Journal of Applied Sciences 8 (20): 3638-3645, 2008
ISSN 1812-5654

Optimal Partitioned State Kalman Estimator for Maneuvering Target Tracking in Mixed Coordinates

[1]A. Karsaz, [2]H. Khaloozadeh, [1]N. Pariz and [1]A. Peiravi
[1]Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
[2]Department of Electrical Engineering, KN Toosi University of Technology, Tehran, Iran

Abstract: A new general two-stage algorithm is originally proposed to reduce the computational effort for maneuvering target tracking in mixed coordinates. The augmented state Kalman estimators, which are based on the jerk modeling, are computationally expensive. The conventional input estimation techniques assume constant acceleration level and there are not covered a generalized input modeling. In this research, an innovative scheme is developed to overcome these drawbacks by using a reduced state Kalman estimator with a new structure, which is optimal for general conditions. In addition, the proposed scheme is an unbiased filtering algorithm applied in mixed coordinates based on the pseudo linear measurements.

Key words: Pseudo linear measurements, optimal reduced state Kalman estimator, input estimation, augmented state Kalman estimators, maneuvering target tracking

INTRODUCTION

The general state estimation problem in a stochastic linear system with unknown input variable, is solved by the well-known augmented state Kalman estimator (AUSKE) or full state method. The AUSKE solves the problems by including the input parameters as a part of an augmented state to be estimated (Khaloozadeh and Karsaz, 2005; Mehrotra and Mahapatra, 1997). However reduced state methods do not augment the state and usually yield a better performance (Mookerjee and Reifler, 1999). The AUSKE suffers from complexity of computational effort and numerical problems when state dimensions are large. The input detection and estimation (IDE) algorithm was first developed by Chan *et al.* (1979) using a simplified batch least square data. Although IDE approach is attractive since it intends to relax restrictive assumptions about input dynamics modeling, it suffers from a major deficiency, being that little prior knowledge is available for dynamics estimation (Li and Jilkov, 2002). For example we can cite (Wang and Varshney, 1993) used the IDE approach in the Maneuvering Target Tracking (MTT) problem. In Wang and Varshney (1993), the predicted states for the maneuvering target are related to the corresponding states without maneuvering assuming constant input or constant acceleration (CA). Therefore, the performance of the estimation is reduced when target moves with non-constant acceleration. According to Lee and Tahk (1999) the unknown input defined as a sum of elementary time functions. Although this input modeling is more general than the constant-input model of the original IDE algorithm, the performance is reduced if there is any input dynamics.

Friedland (1969) introduced a method of separating estimation of the unknown input from the dynamic variables and Blair used this method in the MTT problem (1993). The basic idea was to decouple the Augmented Kalman Filter (AKF) into two-stage filters in order to reduce computation and memory requirements (Hsieh and Chen, 1999, 2000; Kawase *et al.*, 1998; Qiu *et al.*, 2005). Recently, Hsieh and Chen (2000) derived an Optimal Two-Stage Kalman Estimator (OTSKE) for a general case to reduce the computational complexity of the AUSKE. The two-stage filtering method, suggested for MTT problem by Blair (1993) suffers from two major drawbacks. These drawbacks stem from assuming constant acceleration and input term assumed to be observable from the measurement equation (Hsieh and Chen, 1999; Qiu *et al.*, 2005). An alternative solution to overcome theses drawbacks for MTT problem introduced by Karsaz *et al.* (2007a).

In real world applications, target motion is usually modeled in Cartesian coordinates. However, in most systems, measurements of target position are available in terms of range and azimuth with respect to sensor coordinates. There are two major approaches to overcome this problem in target tracking. One method is to use a linear Kalman filter with measurements converted

Corresponding Author: Ali Karsaz, Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran
Tel: 09155199081 Fax: 098-511-8763302

into Cartesian coordinates called Pseudo-Linear Measurements (PLM) (Karsaz *et al*., 2007b; Lerro and Barshalom, 1993; Longbin *et al*., 1998). The other approach is to use nonlinear methods referred to as Extended Kalman Filter (EKF) or unscented Kalman filter (UKF) (Lin *et al*., 2002; Julier and Uhlmann, 2001). Easy of implementation and reduced computational effort for the filter are two important advantages of the PLM method compared with the nonlinear methods (Karsaz *et al*., 2007b). Recently, neural networks and fuzzy logic schemes have been used for maneuvering targets with intelligent adaptation (Duh and Lin, 2004; Lee *et al*., 2004; McGinnity and Irwin, 1998).

The objective of this study is to propose a new partitioned two-stage Kalman estimator which is optimal in the Minimum Mean Square Error (MMSE) sense. The new partitioned dynamic model is proposed for target acceleration vector based on the augmented jerk model has been suggested in Mehrotra and Mahapatra (1997). It is shown that the maneuver tracking algorithm proposed in Wang *et al*. (1993) is a special case of our proposed method. In addition, the obtained relations are in mixed coordinates using PLM method. The motivation of our proposed method is the generation of a two-stage structure to obtain the optimal performance in mixed coordinates. This proposed scheme is named the OPSKE_PLM. In addition, a computer simulation is carried out for the scenario of a maneuvering target tracking by the OPSKE_PLM and the computation cost are compared with the AUSKE_PLM.

STATEMENT OF THE PROBLEM

In MTT problem the target motion can be best modeled in Cartesian coordinates. Unfortunately, in most systems, the target range and azimuth are provided as the target measurements in polar coordinates with respect to the sensor location. Tracking in Cartesian coordinates using polar measurements can be described by the discretized equation set:

$$X_{k+1} = A_k X_k + B_k U_k + W_k^x \tag{1}$$

$$U_{k+1} = C_k U_k + W_k^u \tag{2}$$

$$Z_k^p = h(X_k) + V_k^p \tag{3}$$

where, $X_k \in R^n$ is the system state $U_k \in R^m$ and $Z_k^p \in R^1$ are the input vector and the measurement vector in polar coordinates, respectively. Where the superscript p denotes polar coordinates. Matrices A_k, B_k and C_k are assumed to be known functions of the time interval k and are of appropriate dimensions. Matrix C_k is assumed nonsingular and h(.) is a nonlinear function. The process noises W_k^x, W_k^u and the measurement noise V_k^p are zero-mean white Gaussian sequences with the following covariances $E[W_k^x(W_l^x)'] = Q_k^x\delta_{kl}$, $E[W_k^u(W_l^u)'] = Q_k^u\delta_{kl}$, $E[W_k^x(W_l^u)'] = Q_k^{xu}\delta_{kl}$, $E[V_k^p(V_k^p)'] = R_k^p\delta_{kl}$, $E[W_k^x(V_k^p)'] = 0$ and $E[W_k^u(V_k^p)'] = 0$, where $'$ denotes transpose and δ_{kl} denotes the Kronecker delta function. The initial states X_0 and U_0 are assumed to be uncorrelated with the sequences W_k^x, W_k^u and V_k^p. The initial conditions are assumed to be Gaussian random variables with $E[X_0] = \hat{X}_0$, $E[X_0X_0'] = P_0^x$, $E[U_0] = \hat{U}_0$, $E[U_0U_0'] = P_0^u$, $E[X_0U_0'] = P_0^{xu}$. As we can shown in Eq. 3 the input vector U_k is not observable through the measurement process, despite the assumption in (Blair, 1993; Hsieh and Chen, 1999; Qiu *et al*., 2005). In MTT applications we have:

$$X_k = \begin{bmatrix} x_k & v_k^x & y_k & v_k^y \end{bmatrix}' \tag{4}$$

$$U_k = \begin{bmatrix} u_k^x & j_k^x & u_k^y & j_k^y \end{bmatrix}' \tag{5}$$

where, x_k, v_k^x, u_k^x and j_k^x denote the position, velocity, acceleration and jerk of the target along the x-axis, respectively. Measurements of target position are reported in polar or sensor coordinates as the range measurement (Z_k^r) and the azimuth measurement (Z_k^θ) (Mahapatra and Mehrotra, 2000). Equation 3 for MTT application is rewritten as:

$$Z_k^p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \tan^{-1}(\frac{y_k}{x_k}) \end{bmatrix} + V_k^p \tag{6}$$

where, $Z_k^p = \begin{bmatrix} z_k^r \\ z_k^\theta \end{bmatrix}$, $V_k^p = \begin{bmatrix} v_k^r \\ v_k^\theta \end{bmatrix}$ and $R_k^p = \begin{bmatrix} (\delta_k^r)^2 & 0 \\ 0 & (\delta_k^\theta)^2 \end{bmatrix}$. The errors in range v_k^r and azimuth v_k^θ are assumed to be independent with zero mean and standard deviations δ_k^r and δ_k^θ, respectively. Therefore, Eq. 3 shows a nonlinear relation between the measurements in polar coordinates and system states in Cartesian coordinates.

In MTT problem the target motion can be best modeled in Cartesian coordinates. This situation a standard Kalman Filter (KF) can be used accurately. Tracking in Cartesian coordinates using polar measurements can be handled in two ways. One method is to use a linear Kalman filter with measurements converted to a Cartesian frame of reference. The other approach is to use conventional nonlinear techniques such as EKF and UKF, which incorporates the original measurements in a nonlinear equation in mixed coordinates. In this study, it is use a modified linear

Kalman filter based on PLM. Obtaining the relation between the measurement noise covariance in polar coordinates and the measurement noise covariance in Cartesian coordinates and using PLM technique for the standard Kalman filter implementation is discussed in the following.

It is clear that any measurement expressed in polar coordinates has an exact and equivalent representation in Cartesian coordinates. Therefore the pseudo measurements z_k^x and z_k^y can easily obtained in each iteration from existing observation z_k^r and z_k^θ using classical conversion:

$$\begin{aligned} z_k^x &= z_k^r \cos(z_k^\theta) \\ z_k^y &= z_k^r \sin(z_k^\theta) \end{aligned} \tag{7}$$

Therefore, the measurement equation can take the following linear approximated form in Cartesian coordinates as discussed in Karsaz *et al*. (2007):

$$Z_k = H_k X_k + L_k V_k^p \tag{8}$$

where, $H_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $Z_k = \begin{bmatrix} z_k^x \\ z_k^y \end{bmatrix}$ and

$$L_k = \begin{bmatrix} \cos(z_k^\theta) & -z_k^r \sin(z_k^\theta) \\ \sin(z_k^\theta) & z_k^r \cos(z_k^\theta) \end{bmatrix} \tag{9}$$

The converted measurement covariance matrix R_k in Eq. 8 has been calculated:

$$R_k = E\left\{ L_k \begin{bmatrix} v_k^r \\ v_k^\theta \end{bmatrix} \begin{bmatrix} v_k^r & v_k^\theta \end{bmatrix} L_k^{'} \right\} = L_k R_k^p L_k^{'} \tag{10}$$

For long ranges and/or large azimuth errors a polar to Cartesian unbiased consistent conversion with correction for average bias is given in Lerro and Barshalom (1993) and states as:

$$z_k = \begin{bmatrix} z_k^x \\ z_k^y \end{bmatrix} = \begin{bmatrix} z_k^r \cos(z_k^\theta)[1 - e^{-(\delta_k^\theta)^2} + e^{-\frac{(\delta_k^\theta)^2}{2}}] \\ z_k^r \sin(z_k^\theta)[1 - e^{-(\delta_k^\theta)^2} + e^{-\frac{(\delta_k^\theta)^2}{2}}] \end{bmatrix} \tag{11}$$

THE AUGMENTED STATE KALMAN ESTIMATOR BASED ON THE PSEUDO LINEAR MEASUREMENTS APPROACH (AUSKE_PLM)

One can easily use the AUSKE based on Eq. 1, 2, 8 and 10. Treating X_k and U_k as the augmented system state (Hsieh and Chen, 1999), the AUSKE_PLM is described by:

$$X_{k+1|k+1}^{Aug} = X_{k+1|k}^{Aug} + K_{k+1}^{Aug}(Z_{k+1} - H_{k+1}^{Aug} X_{k+1|k}^{Aug}) \tag{12}$$

$$X_{k+1|k}^{Aug} = A_k^{Aug} X_{k|k}^{Aug} \tag{13}$$

$$K_{k+1}^{Aug} = P_{k+1|k}(H_{k+1}^{Aug})'[H_{k+1}^{Aug} P_{k+1|k}(H_{k+1}^{Aug})' + R_{k+1}]^{-1} \tag{14}$$

$$P_{k+1|k} = A_k^{Aug} P_{k|k}(A_k^{Aug})' + Q_k \tag{15}$$

$$P_{k+1|k+1} = (I - K_{k+1}^{Aug} H_{k+1}^{Aug}) P_{k+1|k} \tag{16}$$

Where:

$$X_k^{Aug} = \begin{bmatrix} X_k \\ U_k \end{bmatrix}, K_k^{Aug} = \begin{bmatrix} K_k^x \\ K_k^u \end{bmatrix}, P_k = \begin{bmatrix} P_k^x & P_k^{xu} \\ (P_k^{xu})' & P_k^u \end{bmatrix},$$

$$\left\langle A_k^{Aug} = \begin{bmatrix} A_k & B_k \\ 0_{m\times n} & C_k \end{bmatrix} \right\rangle, H_k^{Aug} = [H_k \quad 0_{1\times m}], Q_k = \begin{bmatrix} Q_k^x & Q_k^{xu} \\ (Q_k^{xu})' & Q_k^u \end{bmatrix}$$

where, the superscript Aug denotes the augmented system state, I denotes the identity matrix of any dimension and $O_{m\times n}$ is a m×n zero matrix. It is clear from Eq. 12-16 that the computational cost of the AUSKE_PLM increases drastically with the augmented state dimension. The reason for this computational complexity is the extra computation of P_k^{xu} terms in each sample time k (Hsieh and Chen, 1999). Therefore, if this term can be eliminated, one can reduce the complexity of computational effort. In this research, we propose a new optimal two-stage Kalman estimator without calculating the term of P_k^{xu} explicitly. Therefore, the proposed scheme is developed to reduce the computational cost in two ways. First by partitioning the Eq. 12-16 into two partitioned subsystems. The other is to use a linear Kalman filter based on the PLM, instead of using the conventional nonlinear techniques such as EKF and UKF.

DERIVATION OF THE OPTIMAL PARTITIONED STATE KALMAN ESTIMATOR IN MIXED COORDINATES

As mentioned before the conventional IDE approaches are free from input modeling which is the main drawback of these methods (Li and Jilkov, 2002). In contrast, other conventional augmented techniques, which overcome this problem, are computationally expensive. In addition, some of these approaches use conventional nonlinear techniques such as EKF and UKF to solve the nonlinear measurement equation in mixed coordinates. In the view of this fact, the proposed approach in this study intends to relax restrictive assumptions concerning the input dynamics modeling and using a new optimal partitioned Kalman estimator.

The design of a new modified two-stage estimator in mixed coordinates are summarized in the following theorem proved in Karsaz *et al.* (2007b). The major derivation is the relation between the measurement residues of the two different filters. One is the measurement residue of the input-free filter, which does not consider unknown input vector and the other is the measurement residue of the input filter. Based on the measurement residues of the two filters, an input estimation algorithm is derived using the MMSE technique.

Theorem 1: For the linear discrete-time system given by Eq. 1, 2 and 8, let a filter minimize the total covariance $P^x_{k+1|k+1}$ and $P^u_{k+1|k+1}$ at each update. Then, the recursive equations for X_k, $\bar{X}_k$ and U_k are as follows:

$$\hat{X}_{k+1|k} = \hat{\bar{X}}_{k+1|k} + M_{k+1}\hat{U}_{k+1|k} \quad (17)$$

$$\hat{X}_{k+1|k+1} = \hat{\bar{X}}_{k+1|k+1} + N_{k+1}\hat{U}_{k+1|k+1} \quad (18)$$

where, the state vector of the input-free model is denoted by $\bar{X}_k$. The input-free filter is just a Kalman filter based on the model (8) and (1) by ignoring the input term (i.e., $U_k = 0$) as below:

$$\hat{\bar{X}}_{k+1|k+1} = \hat{\bar{X}}_{k+1|k} + K_{k+1}(Z_{k+1} - H_{k+1}\hat{\bar{X}}_{k+1|k}) \quad (19)$$

$$\hat{\bar{X}}_{k+1|k} = A_k\hat{\bar{X}}_{k|k} \quad (20)$$

$$K_{k+1} = P^x_{k+1|k}H'_{k+1}[H_{k+1}P^x_{k+1|k}(H_{k+1})' + L_{k+1}R^p_{k+1}L'_{k+1}]^{-1} \quad (21)$$

$$P^x_{k+1|k} = A_kP^x_{k|k}(A_k)' + Q^x_k \quad (22)$$

$$P^x_{k+1|k+1} = (I - K_{k+1}H_{k+1})P^x_{k+1|k} \quad (23)$$

The desired form of the filtering solution for estimating the unknown vector U_{k+1} are the recursive equations expressing $\hat{U}_{k+1|k+1}$ in terms of $\hat{U}_{k|k}$ and $\tilde{\bar{Z}}_{k+1}$ as below:

$$\hat{U}_{k+1|k+1} = \hat{U}_{k+1|k} + K^u_{k+1}[\tilde{\bar{Z}}_{k+1} - H_{k+1}M_{k+1}\hat{U}_{k+1|k}] \quad (24)$$

$$\hat{U}_{k+1|k} = C_k\hat{U}_{k|k} \quad (25)$$

$$K^u_{k+1} = 2P^u_{k+1|k}M'_{k+1}H'_{k+1}[3H_{k+1}M_{k+1}P^u_{k+1|k}M'_{k+1}H'_{k+1} + P^z_{k+1|k}]^{-1} \quad (26)$$

$$P^u_{k+1|k} = C_kP^u_{k|k}C'_k + Q^u_k \quad (27)$$

$$P^u_{k+1|k+1} = [I - 2K^u_{k+1}H_{k+1}M_{k+1}]P^u_{k+1|k} \quad (28)$$

$$P^z_{k+1|k} = H_{k+1}P^x_{k+1|k}H'_{k+1} + L_{k+1}R^p_{k+1}L'_{k+1} \quad (29)$$

$$M_{k+1} = [A_kM_k + B_k]C^{-1}_k, \qquad k = 2,3,....M_1 = B_0C^{-1}_0 \quad (30)$$

$$N_{k+1} = [I - K_{k+1}H_{k+1}]M_{k+1} \quad (31)$$

The innovations $\tilde{\bar{Z}}_{k+1}$ and $\tilde{Z}_{k+1}$ as the measurement residues of the input-free model and the input model are defined, respectively:

$$\tilde{\bar{Z}}_{k+1} = Z_{k+1} - H_{k+1}\hat{\bar{X}}_{k+1|k} \quad (32)$$

$$\tilde{Z}_{k+1} = Z_{k+1} - H_{k+1}\hat{X}_{k+1|k} \quad (33)$$

The extra computation of the cross covariance matrix $P^{zu}_{k+1|k}$ (which relates to $P^{xu}_{k+1|k}$) is the reason for the computational complexity in the augmented state methods. In view of this fact, the algorithm has been proposed by Wang *et al.* (1993) is a sub-optimal algorithm where, $E\{\tilde{Z}_{k+1}\tilde{U}'_{k+1|k}\} = 0$. Therefore, this term eliminated in the OPSKE_PLM which reduce the complexity of computational effort (Karsaz *et al.*, 2007b).

PERFORMANCE EVALUATIONS

To demonstrate the computational advantage of the OPSKE_PLM over the AUSKE_PLM, the number of arithmetic operations are considered, i.e., multiplications and summations, as suggested in (Hsieh and Chen, 1999). The arithmetic operations of a standard Kalman estimator with state dimension n and measurement dimension l are shown in Table 1. It is clear from the Eq. 12-16 and Table 1, that the arithmetic operations required for the AUSKE_PLM which has state dimension n+m and measurement dimension l are $M(n+m, l)$ for multiplications and $S(n+m, l)$ for summations and $2l^3$ and $2l^3-2l^2$ for multiplication and summation specially required by R_k. So, the Arithmetic Operations Required (AOR) for the AUSKE_PLM are:

Table 1: Standard Kalman estimator arithmetic operation requirements

Variables	No. of multiplications, M(n, l)	No. of summations, S(n, l)
$X_{k+1\|k+1}$	$2nl$	$2nl$
$X_{K+1\|k}$	n^2	n^2-n
K^x_{k+1}	$n^2l+2nl^2+l^3$	$n^2l+2nl^2+l^3-2nl$
$P^x_{K+1\|k}$	$2n^3$	$2n^3-n^2$
$P^x_{K+1\|k+1}$	n^3+n^2l10	$n^3+n^2l-n^2$
Total	$3n^3+2n^2l+2nl^2+l^3+n^2+2nl$	$3n^3+2n^2l+2nl^2+l^3-n^2-n$

Table 2: Input estimation and auxiliary matrices arithmetic operation requirements for the OPSKE_PLM

Variables	No. of multiplications, $M^{OP}(n, m, l)$	No. of summations, $S^{OP}(n, m, l)$
$U_{k+1\|k+1}$	$2ml$	$2ml$
$U_{K+1\|k}$	m^2	m^2-m
K^u_{k+1}	$m^2l+2ml^2+l^3+l^2+ml$	$m^2l+2ml^2+l^3-2ml$
$P^u_{K+1\|k}$	$2m^3$	$2m^3-m^2$
$P^u_{K+1\|k+1}$	$m^3+m^2l+m^2$	$m^3+m^2l-m^2$
$P^z_{k+1\|k}$	$2n^2l$	$2n^2l-2nl+l^2$
$\hat{X}_{k+1\|k+1}$	nm	$nm-n$
$\hat{X}_{k+1\|k}$	nm	nm
M_{k+1}	$n^2m+m^3+nm^2$	$n^2m+m^3+nm^2-nm$
N_{k+1}	n^2m	n^2m-nm
$H_{k+1}M_{k+1}$	nml	$nml-ml$
R_k	$2l^3$	$2l^3--2l^2$
Total	$3ml+2m^2+2m^2l+2ml^2+3l^3+l^2+4m^3+2n^2l+2nm+2n^2m+nm^2+nml$	$-ml-m^2-m+2m^2l+2ml^2+3l^3+4m^3+2n^2l-2nl-l^2-n+2n^2m+nm^2+nml$

$$\begin{aligned}&AOR(AUSKE\_PLM) = M(n+m,l) + S(n+m,l)\\&= [3(n+m)^3 + 2(n+m)^2 l + 2(n+m)l^2 + l^3 + (n+m)^2 + 2(n+m)l] + 2l^3\\&+[3(n+m)^3 + 2(n+m)^2 l + 2(n+m)l^2 + l^3 - (n+m)^2 - (n+m)] + 2l^3 - 2l^2\end{aligned} \tag{34}$$

Table 2 shows the arithmetic operations of the input estimation and the auxiliary matrices needed by the OPSKE_PLM which has state dimension n, measurement dimension l and input vector dimension m. The total arithmetic operations required for the state and input estimation and auxiliary matrices, by the OPSKE_PLM as shown in Table 2 are:

$$\begin{aligned}&AOR(OPSKE\_PLM) =\\&M(n,l) + S(n,l) + M^{OP}(n,m,l) + S^{OP}(n,m,l) =\\&[3n^3 + 2n^2 l + 2nl^2 + l^3 + n^2 + 2nl] +\\&[3n^3 + 2n^2 l + 2nl^2 + l^3 - n^2 - n]\\&+[3ml + 2m^2 + 2m^2 l + 2ml^2 + 3l^3 + l^2 + 4m^3 +\\&2n^2 l + 2nm + 2n^2 m + nm^2 + nml]\\&+[-ml - m^2 - m + 2m^2 l + 2ml^2 + 3l^3 + 4m^3 +\\&2n^2 l - 2nl - l^2 - n + 2n^2 m + nm^2 + nml]\end{aligned} \tag{35}$$

It is clear that the number of the arithmetic operations of the AUSKE_PLM increases drastically with the augmented state dimension, which makes the algorithm computationally inefficient. In contrast, the proposed algorithm based on the two-stage partitioned technique required fewer computations. This enables the proposed algorithm to have much better computational efficiency than the AUSKE_PLM. Using Eq. 34 and 35, the operational savings, denoted by OS^{OPSKE}_{AUSKE}, of the OPSKE_PLM as compared to the AUSKE_PLM are:

$$\begin{aligned}&OS^{OPSKE}_{AUSKE} = AOR(AUSKE\_PLM) - AOR(OPSKE\_PLM) =\\&M(n+m,l) + S(n+m,l) - M(n,l) - S(n,l) - M^{OP}(n,m,l) - S^{OP}(n,m,l)\\&= -2m^3 + 14n^2 m + 17nm^2 - 4n^2 l + 6nml - 2l^3 + 2nl + n - m^2 - 2l^2 - 2nm\end{aligned} \tag{36}$$

Table 3: The OPSKE_PLM compared to the AUSKE_PLM for different input vector dimensions m values

The state vector dimensions	AOR (OPSKE_PLM)	AOR (AUSKE_PLM)	OS^{OPSKE}_{AUSKE}	POS^{OPSKE}_{AUSKE} (%)
n = 4, m = 1, l = 2	888	1085	197	18.15
n = 4, m = 2, l = 2	1090	1738	648	37.28
n = 4, m = 3, l = 2	1414	2623	1209	46.09
n = 4, m = 4, l = 2	1908	3776	1868	49.47
Average	1325	≅ 2305	≅ 980	37.74

It is clear from Eq. 36 that for MTT problem which usually $l \leq n$, the proposed scheme has computational advantage over the AUSKE_PLM.

To measure the relative operational savings of the OPSKE_PLM with respect to the arithmetic operation required by the AUSKE_PLM (AOR (AUSKE_PLM)), the percentage of the operational savings defined as below:

$$POS^{OPSKE}_{AUSKE} = \frac{OS^{OPSKE}_{AUSKE}}{AOR(AUSKE\_PLM)} \times 100 \tag{37}$$

The OPSKE_PLM could be applied to the higher order derivatives of the acceleration for complex maneuvering target (i.e., different input vector dimensions m). In view of this fact and in order to compare the OPSKE_PLM and the AUSKE_PLM in simulation results section, the acceleration vector considered as $U_k = \begin{bmatrix} u^x_k & j^x_k & u^y_k & j^y_k \end{bmatrix}'$. Using Eq. 34-37, the operational savings and the percentage of the operational savings, of the OPSKE_PLM comparing to the AUSKE_PLM for different values of m and n = 4, l = 2 are shown in Table 3.

It can be inferred from Table 3 that the OPSKE_PLM has better overall performance than the AUSKE_PLM (averaged 37.74%).

SIMULATION RESULTS

To evaluate the proposed algorithm, an example of maneuvering target tracking problem which turns, in two-dimensional space is simulated. In this simulation example, the performance of the OPSKE_PLM for the MTT has been compared with the work suggested by Mahapatra and Mehrotra (2000) as an example of the AUSKE method by using the PLM technique. As mentioned before in the augmented state method the state vector includes the input vector i.e., acceleration and jerk parameter in maneuvering target tracking problem. The sampling interval is T = 0.01 (sec) and target maneuver is applied at 9th second (900th sample). The initial conditions are selected similar for the AUSKE_PLM as well as the OPSKE_PLM. The state vectors are:

$$X_k = \begin{bmatrix} x_k & v_k^x & y_k & v_k^y \end{bmatrix}',\ U_k = \begin{bmatrix} u_k^x & j_k^x & u_k^y & j_k^y \end{bmatrix}'$$
$$X_k^{Aug} = \begin{bmatrix} x_k & v_k^x & y_k & v_k^y & u_k^x & j_k^x & u_k^y & j_k^y \end{bmatrix}'$$

where, x_k, v_k^x and j_k^x denote the position, velocity, acceleration and jerk of the target along the x-axis, respectively. We consider the target initial conditions for the state and the acceleration vectors as below:

$$X_0 = [2165\ \text{m} \quad -80\ \text{m/sec} \quad 1250\ \text{m} \quad 25\ \text{m/sec}]$$
$$U_0 = [0\ \text{g} \quad 0\ \text{g/sec} \quad 0\ \text{g} \quad 0\ \text{g/sec}]'$$
$$X_0^{Aug} = [2165\ \text{m} \ -80\ \text{m/sec} \quad 1250\ \text{m} \ 25\ \text{m/sec} \quad 0\ \text{g} \quad 0\ \text{g/sec} \quad 0\ \text{g} \quad 0\ \text{g/sec}]'$$

The target begins to maneuver as U900 = [0 g −0.7 g/sec 0g 0.4 g/sec]' for 9 (sec)$\le t \le$90 (sec).

The measurement standard deviations for the target range and azimuth are δ^r = 100 (m) and δ^θ = 2 (deg), respectively. Thus, the measurement covariance matrix is $R_k^p = \begin{bmatrix} 10000 & 0 \\ 0 & 4 \end{bmatrix}$ in polar coordinates. The system matrices are given by:

$$A_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix},\ B_k = \begin{bmatrix} T^2/2 & T^3/6 & 0 & 0 \\ T & T^2/2 & 0 & 0 \\ 0 & 0 & T^2/2 & T^3/6 \\ 0 & 0 & T & T^2/2 \end{bmatrix}$$

$$C_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix},\ H_k = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}'$$

$$Q_k^u = 2\alpha\sigma_j \begin{bmatrix} T^3/3 & T^2/2 & 0 & 0 \\ T^2/2 & T & 0 & 0 \\ 0 & 0 & T^3/3 & T^2/2 \\ 0 & 0 & T^2/2 & T \end{bmatrix}$$

$$Q_k^x = 2\alpha\sigma_j \begin{bmatrix} T^7/252 & T^6/72 & 0 & 0 \\ T^6/72 & T^5/20 & 0 & 0 \\ 0 & 0 & T^7/252 & T^6/72 \\ 0 & 0 & T^6/72 & T^5/20 \end{bmatrix}$$

$$Q_k^{xu} = 2\alpha\sigma_j \begin{bmatrix} T^5/30 & T^4/24 & 0 & 0 \\ T^4/8 & T^3/6 & 0 & 0 \\ 0 & 0 & T^5/30 & T^4/24 \\ 0 & 0 & T^4/8 & T^3/6 \end{bmatrix}$$

$$P_0^x = 10I_{4\times4},\ P_o^u = 0.1I_{4\times4},\ P_0^{xu} = I_{4\times4}$$

$$A_k^{Aug} = \begin{bmatrix} A_k & B_k \\ 0_{4\times4} & C_k \end{bmatrix},\ H_k^{Aug} = \begin{bmatrix} H_k & 0_{2\times4} \end{bmatrix},$$
$$Q_k = \begin{bmatrix} Q_k^x & Q_k^{xu} \\ (Q_k^{xu})' & Q_k^u \end{bmatrix},\ P_k = \begin{bmatrix} P_k^x & P_k^{xu} \\ (P_k^{xu})' & P_k^u \end{bmatrix}$$

where, $\sigma_j = 0.09$ (m sec^{-3}) is the variance of the target jerk and $\alpha = 0.0123$ (sec^{-1}) is the reciprocal of the jerk time constant $\tau = 1/\alpha$. The measurement standard deviations of the target positions in Cartesian coordinates are selected in each iteration using Eq. 10. The Root Mean Square Error (RMSE) index is used for the results evaluation. The OPSKE_PLM is mathematically equivalent to the AUSKE_PLM. Advantage of the OPSKE_PLM is that it is less computationally intensive than the AUSKE_PLM. Figure 1 and 2 show the actual

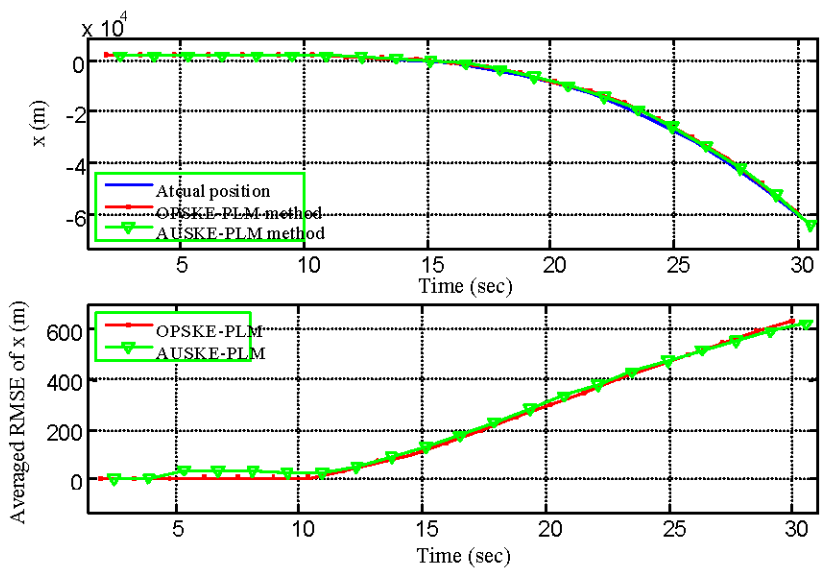

Fig. 1: The actual value and the estimation of the x position and averaged RMS errors estimations by the OPSKE_PLM and the AUSKE_PLM methods

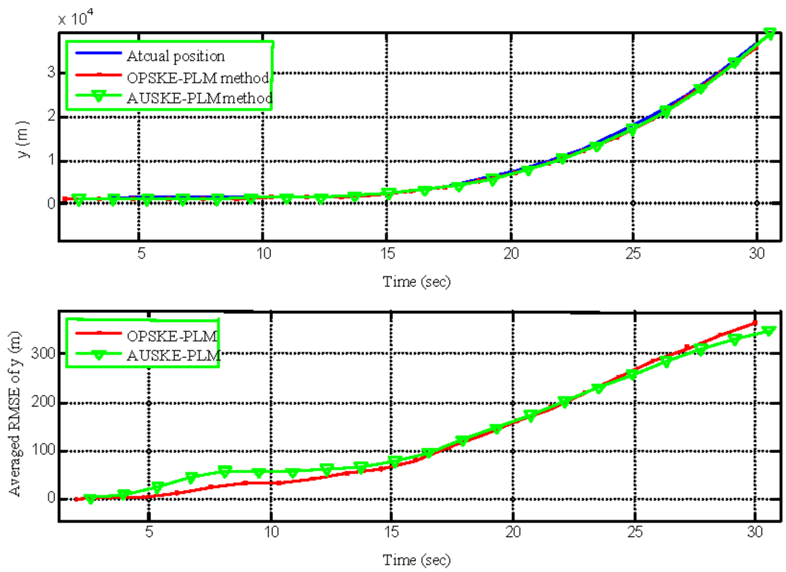

Fig. 2: The actual value and the estimation of the y position and averaged RMS errors estimations by the OPSKE_PLM and the AUSKE_PLM methods

Table 4: Performance of the OPSKE_PLM compared to the AUSKE_PLM in 100 different scenarios of target motion

Performance	OPSKE_PLM	AUSKE_PLM
RMSE x (m)	31.1223	31.1223
RMSE v^x (m sec^{-1})	2.3446	2.3446
RMSE u^x (m sec^{-2})	0.2311	0.2311
RMSE j^y (m sec^{-3})	0.0101	0.0101
RMSE y (m)	21.1433	21.1433
RMSE v^y (m sec^{-1})	1.5245	1.5245
RMSE u^y (m sec^{-2})	0.1204	0.1204
RMSE j^y (m sec^{-3})	0.0054	0.0054

value and the estimation of x and y and RMS errors of x and y positions estimations by the proposed OPSKE_PLM and the AUSKE_PLM, respectively. It is clear that the performance of the proposed OPSKE_PLM is as well as the results obtained by the AUSKE_PLM in the maneuvering target tracking problem.

Table 4 shows that the performance of the proposed OPSKE_PLM is as well as the results obtained by the AUSKE_PLM using Monte-Carlo approach in 100 different scenarios of the MTT problem. Note that in this example n = 4, m = 4 and l = 2 and the operation savings for the OPSKE_PLM over the AUSKE_PLM as shown in Table 3 are 1868 (or 49.47%).

CONCLUSIONS

The proposed scheme is based on a new partitioned dynamic modeling and intends to overcome the computational expensiveness drawbacks of the other works which are based on the augmented methods. The proposed OPSKE_PLM provides the optimal state estimate, which is equivalent to that of the AUSKE_PLM. In addition, the new optimal algorithm relations designed to work in Cartesian coordinates with quickly data processing. Simulation results show the good performance of the proposed algorithm and effectiveness of this scheme in tracking maneuvering targets as well as the AUSKE_PLM.

REFERENCES

Blair, W.D., 1993. Fixed-gain two-stage estimator for tracking maneuvering targets. IEEE Trans. Aerosp. Elect. Syst., 29: 1004-1014.

Chan, Y.T., A.G.C. Hu and J.B. Plant, 1979. A kalman filter based tracking scheme with input estimation. IEEE. Trans. Aerosp. Elect. Syst., 15: 237-244.

Duh, F.B. and C.T. Lin, 2004. Tracking a maneuvering target using neural fuzzy network. IEEE Trans. Syst. Man and Cybernet, Part B, 34: 16-33.

Friedland, B., 1969. Treatment of bias in recursive filtering. IEEE. Trans. Automat. Contr., 14: 359-367.

Hsieh, C.S. and F.C. Chen, 1999. Optimal solution of the two-stage Kalman estimator. IEEE Trans. Automat. Contr., 44: 195-199.

Hsieh, C.S. and F.C. Chen, 2000. General two-stage Kalman filters. IEEE. Trans. Automat. Contr., 45: 819-824.

Julier, S.J. and J.K. Uhlmann, 2001. Unscented filtering and nonlinear estimation. Proceeding IEEE Power Engineering Society, Winter Meeting, March, pp: 401-422.

Karsaz, A., H. Khaloozadeh and N. Pariz, 2007a. A New Algorithm Based on Generalized Target Maneuver Detection. In: 6th IEEE International Conference on Control and Automation (ICCA), May 30, June 1, Guangzhou, China, pp: 3034-3039.

Karsaz, A., H. Khaloozadeh, N. Pariz and A. Khorami, 2007b. Coordinates transformation for target maneuver detection. Proceeding of Control and Application Conference (ICCA), April 2007, pp: 2978-2983.

Kawase, T., H. Tsurunosono, N. Ehara and I. Sasase, 1998. Two-stage Kalman estimator using an advanced circular prediction for tracking highly maneuvering targets. Proceeding International Conference on Acoustics, May 12-15, Speech and Signal Processing, pp: 2453-2456.

Khaloozadeh, H. and A. Karsaz, 2005. A new state augmentation for maneuvering targets detection. Proceeding of 3rd IEEE International Conference on Signal Processing and Communication (SPCOM), Dec. 2005, pp: 65-69.

Lee, H. and M.J. Tahk, 1999. Generalized input-estimation technique for tracking maneuvering targets. IEEE Trans. Aerosp. Elect. Syst., 35: 1388-1402.

Lee, B.J., J.B. Park, Y.H. Joo and S.H. Jin, 2004. Intelligent Kalman filter for tracking a manoeuvring target. IEE Proc. Radar Sonar Navig., 151: 244-350.

Lerro, D. and Y. Barshalom, 1993. Tracking with debiased consist converted measurement versus EKF. IEEE Trans. Aerosp. Elect. Syst., 29: 1015-1022.

Li, X.R. and V.P. Jilkov, 2002. A survey of maneuvering target tracking-part IV: Decision-based methods. Proceeding of the 2002 SPIE Conference Signal and Data Processing of Small Targets, April 2002, Orlando, FL, pp: 1333-1364.

Lin, X., T. Kirubarajan, Y. Bar-Shalom and S. Maskell, 2002. Comparison of EKF, pseudomeasurement and particle filters for a bearing-only target tracking problem. Proceeding of SPIE Conference on Signal and Data Processing of Small Targets Tuesday 02 April, Orlando, FL, USA, pp: 240-250.

Longbin, M., S. Xiaoquan, Z. Yiyu and S. Zhongkang, 1998. Unbiased converted measurements for target tracking. IEEE Conference Aerosp. Electron, July 1998, pp: 1023-1027.

Mahapatra, P.R. and K. Mehrotra, 2000. Mixed coordinates tracking of generalized maneuvering targets using acceleration and jerk models. IEEE Trans. Aerosp. Elect. Syst., 36: 992-1000.

McGinnity, S. and G .W. Irwin, 1998. Fuzzy logic approach to maneuvering target tracking. IEE Proc. Radar Sonar Navig., 145: 337-341.

Mehrotra, K. and P.R. Mahapatra, 1997. A jerk model for tracking highly maneuvering targets. IEEE Trans. Aerosp. Elect. Syst., 33: 1094-1105.

Mookerjee, P. and F. Reifler, 1999. Reduced state estimators for consistent tracking of maneuvering targets. IEEE Trans. Aerosp. Elect. Syst., 41: 608-619.

Qiu, H.Z., H.Y. Zhang and X.F. Sun, 2005. Solution of two-stage Kalman filter. IEE Proceeding Control Theory Applied, 152: 152-156.

Wang, T.C. and P.K. Varshney, 1993. A tracking algorithm for maneuvering targets. IEEE Trans. Aerosp. Elect. Syst., 29: 910-924.