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Ant Colony Optimization Applied to Discrete and Continuous Structural Problems

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Abstract
Ant colony optimization is described and applied on two structural benchmark problems. The problems, being the gear ratio optimization and the geometrical optimization of a composite leaf spring, are chosen of discrete and continuous domain nature to investigate the effectiveness of the algorithm dealing with both types. The results compared with the global optimum obtained by enumeration or with the results of genetic algorithm clearly demonstrate the capabilities of ACO as a promising method to be used in structural optimization.
Keywords: ant colony optimization, structural optimization, gear ratio, composite leaf spring

Introduction

Structural optimization is an important field of mechanical and aerospace engineering which deals with maximally utilizing the geometry and material of an initial design to obtain the optimum amount of objective functions. Weight is the most frequent aim of this process while the design should also satisfy all the structural, functional and manufacturing constraints. Classic methods were the only techniques to solve these problems for years; however metaheuristics such as tabu search (TS), genetic algorithm (GA) and simulated annealing (SA) have improved the quality of solutions in the last two decades. Ant colony optimization is a nature-inspired constructive based method which was first introduced by Dorigo in 1997 [1] and has been extensively applied on various types of combinatorial problems such as traveling salesperson problem (TSP) [1], quadratic assignment, vehicle routing and job-shop scheduling (JSP) [2].

In the field of structural optimization, a few works optimized by ACO mostly on simplified models have been reported. Camp et al. [3] studied the application of ACO for designing steel frames. Christodoulou [4] presented the optimal truss design using ACO. Here, it is attempted to apply it on new types of problems.

Ant colony optimization (ACO)

This algorithm is based on the nature of ants finding their paths by pheromone deposition. Ants usually select the path with more pheromone trail, which is the path that is passed by more ants. In this way, the shorter paths are more desirable and have stronger pheromone trail, because it takes shorter time to march and therefore, they are more frequently visited by ants. These behaviors are simulated by three rules in ACO which can be best applied on TSP problem where it deals with finding the shortest tour. Regarding the nature of problems presented here, the definition of rules is a bit different than those used in TSP-based formulation. Therefore, the design variables are presented by *i* and their divided search domains are shown by *j*. The sections of total solution are chosen in a constructive approach named as "state transition rule":

$$S = \begin{cases} \arg \max\limits_{u \in allowed\, u} \left\{ [\tau(i,j)].[\eta(i,j)]^{\beta} \right\} & if\ q \le q_0 \\ P(i,j) & otherwise \end{cases} \quad (1)$$

where $\tau(i, j)$ shows the amount of pheromone related to the *j*th element of variable *i*, and $\eta(i, j)$ is the heuristic function defined regarding the problem investigated. In this rule, *q* is a random number, and q_0 is a parameter set by the user ($0 \le q, q_0 \le 1$). If $q > q_0$, the next step is selected according to proportional distribution of probability function as in roulette wheels. An important factor in this process is the amount of q_0 which defines the range of randomness and determination of state transition rule. It is clear that the higher amounts of q_0 directs the algorithm towards deterministic decisions, while the lower amounts of it generates more randomness. To avoid stagnation of the algorithm and just the same as what happens in real world due to evaporation of pheromone, after selecting each job, the amount of pheromone level is changed by applying "the local updating rule". The third rule known as "the global updating rule" acts as a positive feedback and accumulates more pheromone around the best solution obtained so far. This process of next step evaluation and updating is repeated till the termination condition which is usually the maximum number of cycles is satisfied.

Benchmark 1: Gear ratio

The problem is taken from Kannan and Kramer [5] where they solved it by a classic method. There are four gears with possible number of teeth ranging from 12 to 60. The objective is to choose proper number of teeth for each gear to obtain a gear ratio as close to 1/6.931 as possible. The problem can be stated as a minimization problem as follows:

$$\text{Minimize } f(x)=\left[\frac{1}{6.931}-\frac{x_1\cdot x_2}{x_3\cdot x_4}\right]^2 \quad (2)$$

Subject to: $12 \le x_i \le 60$

The problem was solved with 10 ants and 10000 iterations which come to 100,000 function evaluations. The results are listed in Table 1 which shows that the algorithm has been able to find the global optimum. It has also successfully approached the global optimum with just 10,000 evaluations. It is notable that the 100,000 evaluations needed to find the global optimum is only about 1.9% of the total search space.

Table 1: ACO optimal results for gear ratio problem

Parameters	10,000 evaluations	100,000 evaluations
x_1	13	19
x_2	16	16
x_3	32	49
x_4	45	43
f(x)	2.7×10^{-8}	2.7×10^{-12}

Benchmark 2: Composite leaf spring

Leaf springs account for 10-20 % of vehicle unsprung weight where the reduction of this weight improves ride characteristics and decreases fuel consumption. Fiber reinforced plastics (FRPs) are proper candidates for replacing steel structures specifically leaf springs, where they have been commercially manufactured. In addition to material replacement that carries the concept of optimization in its nature, size optimization, if applied, can fulfill the capabilities of these advanced materials in weight minimization.

The design problem here is to assign center width and thickness of a mono leaf double tapered composite spring that was previously studied by Rajendran and Vijayarangan [6] with genetic algorithm. In their design, the cross-section area is kept constant but as demonstrated in Figure 1, the thickness decreases toward the leaf end while the width increases with the same taper ratio being constant at the eye for gripping.

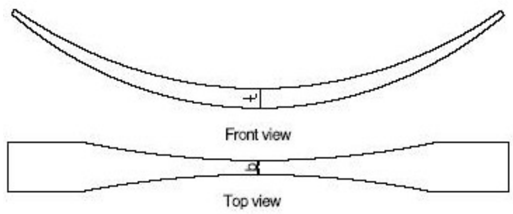

Figure 1: schematic of tapered mono leaf composite spring

The design requirements considering design load, maximum allowable vertical deflection, spring length in straight condition and spring rate are taken to be identical to that of steel leaf spring as stated in Table 2. The material properties and formulation of problem constraints are taken from [6] to allow comparing the results reported by GA and ACO:

$$S_b=\frac{1.5\,WL}{bt^2}\quad,\quad d=\frac{WL^3}{4Ebt^3}\quad,\quad F=\rho Lbt \quad (3)$$

where S_b is the bending stress, d is the vertical deflection, W is the load, L is the spring length, E is the modulus of elasticity and b and t are center width and thickness. The objective function is the weight of spring shown by F where parameter ρ states the density of the FRP structure.

The results presented in Table 3 clearly show that the weight of the spring has decreased from 2.95 kg in its initial design to 1.73 kg in the final design. Also it is shown that the ACO method has outperformed GA as the best GA result is 2.26 kg.

Table 2: Inputs for composite leaf spring optimization

Spring straight length (mm)	1220
Modulus of elasticity (GPa)	32.5
Material density (kg/m^3)	2600
Load (N)	4500
Bounds for allowable stress (MPa)	400-550
Bounds for maximum deflection (mm)	120-160
Bounds for width (mm)	20-50
Bounds for thickness	10-50

Table 3: Optimal design values of composite leaf spring

Parameter	ACO results	GA results
Width (mm)	20	28.48
Thickness (mm)	27.40	25.02
Maximum stress (MPa)	548	462.17
Maximum deflection (mm)	153	141.03
Weight (kg)	1.73	2.26

Conclusion

Ant colony optimization algorithm was applied on two structural problems with discrete and continuous design variables. It was shown that the ACO can be a robust method of optimization in such problems though the modeling process is more intricate in comparison with other metaheuristics and classic approaches.

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