CONNECTIVITY AS A MEASURE OF POWER SYSTEM INTEGRITY

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Received May 1988

Abstract Measures of network stuctural integrity useful in the analysis and synthesis of power systems are discussed. Signal flow methodology is applied to derive an expression for the paths between sources and sinks in a power network. Connectivity and reachability properties of the network are obtained using the minors of a modified connectivity matrix. Node-connectivity, branch connectivity and mixed connectivity of power system are discussed.

چکیده در این مقاله معیارهای یکپارچگی ساختار شبکه‌ها و کاربردآنها در تحلیل و طراحی سیستم‌های قدرت مورد بحث قرارمی‌گیرد. افزایش درجه یکپارچگی ساختار شبکه موجب تنزل درجهٔ صدمه پذیری شبکه در مواقع بروز حوادث احتمالی منتج از پدیده‌های طبیعی یا خرابکاری عمدی شده و درنتیجه قابلیت اطمینان بیشتری برای ارائه سرویس برق به مصرف‌کنندگان فراهم میکند. با بکارگیری روش سیگنال پلوگراف الگوریتم سریعی برای تعیین کلیهٔ مسیرهای موجود برای انتقال انرژی از کلیهٔ مراکز تولید به کلیهٔ مراکز مصرف ارائه می‌گردد که بسادگی به کمک کامپیوتر قابل بکارگیری است. معیارهای یکپارچگی ساختار شبکه از جمله درجه اتصال گره، شاخه و مختلط مورد بررسی قرارگرفته و خواص قابلیت دسترسی شبکه با استفاده از کوفاکتورهای ماتریس درجهٔ اتصال مورد بحث قرار میگیرند. گرچه هدف اصلی این مقاله بررسی و ارائه الگوریتم جدید و سریعی برای محاسبهٔ درجهٔ اتصال، قابلیت دسترسی و قابلیت اطمینان شبکه و کاربرد آنها در طراحی سیستم‌های قدرت است با این الگوریتم میتوان کلیهٔ مسیرهای بین مراکز تولید و مصرف را محاسبه نموده و از آن جهت تهیه برنامهٔ سوئیچینگ در وقوع قطع خطوط یا خروج اضطراری نیروگاههای یک شبکه در بهره‌برداری از سیستم‌های قدرت استفاده نمود.

INTRODUCTION

Quality and continuity of service are two of the most important characteristics of any utility system. The real worth of the service that is provided by a utility to its customers cannot be readily evaluated [1], nor is the cost to customers as a result of a service interruption necessarily related to the income lost by the utility during the interruption.

With the ever present threat of natural disaster and human bungling, power systems are vulnerable to interruptions and the possibility of extensive physical damage exists. As improvements in a power system do not come about without an expenditure which is eventually passed on to the consumer,. the vulnerability of power systems is of utmost concern to the user. Thus any design procedure under normal and emergency conditions must include cost as a constraint.

Most studies of power system reliability are based on probabilistic techniques. More notable among these works are the excellent books on power system reliability by Billinton et al. [2] and Endrenyi [3]. However, in the absence of acourate reliability figures for many of the components of power systems, other measures of network integrity must be identified which can give insight for the design of affordable and reliable power systems.

Furthermore there are some other questions which remain. For instance what does one do when not worried about the probable but the possible major interruption? How does one tell whether a given power network holds together better than another?

We see a need to develop a methodology which can give reasonable insight into network reliability through an assessment of the network's structural integrity. It is for this reason that we apply deterministic measures of network vulnerability which do not require exact information as to the probabilities of failure of system components. This is equivalent to saying that in the absence of accurate reliability figures, we could consider all system components to be equally likely to fail in an environment where the threat to the system is a random function of time and location of system components.

Recently there is increased interest in the application of graph theoretic techniques for planning of power distribuiton networks [4]. The techniques presented here are based on graph theory. They are not only applicable to power systems but also have applications in the areas of computer, communications and transportation networks [5, 6, 7, 8] where structural integrity of the network and the minimization of service interruption in the face of emergencies are desirable characteristics for the system.

Crowin and Miles [9] in their assessment of the 1977 New York City blackout outline the impact of the blackout on many public services such as fire protection, police protection, public health, the sewage disposal system, the transportation system, the water supply system and other sectors of the population. They find the following statement from the Public Services Commision's investigation of the blackout as requested by the Governor of New York State "of particular interest":

"The tragic consequences of the July blackout under-score the special vulnerability of the New York City community to the effects of major power failures. This extreme vulnerability necessitates a higher level of reliability than may be required—and costs greater than may be tolerated—in other service areas."

CONCEPT OF CONNECTIVITY

For the purpose of our discussion we shall equate network structural integrity with network connectivity: how well do its various parts hang together?

Problems of connectivity have been approached in several ways. Most procedures for computing the connectivities of graphs are based on the Max-Flow Algorithm [10]. This outlook is very restrictive. We shall present an alternative approach to exploring connectivity problems based on signal flow methodology [11].

Note that a network can be disrupted because nodes and or branches might fail. Initially we will consider node failures only, later branch failures and the mixed case as well. In this section we regard the power system as a linear graph. In the next section we shall use a signal flow graph model.

Linear graph model

We can model power systems by using linear graphs [5]. A linear graph G(N, B) is a finite collection of basic elements referred to as nodes [N] and a set of pairs of nodes called branches [B]. The branches may be directed or undirected, with the latter always transformable into pairs of oppositely directed branches. The number of nodes in the set [N] is denoted by n and the number of branches in the set [B] is denoted by b. Adjacency information is presented in the connectivity matrix C = [c(i, j)]. Node i is denoted as ni and branch i as bi. Two nodes ni and nj connected via a branch are said to be adjacent, i.e. c(i, j) = 1. Otherwise c(i, j) = 0.

Definition of Node-Connectivity [5, 6]

The global node-connectivity $K^n(G)$ of a connected graph G(N, B) is the minimum number of nodes whose removal from the graph results in a disconnected graph or the presence of a trivial graph (single node). The node-connectivity of a complete graph (in which each node is connected to every other node) is equal to n-1.

The node-connectivity index is a measure of the vulnerability of the power system to node failure (bus, generating station or feeder failures).

Computation of Node-Connectivity

Computationally the global node-connectivity $K^n(G)$ of a graph G(N, B) is the minimum of the pairwise node-connectivities $K^n{}_{ij}(G)$ over all distinct, non-adjacent i, j node pairs, i.e.,

$$K^n(G) = \min_{i \neq j} \left(K^n{}_{ij}(G) \right) \tag{1}$$

where the pairwise node-connectivity $K^n{}_{ij}(G)$ is the number of nodes whose removal from the graph destroys all the paths between the two distinct, non-adjacent nodes i and j.

The key to the computation of the node-connectivity of a graph is in a classical result in graph theory due to Menger [12]. Menger's theorem states that the minimum number of nodes whose removal from a graph disconnects two non-adjacent nodes s and t (say, source and sink) is equal to the maximum number of s-t paths in the graph which are node-disjoint, i.e., which have no nodes in common excluding s and t. For example consider the network of Figure 1. There are three node disjoint paths between the source node and the sink node t. The branches forming these paths are marked as a, b and c.

When applied to a power systems, our interest lies in the transmission of power between generating sources and demand centers. We would therefore compute the global node-connectivities as the minimum of the pairwise node-connectivities over all source-sink pairs of nodes.

Branch Connectivity

Another measure of power system integrity is the branch-connectivity. This index gives an indication of the vulnerability of a power system to branch failure (transmission line or breaker failure). For our application, the branch-connectivity $K^b(G)$ is the minimum of the pairwise branch-connectivities over all source-sink pairs of nodes. Pairwise branch connectivity $K^b{}_{ij}$ is the minimum number of branches in any i-j branch cutset. Cutting the branches in the cut set will disconnect all the paths between nodes i and j.

Mixed Connectivity

In an emergency both nodes and branches might fail, giving rise to the concept of mixed connectivity. Mixed connectivity K^{bn} is defined as the minimum taken over pairwise mixed connectivities. The pairwise mixed connectivity $K^{bn}{}_{ij}$ is the size of the minimum i-j mixed cut set. An i-j mixed cut set is a mixed set of nodes or branches whose removal from the graph destroys all the paths from node i to node j.

CONCEPT OF REACHABILITY

The reachability matrix R = [R(i, j)] is a nonzero whenever there exists at least one path between nodes i and j and is zero otherwise. The matrix R can be found by adding an identity matrix I to the connectivity matrix C and raising the resulting matrix to a power k, k = 1, 2, 3, ..., m until the resultant matrix $(C + I)^m$ is the same as the matrix $(C + I)^{m-1}$. There are faster ways of computing the reachability matrix. For example one could multiply the resultant matrix from above by itself at each step. This speeds up the determination of the reachability matrix. More efficient techniques exist which use row-sweep and row-sum algorithms. This concept has applications in detection and identification of islands in power system networks [13].

SIGNAL FLOW APPROACH

In the approaches mentioned earlier one usually uses a linear graph to model the network, however we approach the connectivity problem by using signal flow methodology. The usual procedure for the calculation of connectivity requires the use of the Max-Flow Algorithm and the efficiency of these procedures depends on the efficiency of the Max-Flow Algorithm [10]. In our approach to connectivity problems there is no need to use the Max-Flow Algorithm. This approach yields a general formula given by

$$P(i, j) = -1^{i+j+1} M_{ij}(C^T\text{-}I) \tag{2}$$

subject to the complementarity condition given as;

$$c(i, j)\, c(j, i) = 0 \tag{3}$$

where;

$M_{ij}(C^T\text{-}I)$ denotes the (i, j)th minor of the n x n matrix $C = C^T\text{-}I$;

I is an n x n identity matrix, and C^T is the transpose of the connectivity matrix. Note that the determinant of the matrix C' is the characteristic equation of the system.

This approach seems to have been overlooked, although related work has appeared in the literature [14, 15, 16, 17].

The application of the general formulas represented by (2) and (3) for path determination and connectivity and reachability evaluation is illustrated by way of examples. But first let us define a signal flow graph.

Signal flow graph model

An alternative model for a power network can be established using signal flow methodology [11, 17]. A signal flow graph consists of a set of nodes [N] and a set of ordered pairs of nodes referred to as directed branches [B]. A node represents a specified quantity or a process variable. For each node j, there is a node signal y(j). Associated with each branch directed from node i to node j there is a branch transmittance x(i, j) that defines the coefficients which relate the node variables. A signal flow graph defines the relationship between the variables at the nodes. These can be expressed as a set of linear algebraic equations.

There exist one or more incoming branches for any dependent node of a signal flow graph. That component of the signal or commodity y(i) at node j which is transmitted through a branch directed from node i to j with branch transmittance, x(i, j) is the product of y at node i and the transmittance of the branch, i.e., y(j) due to branch ij = x(i, j) y(i). At a node the signals entering through several branches are summed: y(j) = sum over i of all x(i, j) y(i).

Path Determination

Given a power system network with a set of source (generating) nodes, a set of transition nodes and a set of sink nodes (load centers), applying (2) to an augmented signal flow model of the power network and then imposing the complementarity condition (3) we can find an expression for all the paths between any subset of source nodes and any subset of sink nodes.

Connectivity Evaluation

From the resulting paths expression the connectivity properties of the network can be obtained. We can find the number of node-disjoint paths between any source-sink pair of nodes. This can be achieved by inspection of the path expressions in simple cases, or by application of a simple algorithm to find the number of disjoint sets among several reduced path lists.

In addition we find the (i-j) branch cut sets and mixed cut sets by manipulating the path expression. Minimal branch and mixed cut sets are identified. Their size indicates the (i-j) pairwise branch and mixed connectivities, respectively.

Thus we find the pairwise connectivities of the graph. The minimum of the pairwise connectivities over all distinct, non-adjacent pairs of source-sink nodes gives the global connectivity indices of the network.

Reachability Evaluation

Reachability properties of the network can also be computed using (2) subject to the complementarity condition given by (3) by inserting the appropriate values of c(i, j) from the connectivity matrix C.

Reliability Evaluation

If the reliabilities of individual components are known, various network reliability figures can be found using the paths expression and standard series-parallel formulas [18]. This includes the probabilities of failure for both nodes and branches. The reliability expressions derived must be modified by reducing to one all powers of those terms which have a power greater than one, thus accounting for terms which appear in several paths between a given pair of nodes. Kim et al. [19] devised an operator which does exactly this operation. This topic will not be pursued any further in this paper.

Signal flow formulation

Given the utility network of Figure 2 with generation source node 1, transition nodes 2 and 3, and sink node 4, we first develop a method to find all the paths from source node 1 to sink node 4. We begin by identifying the elements of a signal flow model for the utility network, augmented by return path 4-1 an shown by dashed lines in Figure 2.

Writing the equations for general node variables y in terms of the transmittances x of the branches we obtain:

$$y(1) = x(4, 1)\, y(4) \tag{4}$$

$$y(2) = x(1, 2)\, y(1) + x(3, 2)\, y(3) \tag{5}$$

$$y(3) = x(1, 3)\, y(1) + x(2, 3)\, y(2) \tag{6}$$

$$y(4) = x(2, 4)\, y(2) + x(3, 4)\, y(4) \tag{7}$$

Putting equations 4-7 in matrix form we obtain

$$Y = XY \tag{8}$$

where:

$$Y = \begin{bmatrix} y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix}$$

and

$$X = \begin{bmatrix} 0 & 0 & 0 & x(4,1) \\ x(1,2) & 0 & x(3,2) & 0 \\ x(1,3) & x(2,3) & 0 & 0 \\ 0 & x(2,4) & x(3,4) & 0 \end{bmatrix}$$

Note that the transmittance matrix X is analogous to the transpose of the connectivity matrix C[5] associated with the graph of Figure 2 where x(i, j) = c(j, i). Equation (8) can be rewritten as;

$$Y = C^T Y \quad (9)$$

Bringing both terms of (9) to the same side and factoring out Y we get:

$$(X\text{-}I)Y = (C^T\text{-}I)Y = C'Y = 0 \quad (10)$$

where I represents an identity matrix of order n.

The matrix C' in (10) is represented here in terms of elements c(i, j) as

$$C' = \begin{bmatrix} -1 & 0 & 0 & c(4,1) \\ c(1,2) & -1 & c(3,2) & 0 \\ c(1,3) & c(2,3) & -1 & 0 \\ 0 & c(2,4) & c(3,4) & -1 \end{bmatrix} \quad (11)$$

The matrix C' is analogous to the "Signal flow matrix" as in Vasudeva [20].

Path Determination Between a Pair of Nodes

The expression for P(1, 4) giving all the paths between nodes 1 and 4 is determined by applying (2) to (11) to find the P(1, 4) seen to be the determinat:

$$P(1,4) = \begin{vmatrix} c(1,2) & -1 & c(3,2) \\ c(1,3) & c(2,3) & -1 \\ 0 & c(2,4) & c(3,4) \end{vmatrix}$$

$$= c(1,2)\,c(2,4) + c(1,2)\,c(2,3)\,c(3,4) + c(1,3)\,c(3,4) + c(1,3)\,c(3,2)\,c(2,4)$$

Each term in (12) represents a path from node 1 to 4. For example, the term c(1, 2)c(2, 4) represents a path from node 1 to 4. The four paths from node 1 to 4 from (12) are shown in Figure 3.

Calculation of Pairwise Node-Connectivity

The P(1, 4) paths expression shown in (12) can be used to find the number of (1-4) node-disjoint paths where 1 and 4 take the roles of a "source" and a "sink" node respectively. We construct for each path a vector which contains the nodes of the path in the order traversed resulting in the path vectors p(1) = [1, 2, 4], p(2) = [1, 3, 4], p(3) = [1, 2, 3, 4] and p(4) = [1, 3, 2, 4].

If we wish to determine the number of paths which are node-disjoint, we first start by excluding the end nodes 1 and 4, leaving us with modified path vectors L(1) = [2], L(2) = [3], L(3) = [2, 3] and L(4) = [3, 2].

The next step is to compare the L vectors to determine the number of disjoint sets. L(1) and L(2) are disjoint sets. L(1) and L(3) are not disjoint, neither are L(2) and L(3) nor are L(1) and L(3). Therefore only paths p(1) and p(2) are node-disjoint. We conclude that the pairwise node-connectivity between nodes 1 and 4 is equal to two, i.e., $K^n{}_{14}(G)=2$.

Determination of Paths Matrix P'

Application of (2) and (3) to the signal flow model of the unaugmented model of the power network yields the paths matrix P' given by

$$P'(i, j) = P(i, j) \text{ subject to } [c(i,j)\,c(j,i)=0] \quad (13)$$

The path matrix P' gives all the paths between every pair of nodes. Indeed it is intuitively obvious that the information contained in P' is also that of the reachability properties of the network.

For the network of Figure 2, the matrix C' from (12) is repeated here, but with 0 substituted for c'(1, 4) to yield the modified connectivity matrix for the unaugmented network as shown:

$$C' = \begin{bmatrix} -1 & 0 & 0 & 0 \\ c(1,2) & -1 & c(3,2) & 0 \\ c(1,3) & c(2,3) & -1 & 0 \\ 0 & c(2,4) & c(3,4) & -1 \end{bmatrix} \quad (14)$$

The terms of the matrix P can be found from (2). Using the matrix C′ of equation (12), calculation of P(i, j) is illustrated for the case i = 1, j = 2:

$$P(1,2) = -1^{1+2+1} \begin{vmatrix} c(1,2) & c(3,2) & 0 \\ c(1,3) & -1 & 0 \\ 0 & c(3,4) & -1 \end{vmatrix}$$

$$= c(1,2) + c(1,3)\,c(3,2) \qquad (15)$$

After working out the remaining terms the results can be gathered in a matrix P as shown:

$$P = \begin{bmatrix} 1 - [c(2,3)\,c(3,2)] & c(1,2) + c(1,3)\,c(3,2) & c(1,3) + c(1,2)\,c(2,3) & c(1,2)\,c(2,4) + c(1,3)\,c(3,4) + c(1,2)\,c(2,3)\,c(3,4) + c(1,3)\,c(3,2)\,c(2,4) \\ 0 & 1 & c(2,3) & c(2,4) + c(2,3)\,c(3,4) \\ 0 & c(3,2) & 1 & c(2,4) + c(3,2)\,c(2,4) \\ 0 & 0 & 0 & 1 - [c(2,3)\,c(3,2)] \end{bmatrix}$$

The terms of the form c(i, j) c(j, i) shown in square brackets above can not occur in any actual paths. Thus we impose the complementarity condition (3). The resultant path matrix P′ is

$$P' = \begin{bmatrix} 1 & c(1,2) + c(1,3)\,c(3,2) & c(1,3) + c(1,2)\,c(2,3) & c(1,2)\,c(2,4) + c(1,2)\,c(2,3)\,c(3,4) + c(1,3)\,c(3,4) + c(1,3)\,c(3,2)\,c(2,4) \\ 0 & 1 & c(2,3) & c(2,4) + c(2,3)\,c(3,4) \\ 0 & c(3,2) & 1 & c(3,4) + c(3,2)\,c(2,4) \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (16)$$

Calculation of Reachability Matrix R

The terms of the reachability matrix R can be found by following these steps:

A) Diagonal elements are all 1 (every node can reach itself).

B) R(i, j) = 1 whenever c(i, j) = 1, where c(i, j) is the corresponding element of the connectivity matrix C. This step could be omitted, but its presence saves computation time.

C) All other terms of the reachability matrix R can be computed by substituting zero or one for the elements c(i, j) from the connectivity matrix C in the paths matrix P′.

R(i, j) = P′(i, j) with values of c(i, j) inserted (17)

If we replace for all c's their appropriate values in (16) we obtain the reachability matrix R as shown:

end node:	1	2	3	4
start node				
1	1	2	2	4
2	0	1	1	2
3	0	1	1	2
4	0	0	0	1

R = (18)

A term R(i, j) = 0 implies that no path exists between nodes i and j, i.e. node j cannot be reached from node i, otherwise node j is reachable from node i. The value of R(i, j) gives the total number of paths from node i to node j. Note that these paths may share nodes, i.e. they are not necessarily node-disjoint. For example the term R(1, 4) = 4 from (18) implies that node 4 can be reached from node 1 via four alternate but not necessarily node-disjoint paths. The paths are given by the P′(1, 4) term of (16). It can be seen that only the two terms c(1, 2) c(2, 4) and c(1, 3) c(3, 4) represent node-disjoint paths from node 1 to 4.

The reachability properties of the power system can be investigated from the matrix P′ once adjacency information has been gathered. There are possible applications in security assessment.

APPLICATION TO GENERAL NETWORKS

The procedure discribed so far can be extended for path determination and connectivity and reachability evaluation of general power networks.

Augmented signal flow model

To obtain the augmented model of a power network we introduce an artificial super source node s and an artificial super sink node t. From the super source node s we craw a "directed super branch" to each of the source nodes j. The super nodes and the super branches are defined to be perfectly reliable and of infinite capacity. Similarly every one of the sink nodes k is connected to the super sink node t.

The directed super branch transmittances c(s, j) and c(k, t) for all j and k can be given as zero-one parameters if subsets of sources and or sinks are to be examined. A value of zero for the transmittance c(s, j) of a directed super branch from the super source node s to the source node j will deselect source node s to the source node j will deselect source node j from inclusion in the path expression. Similarly, c(k, t) = 0 will deselect the sink node k from inclusion in the paths expression. Thus we can find the paths between any subset of the source nodes and any subset of the sink nodes. Connectivity and reachability properties of the network are obtained from the path expression. The following example illustrate this technique.

Signal Flow Formulation for an Example Power Network

Given the power system network of Figure 4 adapted from Billinton et al. [2] with generating stations at nodes 1 and 2, and load centers at nodes 3, 4 and 5, we first find all the paths from any subset of source nodes and any subset of sink nodes using equations (2) and (3). The augmented signal flow model of the power system network of Figure 4 is shown in Figure 5.

The matrix C^T-I for the augmented signal flow graph model of Figure 5 is:

$$C' = C^T - I =$$

	1	2	3	4	5		
−1	0	c(s, 2)	0	0	0	1	s
c(s, 1)	−1	c(2, 1)	c(3, 1)	0	0	0	1
c(s, 2)	c(1, 2)	−1	0	c(4, 2)	0	0	2
0	c(1, 3)	0	−1	c(4, 3)	c(5, 3)	0	3
0	0	2(c, 4)	c(3, 4)	−1	c(5, 4)	0	4
0	0	0	c(3, 5)	c(4, 5)	−1	0	5
0	0	0	c(3, t)	c(4, t)	c(5, t)	−1	t

The s, t minor of C' will give:

P(s, t)=

$$\begin{aligned}
& c(s, 1)\, c(1, 2)\, c(2, 4)\, c(4, 5)\, c(5, t) \\
& + c(s, 1)\, c(1, 2)\, c(2, 4)\, c(4, t) \\
& + c(s, 1)\, c(1, 2)\, c(2, 4)\, c(4, 3)\, c(3, 5)\, c(5, t) \\
& + c(s, 1)\, c(1, 2)\, c(2, 4)\, c(4, 3)\, c(3, t) \\
& + c(s, 1)\, c(1, 2)\, c(2, 4)\, c(4, 5)\, c(5, 3)\, c(3, t) \\
& + c(s, 1)\, c(1, 3)\, c(3, 4)\, c(4, 5)\, c(5, t) \\
& + c(s, 1)\, c(1, 3)\, c(3, 4)\, c(4, t) \\
& + c(s, 1)\, c(1, 3)\, c(3, 5)\, c(5, t) \\
& + c(s, 1)\, c(1, 3)\, c(3, 5)\, c(5, 4)\, c(4, t) \\
& + c(s, 1)\, c(1, 3)\, c(3, t) \\
& - c(s, 1)\, c(1, 2)\, c(2, 4)\, c(4, t)\, [c(3, 5)\, c(5, 3)] \\
& - c(s, 1)\, c(1, 3)\, c(3, t)\, [c(5, 4)\, c(4, 5)] \\
& - c(s, 1)\, c(1, 3)\, c(3, 5)\, c(5, t)\, [c(2, 4)\, c(4, 2)] \\
& - c(s, 1)\, c(1, 3)\, c(3, t)\, [c(2, 4)\, c(4, 2)] \\
& + c(s, 2)\, c(2, 4)\, c(4, 5)\, c(5, t) \\
& + c(s, 2)\, c(2, 4)\, c(4, t) \\
& + c(s, 2)\, c(2, 4)\, c(4, 3)\, c(3, 5)\, c(5, t) \\
& + c(s, 2)\, c(2, 4)\, c(4, 3)\, c(3, t) \\
& + c(s, 2)\, c(2, 4)\, c(4, 5)\, c(5, 3)\, c(3, t) \\
& + c(s, 2)\, c(2, 1)\, c(1, 3)\, c(3, 4)\, c(4, 5)\, c(5, t) \\
& + c(s, 2)\, c(2, 1)\, c(1, 3)\, c(3, 4)\, c(4, t) \\
& + c(s, 2)\, c(2, 1)\, c(1, 3)\, c(3, 5)\, c(5, t) \\
& + c(s, 2)\, c(2, 1)\, c(1, 3)\, c(3, 5)\, c(5, 4)\, c(4, t) \\
& + c(s, 2)\, c(2, 1)\, c(1, 3)\, c(3, t) \\
& - c(s, 2)\, c(2, 4)\, c(4, t)\, [c(3, 5)\, c(5, 3)] \\
& - c(s, 2)\, c(2, 1)\, c(1, 3)\, c(3, t)\, [c(4, 5)\, c(5, 4)] \\
& - c(s, 2)\, c(2, 4)\, c(4, 5)\, c(5, t)\, [c(1, 3)\, c(3, 1)] \\
& - c(s, 2)\, c(2, 4)\, c(4, t)\, [c(1, 3)\, c(3, 1)].
\end{aligned}$$

Path Determination

An analysis of the paths will reveal that the terms shown in square brackets above involve extraneous loops which need to be eliminated. We therefore impose the complementarity condition (3), [c(i, j) c(j, i) = 0], causing the terms indicated in square brackets above to disappear. The remaining terms give the paths P'(s, t) as shown in (19) and listed in Table I.

P'(s,t) =
+c(s, 1) c(1, 2) c(2, 4) c(4, 5) c(5, t)
+c(s, 1) c(1, 2) c(2, 4) c(4, t)
+c(s, 1) c(1, 2) c(2, 4) c(4, 3) c(3, 5) c(5, t)
+c(s, 1) c(1, 2) c(2, 4) c(4, 3) c(3, t)
+ c(s, 1) c(1, 2) c(2, 4) c(4, 5) c(5, 3) c(3, t)
+c(s, 1) c(1, 3) c(3, 4) c(4, 5) c(5, t)
+c(s, 1) c(1, 3) c(3, 4) c(4, t)
+c(s, 1) c(1, 3) c(3, 5) c(5, t)
+c(s, 1) c(1, 3) c(3, 5) c(5, 4) c(4, t)
+c(s, 1) c(1, 3) c(3, t)
+c(s, 2) c(2, 4) c(4, 5) c(5, t)
+c(s, 2) c(2, 4) c(4, t)
+ c(s, 2) c(2, 4) c(4, 3) c(3, 5) c(5, t)
+c(s, 2) c(2, 4) c(4, 3) c(3, t)
+ c(s, 2) c(2, 4) c(4, 5) c(5, 3) c(3, t)
+c(s, 2) c(2, 1) c(1, 3) c(3, 4) c(4, 5) c(5, t)
+c(s, 2) c(2, 1) c(1, 3) c(3, 4) c(4, t)
+c(s, 2) c(2, 1) c(1, 3) c(3, 5) c(5, t)
+c(s, 2) c(2, 1) c(1, 3) c(3, 5) c(5, 4) c(4, t)
+c(s, 2) c(2, 1) c(1, 3) c(3, t). (19)

Table I. All the Paths from the Generating Busses (1, 2) to the Load Centers (3, 4, 5).

end points	path expression	nodes along the path
from node 1 to 3	c(1, 2) c(2, 4) c(4, 3) c(1, 3) c(1, 2) c(2, 4) c(4, 5) c(5, 3)	1, 2, 4, 3 1, 3 1, 2, 4, 5, 3
from node 1 to 4	c(1, 2) c(2, 4) c(1, 3) c(3, 4) c(1, 3) c(3, 5) c(5, 4)	1, 2, 4 1, 3, 4 1, 3, 5, 4
from node 1 to 5	c(1, 3) c(3, 5) c(1, 3) c(3, 4) c(4, 5) c(1, 2) c(2, 4) c(4, 3) c(3, 5) c(1, 2) c(2, 4) c(4, 5)	1, 3, 5 1, 3, 4, 5 1, 2, 4, 3, 5 1, 2, 4, 5
from node 2 to 3	c(2, 1) c(1, 3) c(2, 4) c(4, 3) c(2, 4) c(4, 5) c(5, 3)	2, 1, 3 2, 4, 5, 3 2, 4, 5, 3
from node 2 to 4	c(2, 4) c(2, 1) c(1, 3) c(3, 4) c(2, 1) c(1, 3) c(3, 5) c(5, 4)	2, 4 2, 1, 3, 4 2, 1, 3, 5, 4
from node 2 to 5	c(2, 4) c(4, 3) c(3, 5) c(2, 1) c(1, 3) c(3, 4) c(4, 5) c(2, 1) c(1, 3) c(3, 5) c(2, 4) c(4, 5)	2, 4, 3, 5 2, 1, 3, 4, 5 2, 1, 3, 5 2, 4, 5

The P' (s,t)$_{j}$ paths expression (19) can be used to find the number of paths between any nodes i and j, where i and j take the roles of a "source" and a "sink" node respectively. Having used an artificial super source node s and an artificial super sink node t, we set c(s, i) = 1 and then set all other c(s, k) = 0 in order to select node i as the present source of interest. Similarly we let all c(h, t) = 0, except h = j for which c(j, t) = 1 so that sink node j is selected. The resulting expression gives all the paths between "source" node i and "sink" node j.

For example, if we are interested in finding all the paths from source node 1 to sink node 3 it suffices to set c(s, 1) = 1, c(s, 2) = 0, c(3, t) = 1 and c(4, t) = c(5, t) = 0 in (19). Thus

$$P'(1, 3) = c(1, 2)\ c(2, 4)\ c(4, 3) + c(1, 2)\ c(2, 4)\ c(4, 5)\ c(5, 3) + c(1, 3). \quad (20)$$

If interested in finding all the paths between source node 2 and all the sink nodes, we set c(s, 2) = 1, c(s, 1) = 0, c(3, t) = 1, c(4, t) = 1 and c(5, t) = 1 yielding

$$\begin{aligned} P'(2, t) = {} & c(2, 4)\ c(4, 5) + c(2, 4) + \\ & c(2, 4)\ c(4, 3)\ c(3, 5) + \\ & c(2, 4)\ c(4, 3) + c(2, 4)\ c(4, 5)\ c(5, 3) + \\ & c(2, 1)\ c(1, 3) + \\ & c(2, 1)\ c(1, 3)\ c(3, 5)\ c(5, 4) + \\ & c(2, 1)\ c(1, 3)\ c(3, 4)\ c(4, 5) + \\ & c(2, 1)\ c(1, 3)\ c(3, 4)\ c(4, 5) + \\ & c(2, 1)\ c(1, 3)\ c(3, 4) + \\ & c(2, 1)\ c(1, 3)\ c(3, 5). \quad (21) \end{aligned}$$

Generation of Pairwise Node-Connectivity Table

We construct a vector p for each path which contains the nodes of the path in the order traversed. The node-disjoint paths are found by excluding the source and sink nodes from the path vectors leaving us with modified path vectors L (see Table II).

Table II. p, L Vectors and Pairwise connectivity

p vectors	L vectors	pairwise node-connectivity
[1, 2, 4, 3] [1, 3] [1, 2, 4, 5, 3]	L(1) = [2, 4] L(2) = [Null] L(3) = [2, 4, 5]	(1, 3 are adjacent)
[1, 2, 4] [1, 3, 4] [1, 3, 5, 4]	L(4) = [2] L(5) = [3] L(6) = [3, 5]	(2 disjoint sets) $k^n{}_{14} = 2$
[1, 3, 5] [1, 3, 4, 5] [1, 2, 4, 3, 5] [1, 2, 4, 5]	L(7) = [3] L(8) = [3, 4] L(9) = [2, 4, 3] L(10)=[2, 4]	(2 disjoint sets) $K^n{}_{15} = 2$
[2, 1, 3] [2, 4, 3] [2, 4, 5, 3]	L(11) = [1] L(12) = [4] L(13) = [4, 5]	(2 disjoint sets) $K^n{}_{23} = 2$
[2, 4] [2, 1, 3, 4] [2, 1, 3, 5, 4]	L(14) = [Null] L(15) = [1, 3] L(16) = [1, 3, 5]	(2, 4 are adjacent)
[2, 4, 3, 5] [2, 1, 3, 5] [2, 4, 5]	L(17) = [4, 3] L(19) = [1, 3] L(20) = [4]	(2 disjoint sets) $k^n{}_{25} = 2$

The next step is to compare the vectors L to determine the number of disjoint sets among the L vectors. This is the pairwise source-sink connectivity.

Determination of Node-Connectivity

The minimum of the pairwise connectivities shown in Table II is equal to two and it is by definition the overall source-sink connectivity index. It is this number that we can use to compare two alternative power networks for performance under emergency conditions. For the example network analyzed, there are two nodes whose failure is sufficient to disrupt all the paths available for the transmission of power from the generating stations to the load centers.

So far we have assumed that branches (transmission lines plus breakers) are not subject to failure. In the next section we consider the case where only branches are subject to failure and assume nodes do not fail. Finally we will take up the case where either branches and or nodes could fail.

Generation of Branch-Connectivity Table

From the paths listed in Table I we can find all minimal branch cut sets, then compute the pairwise branch-connectivities K^b_{ij}. This information is listed in Table III.

Table III. All the Paths including branches only, minimal branch cut sets and pairwise branch connectivities

end points	paths	minimal branch cut sets	K^b_{ij}
from n1 to n3	b3, (b2 + b7), b4 (b1 + b6) b3, (b2 + b7), b8, b5	[(b1 + b6), b3]	3
from n1 to n4	b3, (b2 + b7) (b1 + b6), b4 (b1 + b6), b5, b8	[(b1 + b6), b3]	3
from nl to n5	(b1 + b6), b5 (b1 + b6), b4, b8 b3, (b2 + b7), b4, b5 b3, (b2 + b7), b8	[b8, b5]	2
from n2 to n5	b3, (b1 + b6) (b2 + b7), b4 (b2 +b7), b8, b5	[b2 + b7), b3]	3
from n2 to n4	(b2 + b7) b3, (b3 + b6), b4 b3, (b1 + b6), b5, b8	[(b2 + b7), b3] [(b2 + b7), b3]	3
from n2 to n5	(b2 + b7), b4, b5 b3, (b1 + b6), b4, b8 b3, (b1 + b6), b5 (b2 + b7), b8	[b8, b5]	2
Global branch Connectivity			$K^b(G) = 2$

Table IV The Paths including nodes and branches, minimal mixed cut sets and pairwise mixed connectivities

end points	paths	minimal mixed cut sets	$K^{bn}{}_{i}$
from n1 to n3	b3, n2, (b2 + b7), n4, b4 (b1 + b6) b3, n2, (b2 + b7), n4, b8, n5, b5	(b1 + b6), b3 (b1 + b6), n2 (b1 + b6), n4	3
from n1 to n4	b3, n2, (b2 + b7) (b1 + b6), n3, b4 (b1 + b6), n3, b5, n5, b8	(b3, n3) (n2, n3)	2
from n1 to n5	(b1 + b6), n3, b5 (b1 + b6), n3, b4, n4, b8 b3, n2, (b2 + b7), n4, b4, n3, b5 b3, n2, (b2 + b7), n4, b8	(n3, n2) (n3, n4) (n3, b8) (n3, b3)	2
from n2 to n3	b3, n1, (b1 + b6) (b2 + b7), n4, b4 (b2 + b7), n4, b8, n5, b5	(n4, b3) (n4, n1)	2

Determination of Branch-Connectivity

The global branch-connectivity for the power system is equal to the minimum of the pairwise branch-connectivities which equals to two for this example. This implies that there exist two branches whose failure is sufficient to disrupt all the paths available for the transmission of power form the generation sources to the load centers.

Generation of Pairwise Mixed-Connectivity Table

The minimal mixed cut sets and the pairwise mixed connectivities are shown in Table IV.

Determination of Mixed Connectivity

The mixed connectivity $K^{bn}{}_{ij}$ is equal to the minimum of the pairwise mixed connectivities as shown in Table IV. It is equal to two for this example. This implies that there are two components whose failure is sufficient to interrupt the transmission of power.

Having computed the connectivity indices we can go back to the network and think of ways to modify it in order to improve the integrity of the power system.

CONCLUSIONS

It has been shown that the minors of the matrix C^T-I can be used for path determination and connectivity and reachability evaluations, where C is the connectivity matrix associated with the power system.

Note that our procedure will give all the possible paths in a straightforward manner and that it can be readily implemented on a computer. It promises to be far more efficient and consistant than present enumeration schemes partially based on inspection. This could be a basis for planning a switching schedule for planned outages or for preparing contingency plans based on simulated outages for on-line security analysis. In practice the transmission paths would be constrained by power flow.

Power system integrity can be studied using connectivity indices. Reachability properties of the power system can be obtained using the formulas given in this paper in conjunction with adjacency information about the power system.

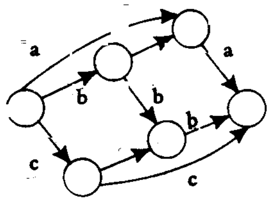

Fig. 1

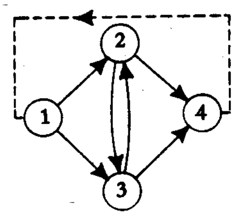

Fig. 2

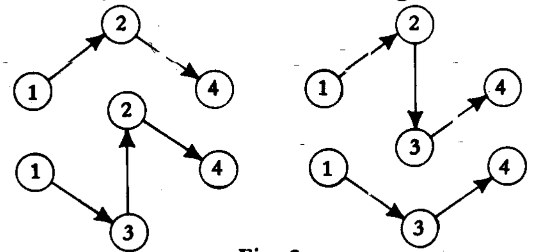

Fig. 3

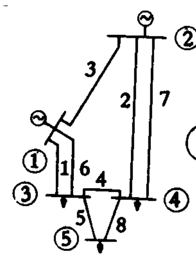

Fig. 4

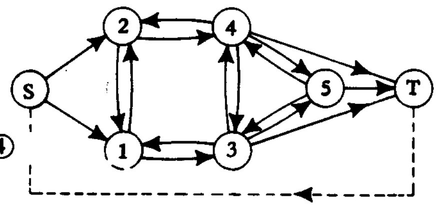

Fig. 5

ACKNOWLEDGMENT

We thank Professor Robin Keller, Lemba Davy Nyirenda, Magdy Saeb, Dennis Brehm and Masoud Tehrani. The research leading to this paper was supported in part by the OFFICE OF WATER RESEARCH AND TECHNOLOGY, USDI under the Matching Grant Program of Public Law 95-467 and by the University of California Water Resources and Technology Project No. A-086-CAL and Water Resources Center Project UCALWRC-W-614.

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