Engineering with Computers 7, 199–208 (1991)

Engineering with Computers

Design of an Optimal Damper to Minimize the Vibration of Machine Tool Structures Subject to Random Excitation

Y.C. Shin
Purdue University, West Lafayette, IN, USA

K.W. Wang
The Pennsylvania State University, University Park, PA, USA

Abstract. The vibration of machine tools during machining adversely affects machining accuracy and tool life, and therefore must be minimized. The cutting forces for stable turning are generally known to be random, and hence excite all the resonance modes. Of all these modes, those that generate relative motions between a cutting tool and a workpiece are of concern. This paper presents a new approach for designing an optimal damper to minimize the relative vibration between the cutting tool and workpiece during stable machining. An approximate normal mode method is employed to calculate the response of a machine tool system with nonproportional damping subject to random excitation. The major advantage of this method is that it reduces the amount of computation greatly for higher-order systems when responses have to be calculated repeatedly in the process of optimization. An optimal design procedure is presented based on a representative lumped parameter model that can be constructed by using existing experimental or analytical techniques. The two-step optimization procedure based on the modified pattern search and univariate search effectively leads the numerical solution to the global minimun irrespectively of initial values even under the existence of many local minima.

1 Introduction

In machining operation, it is known that machine tool structure and cutting dynamics interact as a closed loop system as shown in Fig. 1. The cutting forces generated during the cutting process as well as external excitations will create undesirable vibratory motions of various structural components. In particular, the relative motion between the cutting tool and the workpiece creates undulation on the machined surface, and hence adversely affects the surface accuracy. Furthermore, if the amplitude of the vibration grows substantially, this undulation becomes the source of oscillatory force in the subsequent pass, which again excites the cutting tool more and eventually leads to an unstable situation. This type of vibration is called "regenerative mechanism" and occurs due to the closed loop nature of machining operations. One of the major concerns in designing a machine tool structure is, therefore, reducing the relative amplitude of vibration between the tool and the workpiece. If a machine tool is extremely rigid or the excitation frequency is below its first resonance frequency, the entire system will undergo a rigid body motion with no relative vibration between the tool and the workpiece. Thus, efforts are made to maximize the stiffness of a machine tool structure during design and construction. However, several functional requirements to be satisfied within a limited space and the cost involved in building a very rigid system limit the attainable stiffness of the system, especially that of the cutting tool. Therefore, a certain amount of vibration is unavoidable during operation.

In the past many analytical methods have been developed for the optimization of the machine tool structural response [1–5]. These approaches, however, have mainly dealt with the structural motion of a machine tool itself, not the relative vibration between a cutting tool and a workpiece. Since the cutting tool or workpiece often is the most flexible part of an entire machine tool system, optimizing the rigidity of other structural components might have little effects on reducing the relative vibration at the cutting point.

There are some unique characteristics and difficulties one needs to consider in addressing the underlying problem. Cutting forces in stable machining are known to be random in nature [6,7] and will excite all the resonance modes. This excludes the possibility of using any conventional approaches to tune the structural response based on any particular frequency or a narrow frequency range because these techniques lead to the shifting of natural fre-

Offprint requests: Y.C. Shin, School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907, USA.

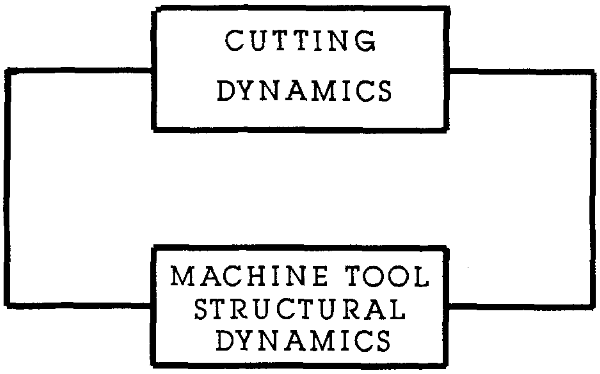

Fig. 1. Machining dynamics as a closed loop system.

quencies away from the specified frequency or frequency range and shifting natural frequencies is not very useful when excitation is random. Instead, vibration amplitudes must be suppressed over the entire frequency range of operation. If a numerical technique such as the finite element method is adopted for optimization, the computation time required to calculate the responses over the entire frequency range will increase exponentially with increasing order of the system. Also, cross couplings among various components will make the system nonproportionally damped [8,9]. The commonly used techniques based on normal modes would not be applicable. Moreover, numerical treatment of the system response becomes a lot more cumbersome when damping is not proportional because mode shapes are represented in complex numbers.

A new approach is presented in this paper based on a small damper attached to the cutting tool or the workpiece holder. After a discrete model with general damping is constructed, the parameters of the damper are tuned such that the power of the amplitude spectrum is minimized within a prescribed frequency range.

2 Fundamental Equations

A mechanical system can be described by a lumped parameter model. Since an actual machine tool structure cannot be regarded as the system with proportional damping, general viscous damping must be used to describe the behavior of the system. The matrix form of equations of motion for a system with general viscous damping can be represented by

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \tag{1}$$

When damping is not proportional, the damping matrix cannot be diagonalized by using real eigenvectors or normal modes. Instead, the system will behave in a complicated manner with complex mode shapes. The method to determine these complex modes was developed by Foss [10]. For a stable, damped system the eigenvalues will be obtained as complex conjugate pairs with corresponding complex conjugate columns representing mode shapes:

$$\lambda_k = \mu_k + i\nu_k$$

$$\lambda_k^* = \mu_k - i\nu_k \tag{2}$$

$$[\Phi] = \begin{bmatrix} [\Psi] & [\Psi^*] \\ [\Psi][\Delta] & [\Psi^*][\Delta^*] \end{bmatrix} \tag{3}$$

Using the orthogonality principle, the equations of motion given in Eq. (1) are uncoupled into $2n$ equations of the first order, which are represented by

$$q_i(s) = \frac{\{\Psi_i\}^T\{F(s)\}}{a_i s + b_i} \tag{4}$$

The total response of the system can be obtained by applying the superposition principle:

$$\{X(s)\} = \sum_{k=1}^{n} \left[\frac{\{\Psi_k\}\{\Psi_k\}^T\{F(s)\}}{a_k s + b_k} + \frac{\{\Psi_k^*\}\{\Psi_k^*\}\{F(s)\}}{a_k^* s + b_k^*} \right] \tag{5}$$

As shown by Eq. (5), a solution for a system with general viscous damping can be obtained if excitation is known and expressed in a mathematical form. However, the numerical solutions are rather cumbersome to deal with when excitation is random. To alleviate this problem, the approximate normal mode method proposed by Seireg [11] is adopted since it leads to a much more simpler mathematical treatment and numerical calculation.

3 Approximate Normal Mode Method

The equations of motion of a system with general viscous damping that is subjected to mass excitation can be expressed in decoupled equations by assuming the fictitious damping ratio [11] and then using approximate normal modes:

$$\ddot{\eta}_{ik}(t) + 2\xi_{ik}\dot{\eta}_{ik}(t) + \omega_k^2\eta_{ik}(t) = \frac{\Sigma\, \phi_{ik} f_i(t)}{M_k} \tag{6}$$

where ϕ_{ik} = kth eigenvectors of undamped system
ω_k = kth eigenvalues

M_k = kth modal mass
ξ_{ik} = kth fictitious damping ratio
η_{ik} = fictitious uncoupled coordinate

The dynamic compliance between mass i and the exciting force at point j is given by

$$\left\{\frac{X_i}{F_j}\right\} = \sum_{k=1}^{n} \frac{\phi_{ik}\phi_{jk}/M_k}{\omega_k^2 - \omega^2 + 2j\xi_{ik}\omega_k\omega} \tag{7}$$

The actual transfer function at the kth natural frequency $\omega = \omega_k$ is

$$Q_{iK}(\omega_k) = \frac{X_i}{F_j} = \vec{\sum_{k=1}}\, \frac{\phi_{ik}\phi_{jk}}{K_k\sqrt{\left(1 - \frac{\omega_K^2}{\omega_k^2}\right)^2 + \left[2\xi_{iK}\frac{\omega_K}{\omega_k}\right]^2}} \tag{8}$$

where $\vec{\Sigma}$ denotes vector summation and K_k is the kth modal stiffness. Now, assuming that the contribution of other modes other than the kth mode at $\omega = \omega_k$ is negligible, the fictitious damping ratio is represented by

$$\xi_{ik} = \frac{\phi_{ik}\phi_{jk}}{2Q_{ik}K} = \frac{\phi_{ik}a_k}{2Q_{ik}} \tag{9}$$

where a_k is the kth modal participation factor and Q_{ik} is the response of mass i excited at point j by a unit magnitude exciting force at natural frequency.

The mean square response of the stationary process can be obtained when either the autocorrelation function $R(\omega)$ or the spectral density $S_x(\omega)$ of the response is known. If the input mean square spectral density $S_f(\omega)$ is known.

$$E[X^2] = \int_{-\infty}^{\infty} S_x(\omega)\, d\omega = \int_{-\infty}^{\infty} |H(\omega)|^2 S_f(\omega)\, d\omega \tag{10}$$

If the input signal is Gaussian white noise, then the spectral density of input, $S_f(\omega)$, is constant over the entire frequency range and can be taken out of the integral.

When random excitation force is applied to mass m and the required response is spring deflection, then

$$|H(\omega)| = \frac{1}{K\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \tag{11}$$

and the integral of the square of the transfer function over the whole frequency domain is [12]

$$\int_0^{\infty} |H(\omega)|^2\, d\omega = \frac{\pi}{4K^2}\frac{\omega_n}{\xi} \tag{12}$$

Since the system response can be obtained by the convolution integral and the convolution integral in time domain is a simple multiplication in frequency domain, the mean square response of the spring deflection in frequency domain is represented by

$$\overline{X}^2 = W_f\frac{\pi}{4K^2}\frac{\omega_n}{\xi} \tag{13}$$

where W_f is the mean square spectral density of the excitation f in the neighborhood of $\omega = \omega_n$.

If the system consists of several degrees of freedom, then the total mean square response at mass i becomes

$$\overline{X_i^2} = \sum_{k=1}^{n} \overline{X_{ik}^2} \tag{14}$$

From Eqs. (8), (13), and (14), the following relationship is obtained:

$$\overline{X_i^2} = \sum_{j=1}^{p} W_{f_j} \sum_{k=1}^{n} \frac{\pi\omega_k\phi_{ik}^2\phi_{jk}^2}{4\xi_{ik}K_k^2} \tag{15}$$

4 Optimum Damper Design of Machine Tool Structure

Since the exciting forces are generated from the cutting process and have random characteristics, the objective of a damper design is to minimize the relative vibration between the workpiece and the cutting tool over the entire frequency range of interest. It is assumed that the forces with equal magnitude but opposite signs are applied to the cutting tool and the workpiece, respectively. Because an actual machine tool cannot be regarded as a system with proportional damping, a model with general viscous damping is used for the formulation of this problem. To illustrate the optimization procedure, an actual machine tool system is simplified into a 4-degree-of-freedom system as shown in Fig. 2.

In Fig. 2 m_1 is the equivalent mass of the entire machine tool structure and m_2 and m_3 represent the inertias of the workpiece and the cutting tool, respectively. Mass m_4, equivalent to that of a damper, is attached to mass 3. Equations of motion of the system shown in Fig. 2 can be formulated as follows:

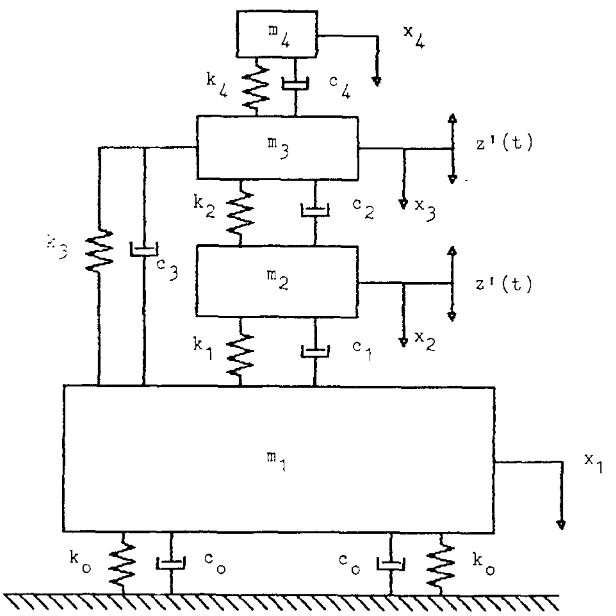

Fig. 2. Simplified model of a machine tool system.

$$
\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \end{Bmatrix}
+ \begin{bmatrix} c_0 + c_1 + c_3 & -c_1 & -c_3 & 0 \\ -c_1 & c_1 + c_2 & -c_2 & 0 \\ -c_3 & -c_2 & c_2 + c_3 + c_4 & -c_4 \\ 0 & 0 & -c_4 & c_4 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix}
+ \begin{bmatrix} k_0 + k_1 + k_3 & -k_1 & -k_3 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 \\ -k_3 & -k_2 & k_2 + k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}
= \begin{Bmatrix} 0 \\ f(t) \\ -f(t) \\ 0 \end{Bmatrix} \tag{16}
$$

In the previous equation $f(t)$ is the stochastic cutting force that is exerted on the workpiece and the cutter during machining and k_4 and c_4 are the damper stiffness and damping constants, respectively. Although this might be a simplified model compared with actual very complicated machine tool systems, it can be expanded without difficulty into an n degree of freedom model that can represent the actual system more precisely.

4.1 *Objective Function*

During machining operations the self-generated forces are quite random in nature. Thus, it is required that a machine tool maintain low-vibration amplitudes over a wide frequency range because most machine tools operate under various cutting conditions. When both input excitation and output response are random, the mean square response is often used to describe its dynamic behavior. Integration of this mean square response over a frequency range represents the total power of the motion within that frequency range. Thus, by reducing the integrated total sum, the system can be made to undergo less vibration. Therefore, the sum of the mean square responses of the tool and the workpiece is taken as an objective function in this study. In the following sections a method to obtain the mean square response over some frequency bandwidth is described and an optimization scheme is illustrated.

4.2 *Numerical Values of System Parameters*

Determining the stiffness and damping coefficients of a real machine tool is very difficult and possible only by experimental means. Analytical techniques encounter a difficulty because the behavior of the most significant source of flexibility such as structural joints is hard to predict. The experiment to obtain these coefficients is proposed as future work; however, the method to construct a lumped parameter system from the experiment has been proposed by others [13–15]. For the purpose of demonstrating the concept, the following parameter values are selected and used in this paper:

$$m_1 = 10 \qquad m_2 = 1 \qquad m_3 = 0.8$$

$$c_1 = 60 \qquad c_2 = 7 \qquad c_3 = 70$$

$$k_1 = 80 \times 10^7 \qquad k_2 = 10 \times 10^7$$
$$k_3 = 100 \times 10^7$$

4.3 *Optimization Procedure*

In the following process the mean square response values are calculated by the approximate normal mode method. The transfer functions of the machine tools without the damper (3-dof system) are plotted for the different values of the base pad damping and stiffness along with corresponding mean square response values obtained by the approximate normal mode method. The results are depicted in Fig. 3 for different sets of parameters. The mean square responses of masses 1 and 2 were ob-

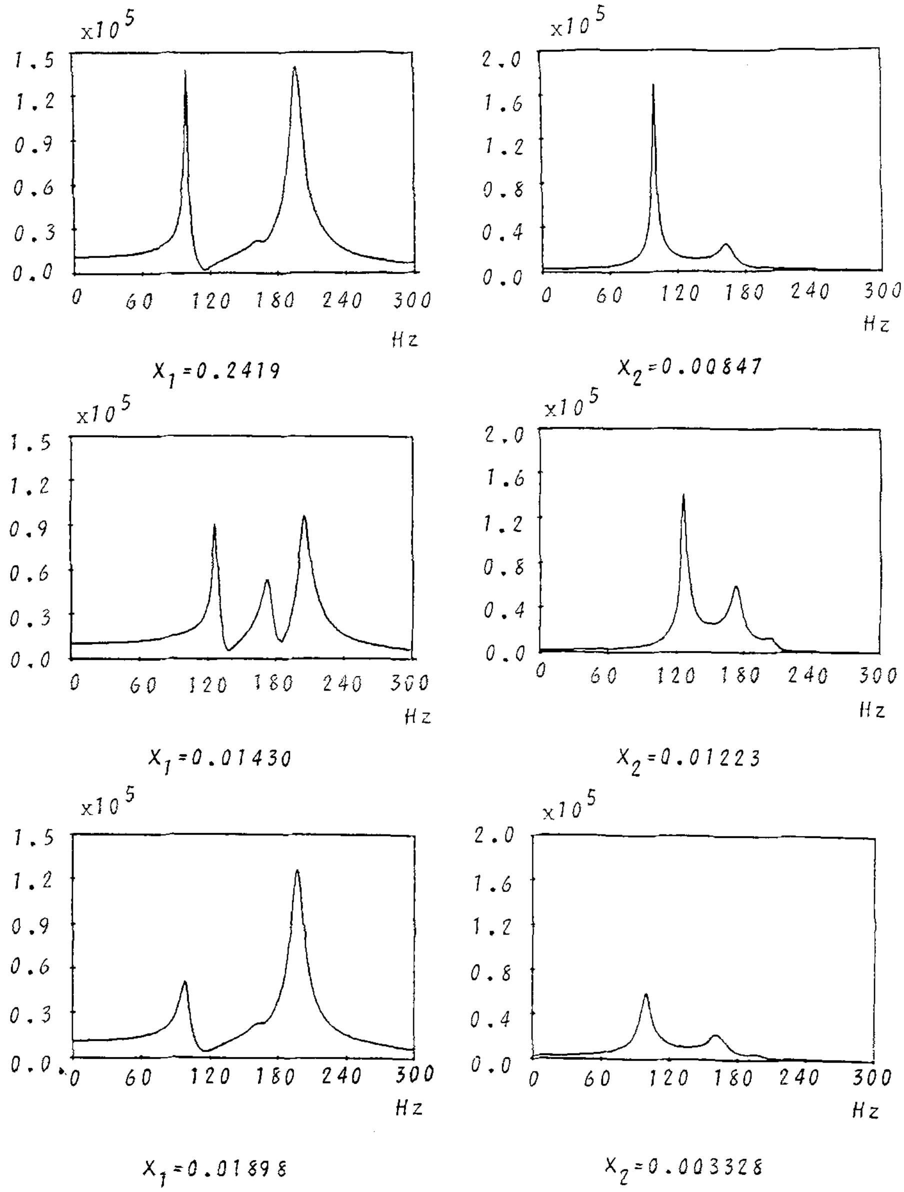

Fig. 3. Transfer function plots at masses 1 and 2 with different mean square response values.

tained by exciting mass 1 with Gaussian white noise. It is quite obvious from Fig. 3 that the method used to calculate the mean square response represents the system vibration amplitude sufficiently well for systems subject to random excitations.

The optimization is performed in two steps. First, optimum base pad coefficients to minimize the relative vibration between the workpiece and the cutting tool are sought. No damper is considered at this stage. The formulation is as follows:

Given parameters: $m_1, m_2, m_3, k_1, k_2, k_3, c_1, c_2, c_3$

Design parameters: C_0, K_0

Objective function: minimize $U = X_2^2 + X_3^2$

Constraints: $C_{0\min} < C_0 < C_{0\max}$

$K_{0\min} < K_0 < K_{0\max}$

In the previous formulation the objective function has been chosen as the sum of squares of the vibration amplitude since the phase difference between two masses is not constant for generally damped systems. Therefore, minimization of relative vibration can only be defined as minimization of the motion of individual components.

The contour for the objective function is shown in Fig. 4. It turned out that only K_0 was a decision variable since C_0 always approaches its maximum value for its minimum objective value. The transfer

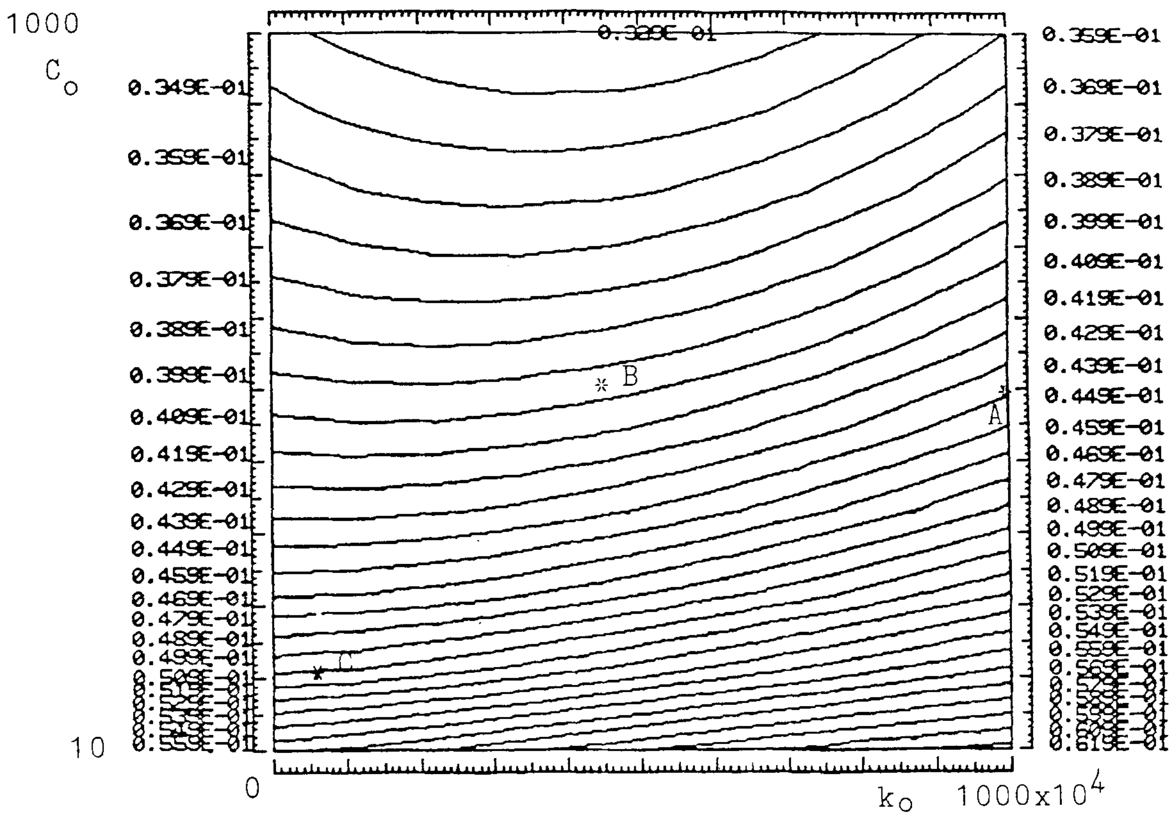

Fig. 4. Isomeric contour of the 3-DOF system.

function plots of the three different points (A, B, C) taken from Fig. 4 are shown in Fig. 5 to illustrate the validity of the optimization results. It is clearly seen that the overall vibration amplitudes at both masses 1 and 2 are the lowest when the value of the objective function is the smallest (point B).

The second phase of the optimal design problem is to design the parameters of the damper. The base spring constant was taken from the optimum value in Fig. 4 and the base damping coefficient was chosen to be a maximum allowable value. The problem is formulated:

Given: m_1, m_2, m_3, k_1, k_2, k_3, c_1, c_2, c_3, c_0, k_0

Decision variable: c_4, k_4

Objective function: minimize $U = X_2^2 + X_3^2$

Subject to: $0 < c_4 < c_{4\max}$ $\quad k_{4\min} < k_4 < k_{4\max}$

The mass of m_4 of the damper was assumed to be 10 percent of m_3. This is to keep the size of the damper small compared with other components of a machine tool and yet reduce the vibration. Therefore, one can first select the size of the damper based on space requirement and then determine spring and damping constants using the optimization procedure described in this paper.

4.4 *Search Method*

As shown in Fig. 6, the contour of the objective function has a narrow and sharp optimal region. Thus, the gradient search technique could not be applied to this problem. Instead, the pattern search scheme, in combination with the univariate search technique for the whole design region, was used to avoid the convergence to local minima.

The original pattern search method proposed by Hooke and Jeeves [16] has a built-in acceleration scheme, that is, increasing the stepsize automatically to facilitate fast convergence to the optimal point. However, in the given problem it was found that search movements according to the original procedure tended to jump over the global minimum zone and reach the local minimum due to the large stepsize generated from the search scheme in a very narrow optimal region. In order to overcome this difficulty and still reduce the convergence time, a maximum allowable stepsize was fixed so that it would give fast convergence but not allow too large jumps. In addition, a gradient search concept was incorporated into the pattern move for faster con-

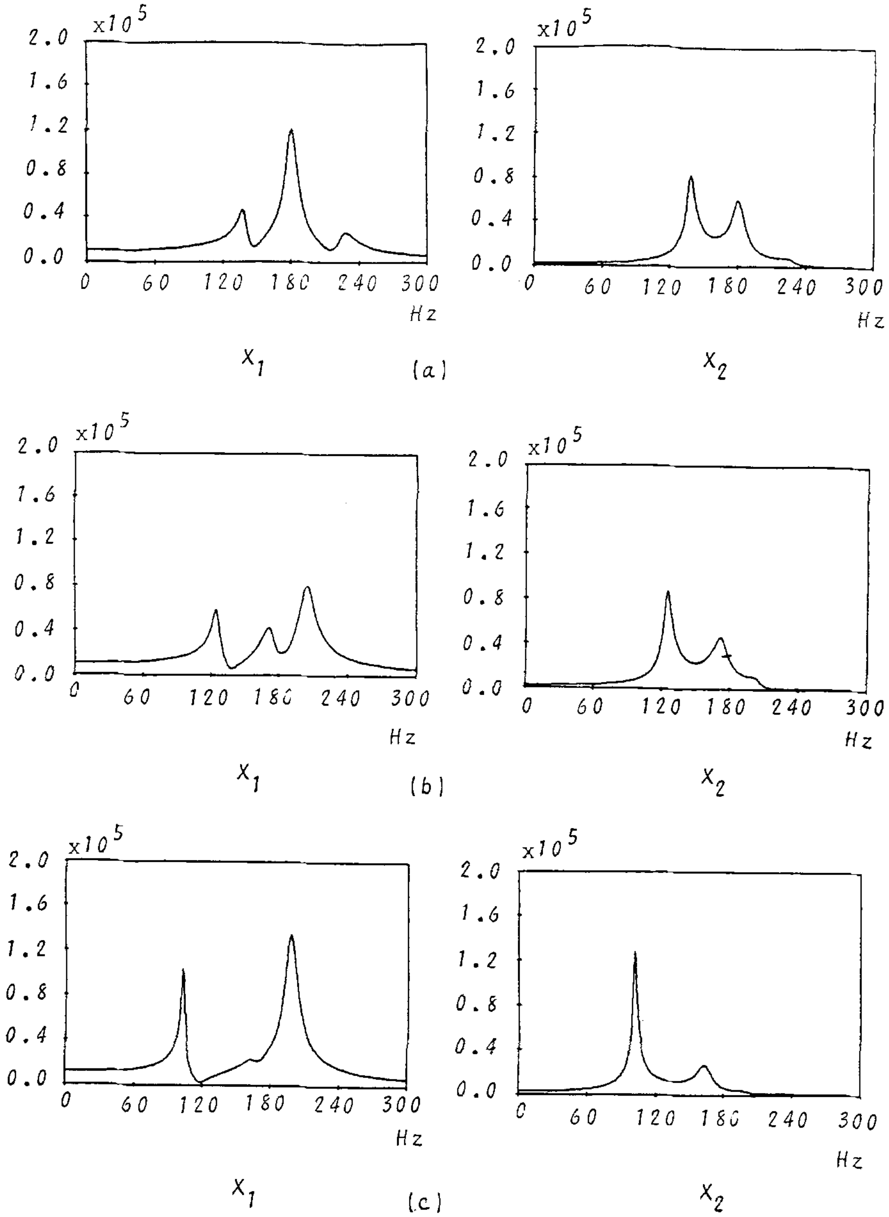

Fig. 5. Transfer functions of the merit values at three different points in Fig. 4.

vergence. As a result, in contrast to the gradient search, a good convergence to the optimum value could be obtained even in the very sharp optimal region, as shown in Fig. 6. The flowchart of this search scheme is given in Figs. 7–9.

The search for the optimal region starts with the described modified pattern move and continues until a minimum value of the objective function is obtained in the neighborhood of the initial point. This convergence point, however, could be one of the local minima. Once the search move is completed, the univariate search is activated over the entire design domain to check whether there is another region where the merit value is lower than the present suboptimal value. If such a point is found, the optimization begins again with the pattern search from that point. Since the univariate search activates only one design variable at a time, it does not involve heavy computational time even for the search in the entire design domain. This dual-step procedure effectively leads the search to the global minimum.

5 Results

The search started with the initial values $k_i = 0.7 \times 10^5$ and $c_i = 80$. The initial merit value was 0.03488

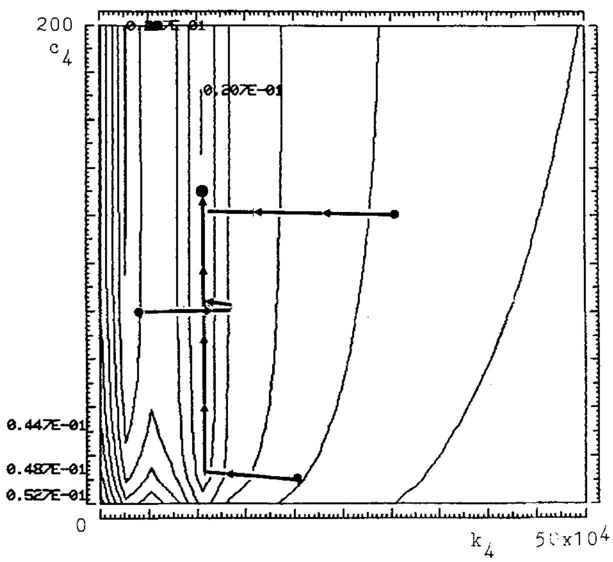

Fig. 6. Search movement to an optimal point from three different initial points.

and the mean square response values of X_2 and X_3 were 0.01546 and 0.008384, respectively. The numerical solution converged to the point where merit value is 0.01095 with $k_f = 9.4 \times 10^4$ and $c_f = 130.16$ and the mean square response values of X_2 and X_3 were 0.004910 and 0.003940, respectively. Two more searches were carried out with different initial points. These also converged to the same optimal point, as shown in Fig. 6. The mean square responses of X_2 and X_3 with the designed damper are given in Fig. 10, along with those without the damper. It is shown that the mean square responses of both m_2 and m_3 are substantially reduced by adding an optimal damper.

6 Conclusion

A new technique has been proposed for synthesizing dampers for machine tools. Unlike previous

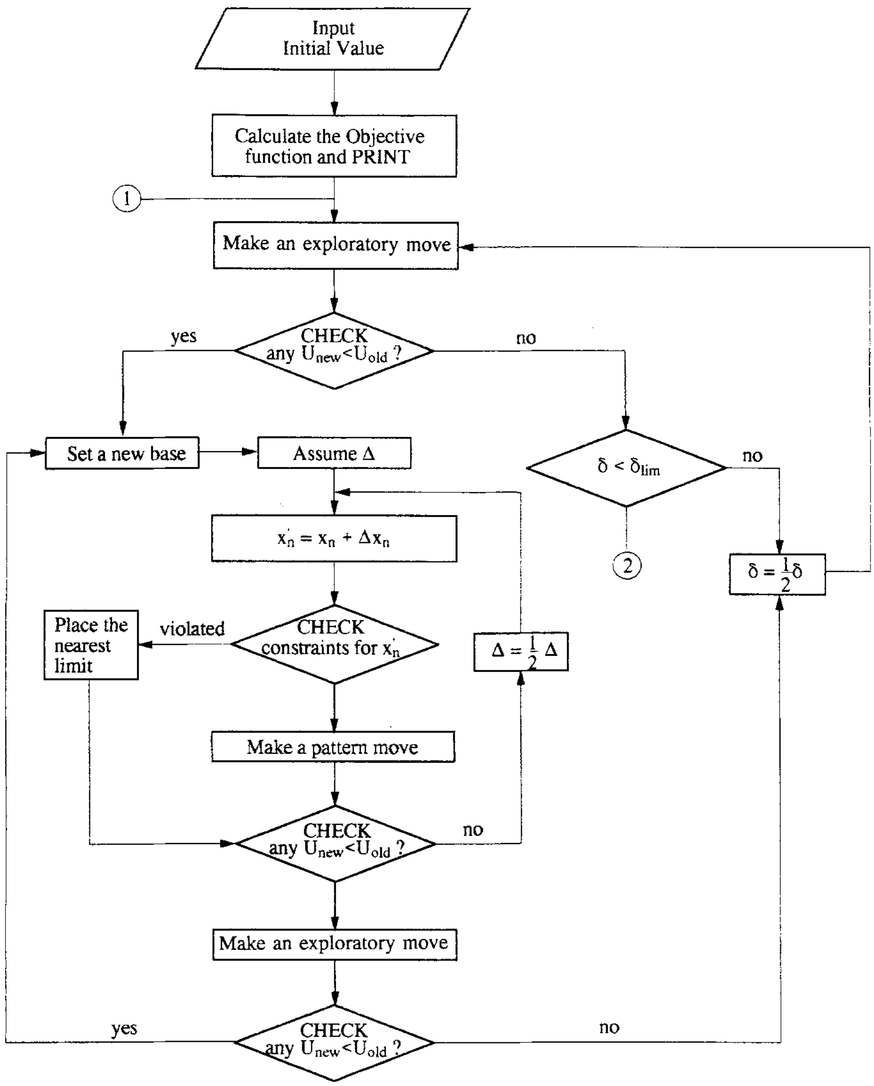

Fig. 7. Flowchart of a pattern search.

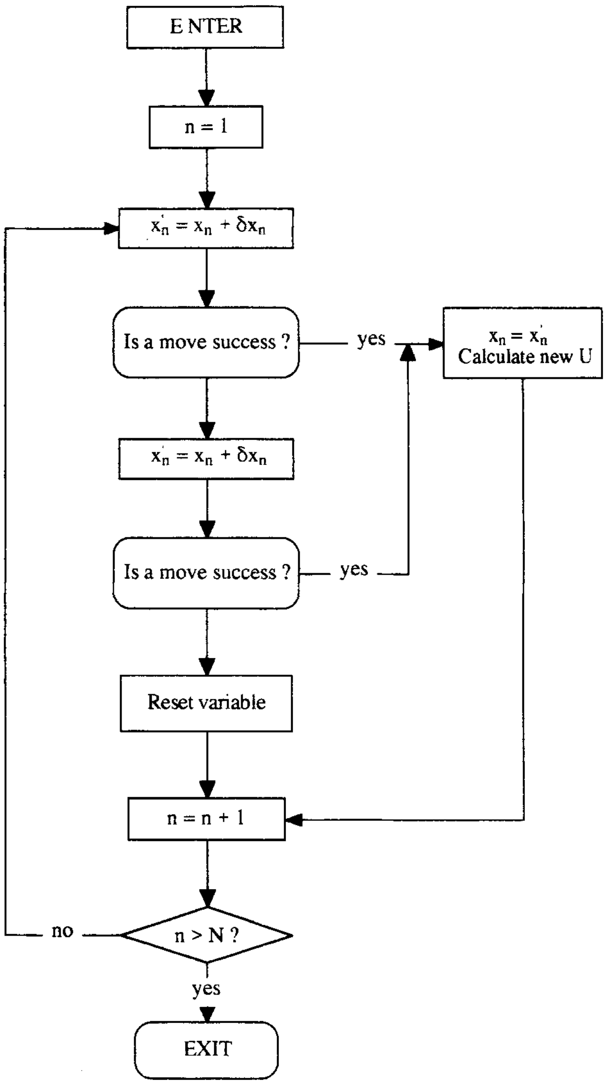

Fig. 8. Flowchart of an exploratory move.

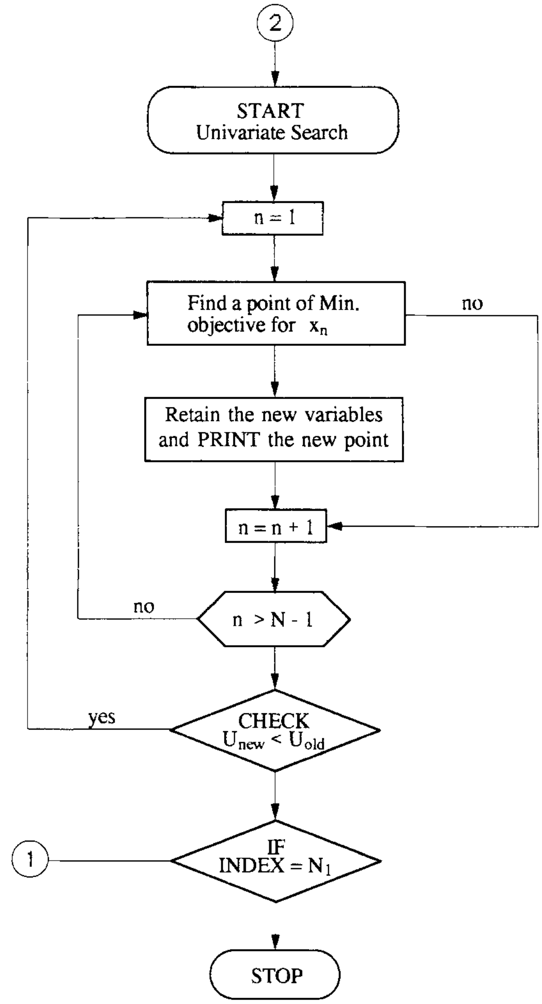

Fig. 9. Flowchart of a univariate search.

studies where excitations are assumed to be deterministic and the total machine tool structure is optimized, this research concentrated on minimizing the cutter and the workpiece vibrations under random excitations. This approach is thus more realistic and addresses the metal cutting excitation problem directly with minimal computation efforts. The technique presented is based on the approximate normal mode method to treat general nonproportional viscous damping and a two-step optimization procedure based on modified pattern search and univariate search techniques. The approximate normal mode method leads to an algebraic sum of responses, which is ideal for the numerical treatment of nonproportionally damped machine tool structures, especially under a large number of degrees of freedom. The modified optimization algorithm im-

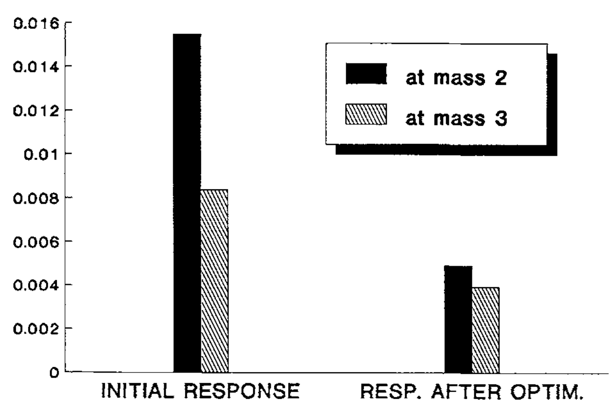

Fig. 10. Mean square responses before and after optimization.

proves convergence in very narrow optimal regions and the univariate search method prevents the numerical solution from getting stuck in local minima.

The concept has been illustrated with a 4-degree-of-freedom machine tool model and promising results have been shown. The optimization method can be easily expanded to more complex machine tool systems with a large number of degrees of freedom.

References

1. Cowley, A. (1972) Co-operative work in computer aided design in the CIRP. Ann. CIRP 21(2), 257–266
2. Reddy, V.R.; Sharan, A.M. (1987) The static and dynamic analysis of machine tools using dynamic matrix reduction technique. Int. J. Mach. Tools Manuf. 27(1), 105–112
3. Haranath, S.; Ganesan, N.; Rao B.V.A. (1987) Dynamic analysis of machine tool structures with applied damping treatment. Int. J Mach. Tools Manuf. 27(1), 43–55
4. Murty, J.R.; Reddy, G.C. (1979) Optimization of functional properties of machine tool structures—A finite element analysis. In: Proceedings of the 20th MTDR Conference, pp. 435–442
5. Opitz, H.; Noppen, N. (1972) A finite element program system and its application for machine tool structural analysis. In: Proceedings 13th MTDR Conference, pp. 55–60
6. Shin, Y.C.; Eman, K.F. (1989) A unified approach to the in-process identification of the closed loop machining dynamics in turning. Trans. NAMRI, SME (May), 235–242
7. Sata et al. (1973) Learning and recognition of the cutting states by the spectrum analysis. Ann. CIRP 22(1), 41–42
8. Yuan, J.X.; Wu, S.M. (1985) Identification of the joint structural parameters of machine tool by DDS and FEM. Trans. ASME, J Eng. Ind. 107, 64–69
9. Shin, Y.C.; Eman, K.F.; Wu, S.M. (1989) Experimental complex modal analysis of machine tool structures. Trans. ASME, J. Eng. Ind. 111 (May), 116–124
10. Foss, K.A. (1958) Co-ordinates which uncouple the equations of motion of damped linear dynamic systems. J. Appl. Mech. (September), 361–364
11. Seireg, A.; Howard, L. (1967) An approximate normal mode method for damped lumped parameter systems. Trans. ASME, J. Eng. Ind. (November), 597–604
12. Crandall, S.H.; Mark, W.D. (1963) *Random Vibration in Mechanical System*. New York: Academic Press
13. Tlusty, J.; Moriwaki, T. (1976) Experimental and computational identification of dynamic structure models. Ann. CIRP 25, 497–503
14. Tlusty, J.; Ismail, F. (1980) Dynamic structural identification tasks and methods. Ann. CIRP 29(1), 251–255
15. Renay, J.P. (1968) Identification of complex structures using near-resonance testing. Shock Vibr. Bull. 38(Part 2), 23–32
16. Cooper, L.; Steinberg, D. (1970) *Introduction to Methods of Optimization*, Philadelphia, PA: W.B. Saunders