A Heuristic Algorithm Approach for Free Vibration Optimization of Functionally Graded Adhesive Single Lap Joints

FARHAD KOLAHAN, MASOUD TAHANI, GOLNAZ SADAT TADAYYON
Department of Mechanical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, Mashhad, P.O. Box 91775-1111, Iran: kolahan@um.ac.ir

Abstract The main objective of this paper is to establish the relationship between effective free vibration parameters of an adhesive functionally graded single lap joints and their design variables. This is done by deriving mathematical models which are developed using finite element method (FEM) and design of experiments (DOE) approach. These mathematical models relate the design variables (adhesive thickness, adherends thickness, overlap length and compositional gradient exponent of FGM plates) to the free vibration response parameters (natural frequency and corresponding modal strain energy). In the next stage, to maximize first natural frequency and minimize modal strain energy, simulated annealing (SA) algorithm is used to determine the best design variables. To investigate the performance of proposed procedure, the results of heuristic algorithm are compared with the global optimum obtained by complete enumeration. This comparison shows that the proposed optimization algorithm can efficiently solve such complicated problems. The evaluations of FEM outputs and mathematical models also reveal that the adhesive thickness has a negligible effect on natural frequency and modal strain energy, while other design variables have more significant effects on these design criteria.

Key words adhesive single lap joint, functionally graded materials, free vibration, optimization

1. Introduction

The adhesive joints have increasingly become popular in various industrial applications due to their superior characteristics such as lighter weights and uniform stress distributions [1]. On the other hand, in laminated composite structures, large interlaminar stresses are usually developed due to the differences between material properties of laminate constituents. To overcome this shortcoming, functionally graded materials (FGMs) have been developed. Generally, a FGM is a composite consisting of two or more phases; typically, a ceramic for thermal characteristics and a metal for mechanical properties [2]. Many studies exist on the various aspects of FGM including vibration, elastic, plastic and thermal analysis of such materials. In the field of vibration analysis, Loy et al. [3] investigated the natural frequency of simply supported FGM cylindrical shells. In their study, the derived governing equations based on the classical shell theory assumption are solved using Rayleigh-Ritz method. Vel and Batra [4] presented a three-dimensional exact solution for free and forced vibrations of simply supported functionally graded rectangular plates which have a good agreement with the results from the first order shear deformation theory. Kim [5] developed a theoretical method to investigate the vibration characteristics of initially stressed of FGM rectangular plates made up of metal and ceramic in a thermal environment. They assumed the temperature to be constant in the plane of the plate and to vary in the thickness direction only. Woo et al. [6] proposed an analytical solution for the nonlinear free vibration behavior of plates made of functionally graded materials using the von Karman theory and showed the importance of nonlinear coupling effects on the vibration response of FGM plates.

Optimization is one of the most important issues in designing different structures. Although there is a rich body of research in the area of stress and vibration analysis, there are very few studies on optimal design of FGM. To the best of our knowledge, the only optimization study on FGM structures under free vibration was done by Gunes et al. [7]. They carried out three-dimensional free vibration and stress analyses on an adhesively bonded functionally graded single lap joint (SLJ) using both finite element and artificial neural network (ANN) methods. In their studies, the FEM was employed to generate several sets of design outputs. These results were then used to train ANN procedure which, in turn, predicts free vibration parameters of the FGM structure under study. The training season for ANN, however, required a large amount of CPU time, and therefore was inefficient. They also employed a genetic algorithm procedure to determine proper configuration of their ANN.

Because of the lack of investigations on free vibration optimization of FGM, in this paper an optimization study is made towards free vibration of functionally graded single lap joints. In this regard, the first natural frequency and corresponding modal strain energy values of adhesively functionally graded single lap joints are determined by applying finite element method. The design variables considered here include geometry factors and mechanical properties of FGM plates. To generate required data for mathematical models, design of experiment (DOE) procedure has been used. Then, in order to relate important design variables to free vibration response parameters, various regression functions have been fitted on these data and the best set of regression models is selected. By using these models, there would be no need to run additional FEM analysis and therefore large CPU times may be saved. Finally, using a simulated annealing algorithm, the proposed models have been solved to minimize a weighted objective function consisting of free vibration response parameters.

2. Problem statement

Free vibration of FGM single lap joint structures is mainly affected by geometric characteristics and mechanical properties of materials. The important design variables in a functionally graded single lap joint are adherends thickness (*t*), adhesive thickness (*h*), overlap length (*C*) and volume fraction distribution of components in FGM (see Figure 1).

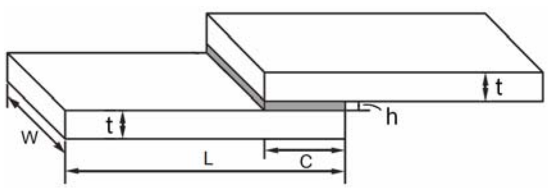

Fig. 1 The adhesive single lap joint

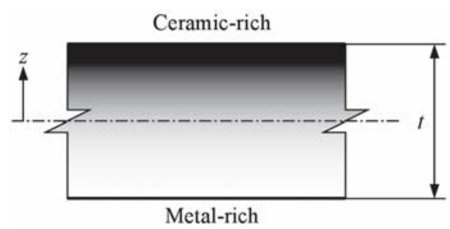

Fig. 2 The functionally graded adherend

One of the most important issues in functionally graded materials is their mechanical properties. Figure 2 illustrates a possible composition distribution of metal-ceramic functionally graded materials, where the metal volume fraction V_m varies from 100% at the bottom surface to 0% at the top one. Among many proposed functions for defining volume fractions, the most common equations are as follows [7]:

$$V_m(z) = (1 - \frac{z}{t})^n \quad (1)$$

$$V_C(z) = 1 - V_m(z) \quad (2)$$

In the above formula, *n* is a non-negative real number representing the compositional gradient exponent of FGM. The mechanical properties of the adherends are determined by a linear rule of mixtures and the constituents' material properties given in Table 1 [2]:

Table 1 Mechanical properties of adhesive single-lap joint components.

SLJ components		Mechanical properties		
	SLJ constituents	Young's modulus, E (GPa)	Poisson's ratio, υ	Density, ρ (kg/m^3)
FGM adherends	Ni (Metal)	199.5	0.3	8908
	Al_2O_3 (Ceramic)	393	0.25	3900
Adhesive	Epoxy	4.39	0.34	2500

The adhesive joints are usually designed to have a high strength with maximum possible first bending natural frequency and minimum corresponding modal strain energy [7]. As pointed out, in this paper free vibration optimization analysis of a FGM adhesive single lap joint structure is performed in terms of both geometry and mechanical properties.

3. Model development and optimization procedure

To generate required data for model development, a taguchi matrix with minimum trial runs has been employed. The selected taguchi matrix has four-factor five-level variables resulting in 18 experimental runs. The ranges of input data for the experiments are: $0.1 \le h \le 0.5$, $0.5 \le t \le 5$, $10 \le C \le 50$ and $0.1 \le n \le 10$. Also constant length (L) and width (W) of adherends are considered to be 160 and 20 mm respectively. Using the FEM results of these 18 trial runs and regression analysis, different mathematical models have been developed to relate the design variables to the free vibration parameters (first natural frequency and corresponding modal strain energy). Analysis of Variance (ANOVA) technique has shown that logarithmic models provide the best fit to our data. Figure 4 illustrates that there are excellent agreements between the results of mathematical model and those of FEM analysis, for both natural frequency and modal strain energy. The proposed logarithmic models are presented below:

$$\omega = \exp(2.082 + 0.086 \ln n + 0.152 \ln C + 1.002 \ln t - 0.001 \ln h) \qquad (3)$$

$$U = \exp(7.145 + 0.173 \ln n + 0.305 \ln C + 2.003 \ln t - 0.001 \ln h) \qquad (4)$$

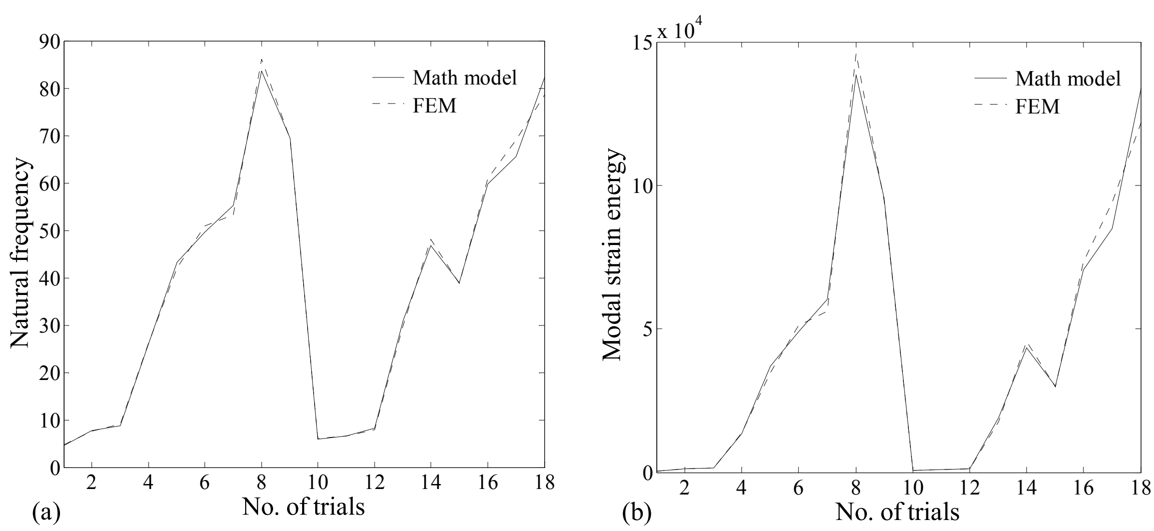

Fig. 4 Comparison between the results of mathematical models and FEM analysis for all trial runs, (a) Natural frequency, (b) Modal strain energy.

The above models can predict the response parameters for any given set of design variables. However, in most real applications, it is required to determine the design variables in such a way that the response parameters are optimized. This calls for optimization of the above models to obtain the best set of design variables. The objective is to determine design variables so that the following weighted mini-max objective function is minimized:

$$f = A \times U - B \times \omega \tag{5}$$

Simulated annealing (SA) algorithm, first proposed by Kirkpartick [9], is a powerful optimization technique suitable for solving large-sized complicated problems. This method is based annealing process, in which the temperature of a molten metal is lowered slowly to reach the solid state. In SA, the optimization procedure begins by generating an initial solution at random. At each stage, a small random change is made to the current solution. Then the objective function value is calculated and compared with that of current solution. A move is made to the new solution if it has a better value. A non-improving solution is also accepted with the probability based on Boltzmann distribution. This helps the algorithm jump out of local minima. The details of this method and its applications are well documented in related literature [e.g.8, 9].

4. Results and discussions

In order to evaluate the effects of the relative importance of the first natural frequency against modal strain energy on the design variables, in the optimization process, *B* is set to unity and *A* is given different values (see Eq. (5)). The algorithm aims at determining the design variables so that the minimum value for the weighted objective function given in Eq. 5 is obtained. To investigate the performance of the proposed solution procedure, in Table 2 the results of 5 sample runs of SA algorithm are compared to the global optima obtained by complete enumeration (CE) of entire solution space.

Table 2: Optimum design variables for different coefficients of A in Eq. (5).

$A(\times 10^{-4})$	f		n		C		t		h	
	CE	SA	CE	SA	CE	SA	CE	SA	CE	SA
2	-57.560	-57.537	10	9.9	50	50	5	5	0.5	0.49
2.9	-43.524	-43.520	7.4	7.9	50	49	5	5	0.5	0.47
3	-42.089	-42.087	5	4.8	50	50	5	5	0.5	0.48
5	-25.381	-25.370	0.1	0.1	17	18	5	4.9	0.5	0.39
7	-18.134	-18.129	0.1	0.1	10	12	3.9	3.8	0.5	0.5

These results illustrate that the proposed procedure can be efficiently used to determine optimal design variables for any set of response parameters weights.

5. Conclusions

In this paper, free vibration characteristics of an adhesive functionally graded single lap joint are modeled and an optimum design procedure for such structures is developed. The good agreement between the results of mathematical models and those of FEM analysis proves the effectiveness of the proposed method. With respect to the outputs from both mathematical models and FEM results, it can be observed that the most important variable is adherend thickness; while adhesive thickness has a negligible effect on the free vibration characteristics. The results also demonstrate that the optimal design parameters may vary significantly if the relative importance (A and B) of the first natural frequency and corresponding modal strain energy is changed. To investigate the performance of proposed optimization procedure, the results of simulated annealing algorithm are compared to global optima obtained by complete enumeration of all possible solutions. This comparison shows the proposed optimization procedure is quite capable of solving such complicated problems effectively and efficiently.

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