal truss design using ACO. Kolahan et al. [17] also optimized a helical compression spring to achieve the minimum weight. As its application in structural optimization is a new topic of investigation, the results are compared to GA reported by Grosset et al. [18].

The remainder of this paper is organized as follows. In section 2, a brief description of free vibration analysis in laminated plates is presented. Ant colony optimization is introduced in section 3 and the optimization problem is defined in section 4. In section 5, multi-objective optimization is summarized and section 6 reports the numerical results. Finally, conclusions are given in section 7.

2. FREE VIBRATION ANALYSIS

Consider a simply supported symmetric hybrid laminated plate of length *a*, width *b* and thickness *h* in the *x*, *y* and *z* directions, respectively. Each of the material layers is of equal thickness *t* and idealized as a homogeneous orthotropic material. The total thickness of the laminate is equal to $h = N \times t$ with *N* being the total number of the layers.

The hybrid laminate is made up of N_i inner and N_O outer layers so that $N = N_i + N_o$. The governing equation of motion within the classical laminated plate theory for the described symmetric laminate is given by (see [19]):

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = \rho h \frac{\partial^2 w}{\partial t^2} \tag{1}$$

where *w* is the deflection in the *z* direction, *h* is the total thickness and ρ is the mass density averaged in the thickness direction which is given by:

$$\rho = h^{-1}\int_{-h/2}^{h/2} \rho^{(k)} dz = \frac{1}{N}\sum_{k=1}^{N} \rho^{(k)} \tag{2}$$

where $\rho^{(k)}$ denotes the mass density of material in the *k*th layer.

The bending stiffnesses D_{ij} in Eq. (1) are defined as:

$$D_{ij} = \sum_{k=1}^{N}\int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} z^2 dz \tag{3}$$

where $\overline{Q}_{ij}^{(k)}$ is the transformed reduced stiffness of the *k*th layer.

The boundary conditions for the simply supported plate are given by:

$$\begin{aligned} &w = 0,\ M_x = 0 \quad \text{at} \quad x = 0, a \\ &w = 0,\ M_y = 0 \quad \text{at} \quad y = 0, b \end{aligned} \tag{4}$$

where the moment resultants are defined as:

$$(M_x, M_y) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y)\, dz \tag{5}$$

It is shown by Nemeth [20] that in buckling problems, the terms D_{16} and D_{26} which demonstrate the bending-twisting interactions in composite laminates, can be safely neglected if the non-dimensional parameters

$$\gamma = D_{16}(D_{11}^3 D_{22})^{-1/4}, \quad \delta = D_{26}(D_{11} D_{22}^3)^{-1/4} \tag{6}$$

satisfy the constraints

$$\gamma \le 0.2 \quad , \quad \delta \le 0.2 \tag{7}$$

Because of the analogy between buckling and free vibration analysis, the same constraints are used to reduce the complexity of the problem.

Taking into account the governing equation (1) and the boundary conditions in (4), a general form of solution for *w* in the natural vibration mode (*m*,*n*) is presented as:

$$w(x, y, t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{i\omega_{mn} t} \tag{8}$$

where ω_{mn} is the natural frequency of the vibration mode (*m*,*n*) and $i = \sqrt{-1}$.

Substituting Eq. (8) into (1) yields:

$$\omega_{mn}^2 = \frac{\pi^4}{\rho h}\left(D_{11}\left(\frac{m}{a}\right)^4 + 2(D_{12} + 2D_{66})\left(\frac{m}{a}\right)^2 \left(\frac{n}{b}\right)^2 + D_{22}\left(\frac{n}{b}\right)^4 \right) \tag{9}$$

Different mode shapes are obtained by inserting different values of *m* and *n* where for the fundamental frequency, both are put equal to one.

3. ANT COLONY OPTIMIZATION

Real ants are biologically blind; however, they are capable of finding the shortest path from a food source to their nest without using any visual cues. This amazing capability is based on a simple fact. Ants secrete pheromone, a chemical substance, with a constant rate on the paths they march on and also are able to sense the intensity of pre-deposited pheromone in the environment. They also prefer to follow the paths with higher amount of pheromone but this tendency is not deterministic. Therefore in general, if more ants march on a certain path, more pheromone will be accumulated and the path will be even more desirable. This behavior is comprehensively shown in Fig. 1.

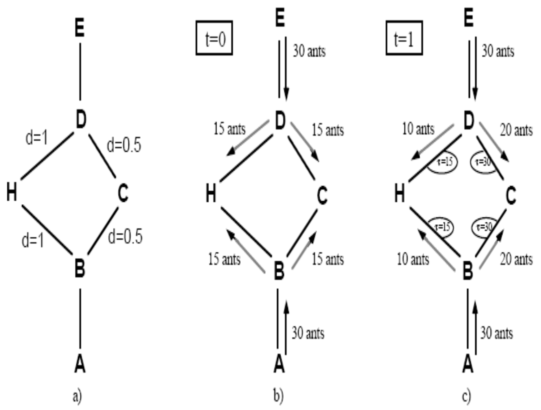

Figure 1. The behavior shown by ants when encountering an obstacle on their path [12]

Let's suppose that due to an obstacle between points A and E, two different paths namely ABCDE and ABHDE are identifiable. Two groups of 30 ants arrive at points D and B and they decide either going through BCD or BHD. As this is the start of path exploration, no pheromone exists on each of the paths and therefore there are no preferences. After a time step, as the path BCD is shorter and the rate of pheromone secretion is constant, more pheromone is accumulated on it and therefore it will be more desirable for next groups. The process continues till path BCD is so highlighted that practically path BHD looses all its preference.

The behavior is very simple and not even fully deterministic but effective enough as it is cooperative. All this behavior is simulated with a little difference in the ACO algorithm [12]. The artificial ants employed in ACO are not fully blind, i.e. they have general information about the search space, have a memory of the length of the path they have explored and also live in an environment where time is discrete so the decisions are made in a step-by-step procedure. To understand the ACO algorithm, knowing the application of it on the TSP problem is essential as the algorithm have three distinct operators first defined and best described based on this problem.

TSP problem is in fact a group of problems being one of the most distinguished challenges in the history of applied mathematics, making it a reliable benchmark for optimization methods. In this problem, there are *n* cities where finding the shortest tour including all cities being visited for just one time and ending in the first city is desired. The ACO algorithm employs *m* ants which are spread randomly on the cities. The ants start building their tour individually and come to the end of iterations altogether. Three basic rules called "state transition rule", "global updating rule", and "local updating rule" build the foundation of this algorithm.

3.1 State Transition Rule

Unlike other heuristic methods such as tabu search, genetic algorithm and simulated annealing where the coded solution candidate is built altogether and then evaluated, the ants construct the solution in a step-by-step procedure in the ACO. It means each ant should decide where to go for its next step by selecting among all unvisited candidate elements. The mechanism used in the ACO is a combination of directed greedy behavior and Rolette wheel known as state transition rule. The ant arrived at the city *i* chooses the next city among unvisited cities according to the following mechanism:

$$S = \begin{cases} \arg\max_{s} \left\{ [\tau(i,j)].[\eta(i,j)]^{\beta} \right\} & \text{if } q \le q_0 \\ s & \text{otherwise} \end{cases} \tag{10}$$

$$s = \begin{cases} \dfrac{[\tau(i,j)].[\eta(i,j)]^{\beta}}{\sum\limits_{u \,\in\, allowed\ u} [\tau(i,u)].[\eta(i,u)]^{\beta}} & \text{if } j \in allowed\ j \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where $\tau(i,j)$ is the amount of pheromone related to the path between cities *i* and *j*, and $\eta(i,j)$ is the heuristic function defined here as the inverse of distance between these two cities. The heuristic function is an operator in ACO which is defined according to the nature of the involved problem as it can be distance like in TSP problem or any other concepts such as cost or time. It has the rule of guiding and accelerating the convergence but is not vital to the concept of ACO. As can be seen, the total decision term is a combination of both pheromone and heuristic functions with the latter having a power of β.

The state transition rule consists of two sub-rules, while q and q_0 determine which one to be used. The constant parameter q_0 demonstrates the relative importance of sub-rules; however, q is a randomly generated number, uniformly distributed in domain [0,1].

If there comes $q \le q_0$ which is the case of exploitation, the city with the largest combination of pheromone and heuristic is chosen. Otherwise, the algorithm does not decide deterministically but only gives chances to the elements in proportion to their values as it is in a Rolette wheel; which means the city with the largest calculated term is not necessarily chosen. Thus, exploration of candidates with smaller function values is made feasible. In general, the ants act in a parallel manner. Their first elements of solution are assigned randomly and then to the end of constructing the solution, state transition rule is repeated.

3.2 Global Updating Rule

In ACO, the globally best ant which is the ant that has constructed the best solution from the beginning of the trial is allowed to deposit pheromone on its trail, even though no better tour is found in several consequent iterations. This rule which acts as positive feedback makes the search for the real best solution more directed. The rule is given by:

$$\tau(r,p) = (1-\alpha).\tau(r,p) + \alpha.\Delta\tau(r,p) \qquad (12)$$

$$\Delta\tau(r,p) = \frac{1}{F} = (L_{gb})^{-1} \qquad (13)$$

where α is a coefficient that acts to decrease the amount of pheromone inspired by evaporation in nature. The added amount of pheromone is obtained by inversing the objective function F which is the globally best tour length L_{gb} obtained from the beginning of the optimization.

3.3 Local Updating Rule

To avoid premature convergence and just like the natural phenomenon happening in nature due to evaporation, a local pheromone trail updating is performed on the value of pheromone related to the pair of cities just chosen by state transition rule:

$$\tau(r,p) = (1-\rho).\tau(r,p) + \rho.\Delta\tau(r,p) \qquad (14)$$

$$\Delta\tau(r,p) = \tau_0 \qquad (15)$$

It should be noted that the decay parameter ρ is chosen from domain $[0,1]$. Also in general, τ_0 which is the initial amount of pheromone, is calculated as the inverse of a rough estimate of the objective function multiplied by *n*, the problem dimension. In TSP, this approximation can be obtained by greedy methods.

3.4 Modifications in the ACO Concept

The TSP problem, on which the ACO was described, belongs to a group of problems in which the final solution is a string that includes all the elements of search space and therefore the design variables are not defined or even exist in the familiar way as is known in classical optimization. In contrary, most of the problems defined in the field of mechanical and structural engineering have distinct design variables that should be clearly and arithmetically stated at the end of optimization process. Consequently, the number of design variables should be defined in analogous to the concept of cities in TSP. This indicates that the pheromone function (or probably the heuristic function) that was defined between the elements of solution (cities in TSP) should now be assigned *on* them.

It also means that reminding the analogy to TSP, the allowable numerical values of discrete variables can be modeled as the zones of cities which no movement between the zones are permitted and only an ant can go from a zone to a zone in another city.

Defining the heuristic function brings another dimension of intricacy to the applying of ACO on most problems. This is because of the fact that defining a measurable and meaningful concept between any two elements of search space during the solution (before completing its construction), and meanwhile giving it a clear relevance to the objective function is generally very complex. Therefore, as done in this paper, and without damaging the overall effectiveness of ACO [12], this function is neglected.

In constrained optimization that is the case here, the solutions which do not satisfy the constraint should be omitted. Regarding the programming strategies, this can be done by detecting these solutions and omit them before evaluation or check their status and add a considerably big amount to them after that. The latter case which is known as penalty function approach is used here as involves simple modifications in the original code.

At the end of this section, it might be helpful to summarize all the above explanations in the following pseudo-code:

The ACO Pseudo-Algorithm:
Initial parameter setting
Repeat for each iteration
    Repeat for each ant
        Set the Initial point for each ant
        Repeat for (No. of design variables-1)
            Perform "the state transition rule"
            Perform "the local updating rule"
        Update the best global solution
    Perform "the global updating rule"
    Check the end condition
Report the best solution

4. Problem Description

The design problem here is the selection of the optimal stacking sequence to obtain the simultaneous minimization of the weight and cost of a rectangular laminated plate of length $a = 36$ in. and width $b = 30$ in., later converted to metric scale, subjected to a constraint on the first natural frequency having the lower bound of 25 Hz. This frequency is calculated based on the formulations presented in section 2.

In this problem, the concept of hybridization using a two-material composite in which high-stiffness and more expensive graphite-epoxy is used in the outer layers and inexpensive low-stiffness glass-epoxy in the inner layers is considered. This way besides providing suitable structural rigidity, cost reduction which is always a significant and worthy goal can be achieved.

The stiffness-to-weight ratio of graphite-epoxy is about four times higher than that of glass-epoxy, with $E_1/\rho = 345$ against $E_1/\rho = 87.5$. However it is also more expensive, with a cost per kilogram that is 8 times higher than that of glass-epoxy. If the first priority is weight, then graphite-epoxy will be preferred; while if cost is paramount the optimum laminate will obviously contain glass-epoxy plies. The design of this simple rectangular plate leads us to study the trade-off between these two objective functions.

The problem investigated here, as mentioned earlier is taken from [18] where the results are given using GA. No modifications in the problem are done in order to compare the reported results with the ones obtained by ACO.

The initial design is supposed to have 44 layers but this may vary by the algorithm in order to achieve the optimal design. The fiber orientation can take any value from a set of 19 angles ranging from 0° to 90° in steps of 5°. The laminate is considered symmetric and balanced. Being symmetric is a practical assumption which is of great advantage in problem simplification as only half of the laminate is needed for optimization. In addition, the requirement that the laminate be balanced can be easily enforced by using pairs of $\pm\theta$ layers. This supposition is taken in order to minimize shear-extension and bending-twisting effects. Although 0° plies and 90° layers do not need to come in pairs, they are treated like other angles due to programming necessities but with half the normal thickness to simulate a single ply.

5. MULTI-OBJECTIVE APPROACH

The purpose of multi-objective optimization is different from that of single-objective optimization. In the latter, the goal is to find *the best solution*, which is the design that minimizes (or maximizes) the objective function. In contrast, in multi-objective optimization there is no single solution that minimizes (maximizes) all the objective functions. Indeed, the objective functions often conflict, as a design that decreases one objective will increase another. The interaction between the objective functions gives rise to a set of compromise solutions called *Pareto set*. A solution belongs to the Pareto set if there is no other design such that all the objective functions are lower at the same time. The designer will then need to use additional information to prioritize the objective functions in order to choose between the elements of the Pareto set. In this paper, the Pareto set is generated by optimizing a convex combination of the two objectives, weight W and cost C for a series of values of the multiplier α as:

$$F = \alpha\, W + (1-\alpha)\, C \tag{16}$$

Several values of α are chosen successively and the combined objective function is minimized using a single-objective optimizer based on ant colony optimization. If the Pareto set is convex, this procedure yields points that belong to the Pareto set.

6. NUMERICAL RESULTS

The problem described in previous sections is solved by ant colony optimization code written in Matlab software and run on Pentium IV 2400 GHz CPU. The properties of glass-epoxy and graphite-epoxy laminates are taken from [21] and presented in Table 1.

Table 1. Glass-epoxy and graphite-epoxy mechanical properties

Parameters	Graphite-epoxy	Glass-epoxy
Longitudinal modulus (GPa)	137.9	43.4
Transverse Modulus (GPa)	8.96	8.89
In-plane shear modulus (GPa)	7.1	4.55
Poisson ratio	0.3	0.27
Material density (kg/m^3)	1587	1970
Layer thickness (mm)	0.127	0.127
Cost factor	8	1

The parameter setting employed in ACO code is also given in Table 2.

Table 2. ACO parameter settings

n	α	ρ	τ_0	q_0	Cycles
10	0.1	0.1	0.03	0.5	2000

In order to construct the Pareto front, the weighting factor α is varied from 0.0 to 1.0 for certain amounts and the objective function F is minimized for each state. Running the program for at least 5 times in order to make sure about the convergence, the results are obtained and compared to GA as summarized in Table 3. It is notable that with the intention of illustrating the material of each layer in the final stacking sequence notation, the graphite layers are shown by plain numbers while the glass layers are represented by underlined numbers.

In a general view, it can be easily detected that for the seven different values of α for which the objective function is minimized, the ACO and GA results are identical in 5 cases, where it seems both have come to the global optimums. In the other two cases, the ACO has outperformed GA. This definitely demonstrates that the ACO method can be considered a promising and powerful algorithm in comparison to the prominent GA which is known as the most flexible heuristic approach.

The second general issue is associated with orientation patterns obtained by GA and ACO where it seems that the GA has resulted in better arranged sequences with less diversion in angles. In fact, if the natural frequency was an objective function besides cost or weight, the sequence and value of angles would be much more important. In the case investigated here, the natural frequency is just a constraint that should be satisfied. Even if obtaining the stacking sequence with the lowest amount of objective function, satisfied constraint of the first natural frequency and also maximum amount of this frequency is considered in the problem, which seems is the case in [18], the difference between frequencies of the optimums is less than 2% which can be easily neglected and is not practically significant. In fact, the number of layers and the material of which the layers are made are the only two factors that play a role in minimization of the objective function which is the weighted combination of the cost and weight of the laminate.

The results related to situations with the weighting factor equal to 0.0 and 1.0 are the first two which are important to be explained. In the case of $\alpha = 0.0$, the problem is reduced to single-objective optimization for cost minimization. The code is expected to result in a laminate with the layers all made of glass plies. The result confirms the above deduction as a laminate with 42 layers of glass plies is obtained.

For $\alpha = 1.0$, the only active objective is the weight and consequently a laminate with all layers made of graphite-epoxy is expected. The results of both GA and ACO prove this assumption like the previous case with the optimum being a 22-ply laminate. It is obvious that graphite-epoxy is stiffer than glass-epoxy and can fulfill the requirement for the minimum value of the first natural frequency with less number of plies.

For the other values of the weighting factor, it is the interaction of objectives that form the optimum designs and thus the interpretations can not be straightforward as before; however, there are some general points to be noted. Without any implementation in the codes, both GA and ACO have achieved designs in which the layers made of graphite-epoxy have appeared in the outer layers and those made of glass-epoxy in the inner ones. This creates a sandwich-type composite where the structural function is assured by the stiff graphite layers, placed on the outside, where their contribution to the flexural properties of the laminate is maximal, while inner layers are merely used to increase the distance of the outer plies from the neutral plane and to reduce the total cost.

Contribution of layers with angles ranging from $\pm 40^\circ$ to $\pm 60^\circ$ is in order to maximize the first natural frequency of the plate. The appearance of 0° or 90° plies is due to a different reason. Although these plies do not contribute much to the frequency, it is advantageous to use them as unlike other angles, they do not come in pairs which saves unnecessary additional weight and cost. In addition, the 0° plies always come into view in the inner layers where they are the least damaging for the performance of the plate.

Figs. 2-4 show the convergence histories related to the optimization process done for weighting factors of $\alpha = 0.0$, $\alpha = 0.87$ and $\alpha = 1.0$, respectively. They are selected as the first and the third demonstrate the capabilities of ACO in single-objective and the second one in multi-objective optimization. It is easily identified that the ACO has converged very quickly in the first few iterations.

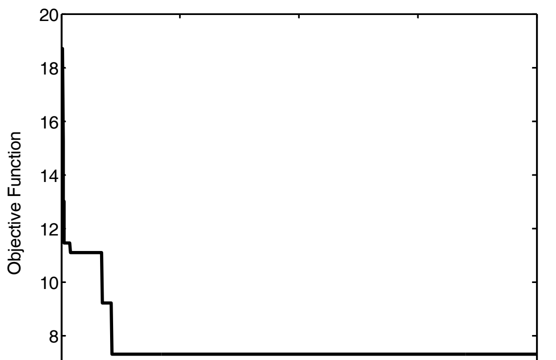

Figure 2. Convergence history for $\alpha = 0.0$

Unfortunately, no time history is reported in [18] to make the comparison of convergence rate possible. It can just be reported that the ACO has obtained the optimum design in a period of time ranging from 0.15 to 15 seconds which is reasonable regarding the size of the problem. To think of the importance of employing optimization methods especially robust heuristics such as the ACO in acquiring the optimum stacking sequence in such

Table 3. Optimum stacking sequence for minimum weight and cost

α	θ_{best} by ACO	θ_{best} by GA	ω by ACO (Hz)	ω by GA (Hz)	Weight by ACO	weight by GA	Cost by ACO	Cost by GA	Objective function by ACO	Objective function by GA
0.0	$[\pm\underline{55}/\pm\underline{50}/\pm\underline{65}/\underline{90}/\pm\underline{25}/\pm\underline{85}_2/\pm\underline{75}/\pm\underline{85}/\pm\underline{60}/\pm\underline{50}]_s$	$[\pm\underline{50}_{10}/\underline{0}]_s$	25.07	25.82	7.32	7.32	7.32	7.32	7.32	7.32
0.7	$[\pm 50/\pm\underline{50}_2/\pm\underline{30}/\pm\underline{65}/\pm\underline{40}/\pm\underline{70}/\pm\underline{40}]_s$	$[\pm 50/\pm\underline{50}_7]_s$	25.09	25.10	5.44	5.44	9.37	9.37	6.62	6.62
0.8	$[\pm 60/\pm 40/\pm\underline{45}/\pm\underline{85}/\pm\underline{65}/\pm\underline{85}/\pm\underline{5}]_s$	$[\pm 50_2/\pm\underline{50}_5]_s$	25.42	25.88	4.61	4.61	12.52	12.52	6.19	6.19
0.87	$[\pm 50/\pm 40/90/\pm\underline{50}/\pm\underline{65}/\pm\underline{75}/\pm\underline{45}]_s$	$[\pm 45_2/90/\pm\underline{50}_3/\pm\underline{80}]_s$	25.11	25.08	4.19	4.19	14.02	14.02	5.47	5.47
0.93	$[\pm 55_2/\pm 40_2/90/\pm 65]_s$	$[\pm 50_3/90/\pm\underline{50}_2/\underline{0}]_s$	25.02	25.38	3.09	3.71	24.72	17.47	4.60	4.67
0.96	$[\pm 55/\pm 50/\pm 55/\pm 60/0/\pm 5]_s$	$[\pm 50_4/\pm\underline{50}_2]_s$	25.07	26.07	3.09	3.64	24.72	19.37	3.95	4.27
1.0	$[\pm 55/\pm 50/\pm 55/90/\pm 55/\pm 35]_s$	$[\pm 50_5/0]_s$	25.10	25.14	3.09	3.09	24.72	24.72	3.09	3.09

problems, it can be noted that in the present problem, the average ratio of the total number of feasible stacking sequences needed to be evaluated by direct enumeration to the number of evaluated designs by the ACO is approximately of the order of 10^{13}.

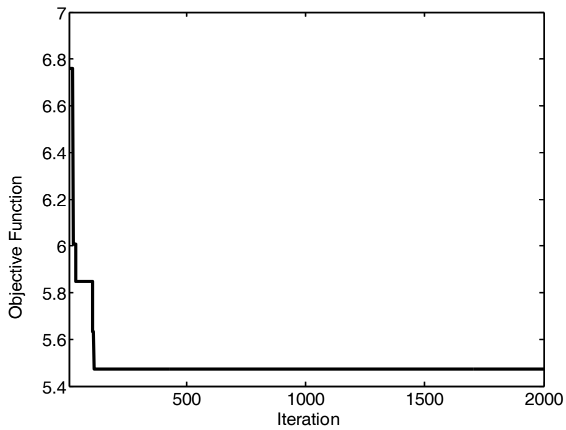

Figure 3. Convergence history for $\alpha = 0.87$

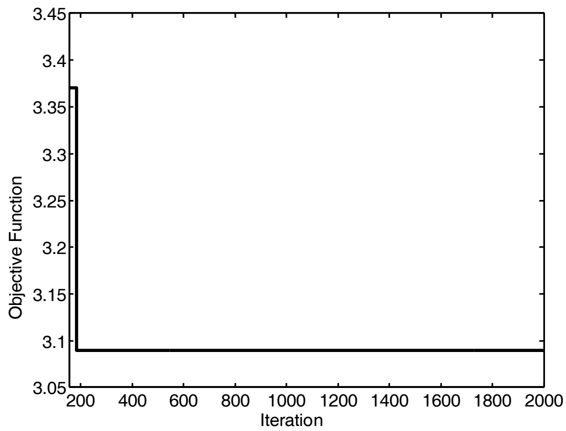

Figure 4. Convergence history for $\alpha = 1.0$

Finally, the Pareto front obtained by employing the ACO and using the weighted sum method is shown in Fig. 5. The solid line represents the set of solutions of the composite objective function for the different values of α. The various symbols show all the feasible designs that were generated during the search. The Pareto front is the set of all the non-dominated solutions, which corresponds to the lower envelope of all the design points in the weight/cost plane. This confirms the validity of our method for constructing the Pareto front.

The Pareto trade-off curve can be used to help the designer determine the optimal configuration for his problem. The final choice of the best design will depend on additional information that will enable him to assign priorities to the two objectives. There is no single best design and depending on the application that is considered, the choice will be different.

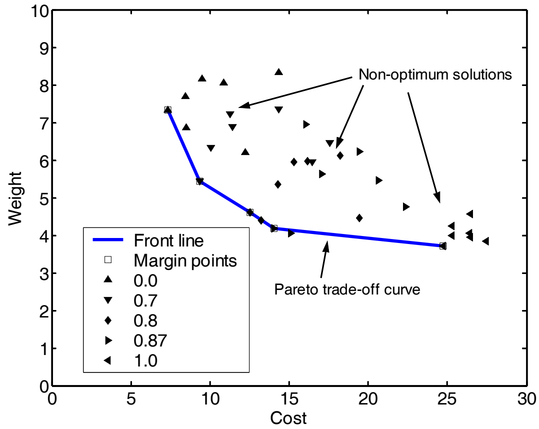

Figure 5. Pareto set of non-dominated solution obtained by the weighted sum method. The solid line shows the Pareto front. The symbols represent all the feasible solutions calculated by the ACO for the different values of the weighting factor.

7. CONCLUSIONS

The problem of obtaining minimum cost and weight in hybrid laminates was investigated. The laminate chosen with certain geometrical specifications was symmetric balanced and made of glass-epoxy and graphite-epoxy layers. The design variables were the number of layers made of glass or graphite fibers as well as the fiber orientations. The optimization process was constrained by the first natural frequency of the plate to be not less than a predefined value. The approach chosen for doing the multi-objective optimization was also the weighted sum method.

The ant colony optimization (ACO) was the method employed to solve this problem. The results were presented for different weighting factors and finally the Pareto front curve was constructed. The results were also compared to those obtained by the GA reported in the literature. This comparison not only confirmed the ACO results, but also showed that the ACO has outperformed GA in some cases or at least has resulted in identical designs. Considering the fact that GA is referred to as the most flexible and versatile heuristic method, it can be proposed that the ACO can perform as a robust and promising algorithm in different fields such as structural optimization and composites design.

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