Analysis of Interlaminar Stresses in General Cross-Ply Composite Laminates Subjected to Transient Vibration

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Abstract In this paper, within the displacement field of the first-order shear-thickness deformation theory (FSDT), transient vibrations of rectangular cross-ply composite plates are studied and dynamic interlaminar stresses are obtained. In the theoretical formulations the effects of all the rotational inertia terms are considered. Also the change in the plate thickness is taken into account due to its important role in the edge effects. The equations of motion are derived by using Hamilton's principle. It is assumed that the plates have two simply supported opposite edges and the remaining boundary conditions are arbitrary. The obtained equations are solved analytically using Levy's formulations, the orthogonality relation, and Laplace transform. The function of time is obtained using the results of free vibration and convolution integral. First, time responses are obtained for the case of transient vibration and then the interlaminar stresses are determined by integrating the three-dimensional local equations of motion and utilizing given boundary conditions. The accuracy and effectiveness of the present theory in describing the localized three-dimensional effects are demonstrated by comparing the results of the first-order theory with those obtained from the finite element method.

Key words cross-ply laminates, transient vibration, first-order shear-thickness deformation theory, interlaminar stresses

1. Introduction

Laminated composite plates are being increasingly used in aeronautical and aerospace industry as well as in other fields of modern technology. As an efficient use a good understanding of their structural and dynamical behavior and also a verified consideration of the deformation characteristics, stress distribution, natural frequencies, and buckling loads under various load conditions are expected. Several representative researchers have surveyed the development of the study on free vibrations of composite laminated plates but the studies on forced vibration of composite plates are very limited. Ribeiro [1] studied the large amplitude, geometrically nonlinear periodic vibrations of shear deformable composite laminated plates and found the mode shapes of vibrations. Onkar and Yadav [2], using basic analytical techniques, studied the nonlinear random vibration of a simply supported cross-ply laminated composite plate. To the extent of the author's knowledge, no work has been reported for analysis of interlaminar stresses of laminated plates under forced vibrations. In this study, forced vibrations of multilayer composite plates are investigated based on first-order shear-thickness deformation theory. In the theoretical formulations the effects of all the rotational inertia terms are considered. Also the change in the plate thickness is taken into account due to its important role in the edge effects. It is assumed that the plates have two simply supported opposite edges and there is no limitation for the remaining boundary conditions. Displacement components are separated as products of position and time functions. The function of position components of displacement is obtained in the form of Fourier series. Equations of free vibration of the plate are solved analytically using the state-space approach. The function of time is obtained using the results of free vibration and convolution integral.

2. Mathematical formulations

It is intended here to determine the interlaminar stresses in a general cross-ply laminate subjected to transient vibration. The formulation is restricted to linear elastic material behavior and small strain and displacements.

In order to approximate the three-dimensional elasticity problem to a two-dimensional plate problem, the displacement components $u_1(x, y, z, t)$, $u_2(x, y, z, t)$, and $u_3(x, y, z, t)$ at any point in the plate space are expanded in a Taylor's series in terms of thickness coordinate. The elasticity solution indicates that the transverse shear stress vary parabolically through the plate thickness. This requires the use of shear correction factors for theories with constant transverse shear stresses. The displacement field which satisfies the above criteria may be assumed in the form

$$\begin{aligned} &u_1(x,y,z,t) = u(x,y,t) + z\psi_x(x,y,t)\,, \quad u_2(x,y,z,t) = v(x,y,t) + z\psi_y(x,y,t) \\ &u_3(x,y,z,t) = w(x,y,t) + z\psi_z(x,y,t) \end{aligned} \tag{1}$$

where u_1, u_2, and u_3 are the displacement components in the *x*, *y*, and *z* directions respectively, *u* and *v* are the in-plane displacements and *w* is the transverse displacement of a point (*x*, *y*) on the middle plane. The functions ψ_x and ψ_y are the rotations of a normal transverse to the middle plane about *y*- and *x*-axes respectively and ψ_z is the thickness change parameter.

By substituting the displacement field in (1) into the strain-displacement relations [3] of elasticity, the following results will be obtained

$$\varepsilon_x = \frac{\partial u_1}{\partial x} = \frac{\partial u}{\partial x} + z\frac{\partial \psi_x}{\partial x}, \quad \varepsilon_y = \frac{\partial u_2}{\partial y} = \frac{\partial v}{\partial y} + z\frac{\partial \psi_y}{\partial y}, \quad \varepsilon_z = \frac{\partial u_3}{\partial z} = \psi_z$$
$$\gamma_{yz} = \frac{\partial u_2}{\partial z} + \frac{\partial u_3}{\partial y} = \psi_y + \frac{\partial w}{\partial y} + z\frac{\partial \psi_z}{\partial y}, \quad \gamma_{xz} = \frac{\partial u_1}{\partial z} + \frac{\partial u_3}{\partial x} = \psi_x + \frac{\partial w}{\partial x} + z\frac{\partial \psi_z}{\partial x} \tag{2}$$
$$\gamma_{xy} = \frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial x} = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + z\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right)$$

The displacement field in Eqs. (1) can be used to drive the governing equation of motion by means of Hamilton's principal. The Hamilton principle for an elastic body is [4]

$$\int_{t_1}^{t_2} (\delta U + \delta V - \delta T)dt = 0 \tag{3}$$

where δU is the variation of the total strain energy, δV is the variation of the potential energy of the applied forces on the external surfaces of the plate, and δT is the variation of the total kinetic energy. Using Eq. (3) the governing equations of motions (Euler-Lagrange equations) are obtained as follows

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1\ddot{u} + I_2\ddot{\psi}_x, \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_1\ddot{v} + I_2\ddot{\psi}_y, \quad \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = I_2\ddot{v} + I_3\ddot{\psi}_y$$
$$\frac{\partial R_x}{\partial x} + \frac{\partial R_{xy}}{\partial y} - N_z = I_2\ddot{w} + I_3\ddot{\psi}_z, \quad \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q(x,y,t) = I_1\ddot{w} + I_2\ddot{\psi}_z, \quad \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2\ddot{u} + I_3\ddot{\psi}_x \tag{4}$$

where

$$\left(N_x, N_y, N_z, N_{xy}\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \sigma_z, \sigma_{xy}\right)dz, \quad \left(M_x, M_y, M_{xy}\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \sigma_{xy}\right)zdz$$
$$\left(Q_x, Q_y\right) = \int_{-h/2}^{h/2} \left(\sigma_{xz}, \sigma_{yz}\right)dz, \quad \left(R_x, R_y\right) = \int_{-h/2}^{h/2} \left(\sigma_{xz}, \sigma_{yz}\right)zdz, \quad \left(I_1, I_2, I_3\right) = \int_{-h/2}^{h/2} \rho\left(1, z, z^2\right)dz \tag{5}$$

In Eqs. (5) N_x, N_y, N_z, N_{xy}, M_x, M_y, and M_{xy} are the stress resultants and Q_x and Q_y are the transverse shear force resultants, and I_1, I_2, and I_3 are the corresponding mass terms. The linear constitutive relations for the *k*th orthotropic lamina, with fiber orientations of 0° and 90° only, with respect to the laminate coordinate axes are given by [5]

$$\{\sigma\}^{(k)} = \left[\overline{C}\right]^{(k)} \{\varepsilon\}^{(c)} \tag{6}$$

where the matrix $\left[\overline{C}\right]$ is called the off-axis stiffness matrix. By substituting Eqs. (2) into Eq. (6) and the subsequent results into Eqs. (4), the governing (i.e. displacement) equations of motion are obtained.

3. Analytical solutions

Here the exact solution of Eqs. (4) for cross-ply rectangular plates are developed. The process of solving the governing differential equations consists of Levy's formulations [5]. Levy's solution exists when at least two opposite edges of the plate have simple supports. To this end, it is assumed here that the edges of the plate (for the case of cross-ply laminate) at $x=0$ and $x=a$ have the following boundary conditions

$$N_x = v = M_x = \psi_y = \psi_z = w = 0 \tag{7}$$

It is noted that the boundary conditions in (7) will identically be satisfied if the following expressions for the displacement components are assumed

$$\begin{aligned}
u(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} U_{mn}(y)\cos\alpha_m x\,\Omega_n(t), & v(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} V_{mn}(y)\sin\alpha_m x\,\Omega_n(t) \\
w(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}(y)\sin\alpha_m x\,\Omega_n(t), & \psi_x(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Psi_{xmn}(y)\cos\alpha_m x\,\Omega_n(t) \\
\psi_y(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Psi_{ymn}(y)\sin\alpha_m x\,\Omega_n(t), & \psi_z(x,y,t) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \Psi_{zmn}(y)\sin\alpha_m x\,\Omega_n(t)
\end{aligned} \tag{8}$$

where $\alpha_m = m\pi/a$ with m being the Fourier integer and U_{mn}, V_{mn}, … are the eigenfunctions in free vibration analysis. The function $\Omega_n(t)$ is known as the generalized displacement coordinate and can be obtained by using the orthogonality relation as below

$$\Omega_n(t) = \Omega_n(0)\cos\omega_n t + \frac{1}{\omega_n}\dot{\Omega}_n(0)\sin\omega_n t + \frac{1}{\omega_n}\int_0^t \frac{Q_{mn}(\tau)}{N_{mn}}\sin\omega_n(t-\tau)\,d\tau \tag{9}$$

where N_{mn} is the generalized mass term and is defined as follows

$$N_{mn} = \int_{-b}^{b}\left[I_1\left(U_{mn}^2 + V_{mn}^2 + W_{mn}^2\right) + 2I_2\left(\Psi_{xmn}U_{mn} + \Psi_{ymn}V_{mn} + \Psi_{zmn}W_{mn}\right) + I_3\left(\Psi_{xmn}^2 + \Psi_{ymn}^2 + \Psi_{zmn}^2\right)\right]dy \tag{10}$$

4. Numerical results and discussion

In the present paper, several numerical examples are studied for symmetric and antisymmetric cross-ply laminates subjected to transient vibration. The laminates have length $a=0.1$ m, width b, and thickness $h=0.01$ m with aspect ratio $a/b=1$. Each lamina is assumed to be of the same thickness $h_k = h/N$, where N is the number of laminae and the following lamina properties are used in all numerical examples, $E_1/E_2 =$ open, $E_2 = E_3$, $G_{12} = G_{13} = 0.6E_2$, $G_{23} = 0.5E_2$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$. It is assumed that the plate is subjected to a uniform transverse pressure during the time $t=0.2$ ms.

Fig. 1 shows the distributions of interlaminar shear stress σ_{yz} through the y direction in an antisymmetric $[0/90]_2$ SSSC cross-ply laminate subjected to pressure 0.1 kPa. Fig 2 presents the time history of interlaminar shear stress σ_{yz} of a [0/90/0] SSSC laminate at $x=a/3$, $z=h/3$, and various values of the width coordinate (y). The applied pressure is 0.2 kPa. All stress distributions in Figs. 1 and 2 are compared with the finite element analysis (FEA) and excellent agreements between the FSDT and FEA are found.

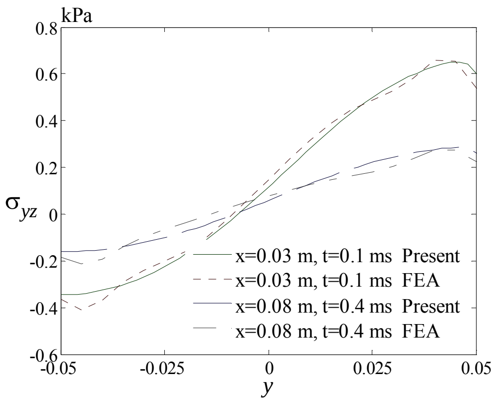

Fig. 1 Distributions of interlaminar shear stress σ_{yz} through the y direction of a $[0/90]_2$ SSSC cross-ply laminate

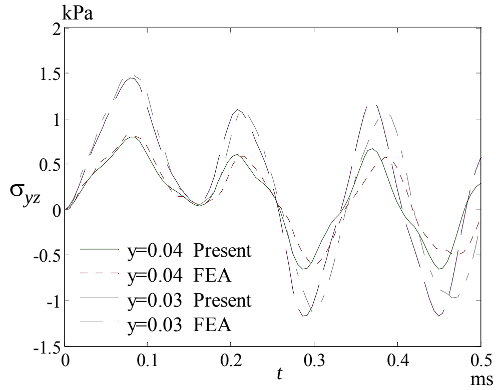

Fig. 2 Time history of the interlaminar shear stress σ_{yz} at $z=h/3$ of a [0/90/0] SSSC cross-ply laminate

Fig. 3 demonstrates the time history of interlaminar normal stress σ_z of symmetric SCSC cross-ply laminates at $x=a/2$, $y=b/4$, and $z=h/3$ for two different configurations. It confirms that the [0/90/0] laminate has greater maximum interlaminar normal stress σ_z than that [90/0/90] laminate when the applied pressure is 0.5 kPa. Fig. 4 presents the distributions of interlaminar normal stress σ_z of fully simply supported cross-ply laminates subjected to pressure 0.2 kPa at $x=a/3$, $z=h/3$, and various values of the a/h ratio. It is seen that as the a/h ratio is increased, the numerical value of normal interlaminar stress σ_z is also increased.

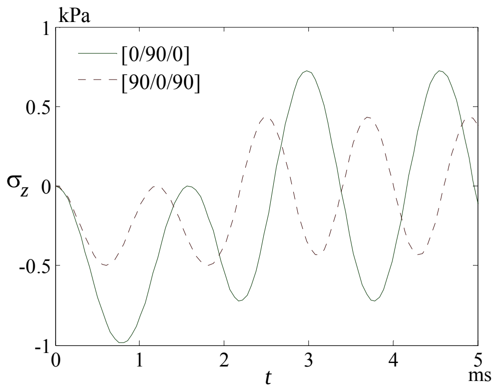

Fig. 3 Time history of interlaminar normal stress σ_z of SCSC cross-ply laminates

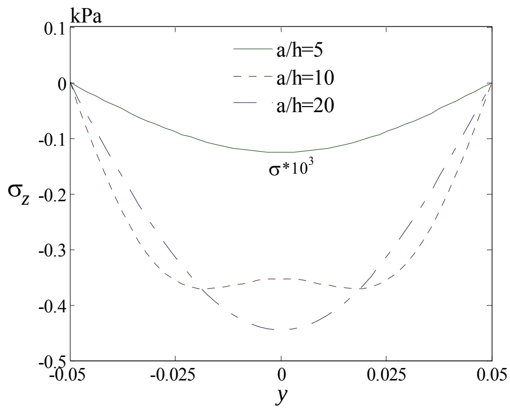

Fig. 4 Distributions of interlaminar normal stress σ_z of fully simply supported [0/90/0] cross-ply laminates

Fig. 5 illustrates the time history of in-plane normal stress σ_x at $y=b/4$, $z=h/3$, and various values of the length coordinate (x) of a fully simply supported [0/90] antisymmetric laminate subjected to pressure 0.5 kPa. The magnitude of σ_x is naturally increasing when approaching the middle of the plate. Fig. 6 presents the time history of interlaminar normal stress σ_z at $x=a/10$, $y=b/4$, $z=h/3$, and various values of the E_1/E_2 ratio. As the E_1/E_2 ratio is increased, the numerical value of interlaminar normal stress σ_z is decreased.

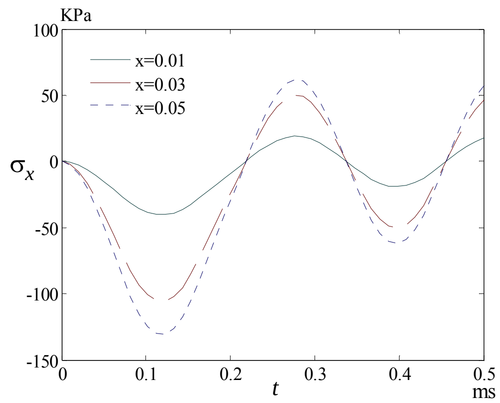

Fig. 5 Time history of in-plane normal stress σ_x for fully simply supported [0/90] laminate

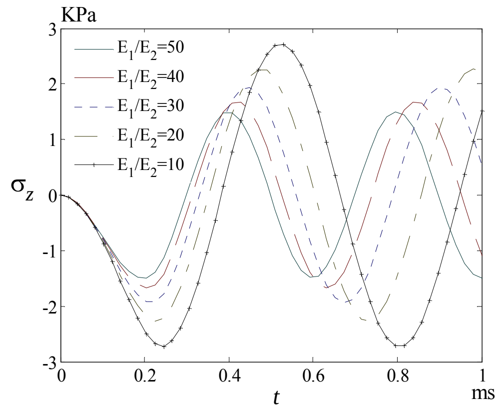

Fig. 6 Time history of interlaminar normal stress σ_z for SFSF [90/0/90] laminate

5. Conclusions

In this paper, an analytical method is developed to calculate the response and distribution of dynamic interlaminar stresses in composite laminated plates with two opposite simply supported edges, subjected to transient vibration. The equations of motion are derived by using Hamilton's principle. The obtained equations are solved analytically using Levy's formulations, the orthogonality relation, and Laplace transform. It is found that the present results have excellent agreements with those obtained by using finite element method. These close agreements verify the accuracy of the first-order shear-thickness theory which is used in this case study.

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