DETERMINING ECONOMIC ORDER QUANTITY IN A TWO-LEVEL SUPPLY CHAIN WITH TRANSPORTATION COST

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ABSTRACT

This paper considers a two-level supply chain consisting of one warehouse and a number of identical retailers. Unlike the common practice which determines the optimal ordering policy according to inventory costs only, in this model we consider the transportation cost as well. We suppose that the delivery of each order from the warehouse to any retailer is made by a single vehicle without splitting. We also assume that there are three types of vehicles which are defined as small, medium and large. Each type has its own fixed cost, variable cost, and the capacity size. We assume that the demand rate at each retailer is known and the demand is confined to a single item. Shortages are allowed neither at the retailers nor at the warehouse. First, we obtain the total cost which is the sum of the holding and ordering cost at the warehouse and retailers as well as the transportation cost from the warehouse to retailers. Then, we find the economic order quantities for the warehouse and retailers which minimize the total cost.

KEYWORDS

Economic Order Quantity, Supply chain, Inventory control, Transportation

1. INTRODUCTION

The goal of most research efforts related to the supply chain management is to present mechanisms to reduce operational costs. The most important operational costs in a supply chain are the inventory holding cost and the transportation cost.

In the replenishment process, other than the inventory cost the transportation cost is a major cost factor that affects the optimal shipment size. Some articles in supply chain consider the transportation cost as a part of the ordering cost and assume it is independent of the shipment size; e.g. Hill, R.M. (1997), Goyal, S.K. and Nebebe, F. (2000), and Hoque, M.A. and Goyal, S.K. (2000).

In many practical cases, the transportation cost is affected by the shipment size and vice versa. So, it is important to determine the economic order quantity which minimizes the overall logistics costs.

Ganeshan, R. (1999) introduces a three-level supply chain consisting of a number of identical retailers, one central warehouse, and a number of identical suppliers. In his model, the objective function consists of the ordering, the holding, and the transportation costs. He considers the transportation cost as a function of the order quantity but ignores the capacity of the vehicle. Ertogral, K. et al. (2007) consider a vendor-buyer supply chain model and incorporate the transportation cost. In their work, the transportation is made by one type of vehicle whose cost is a function of the shipment size; this function has an all-unit-discount structure. Our model differs from the one proposed by Ertogral, K. et al. (2007) in the sense that we assume there are three types of vehicles which are defined as small, medium, and large. Each type has its own fixed cost, variable cost, and the capacity size.

2. THE MODEL

In this paper we consider a two-level supply chain consisting of one warehouse and a number of identical retailers, (Fig1). We assume that the demand rate at each retailer is known and the demand is confined to a single item. Shortage is allowed neither at the retailers nor at the warehouse.

The transportation time for an order to arrive at a retailer from the warehouse is assumed to be constant. The warehouse orders to an external
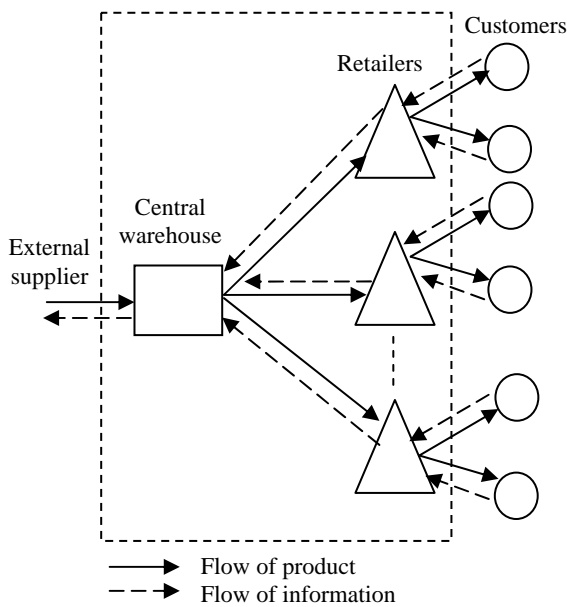
Table 1. Transportation Scheme

Vehicle Type	Capacity	Fixed Cost	Variable Cost
S	q_1	F_1	v_1
M	q_2	F_2	v_2
L	q_3	F_3	v_3

Fig. 1. A two-level supply chain

Fig. 2. Variations of transportation cost

supplier. The lead time for an order to arrive at the warehouse is assumed to be constant. We assume that the retailers are identical; i.e., the parameters related to the retailers such as the demand rate, the rate of holding cost, the ordering cost and the transportation time are same for all the retailers. The objective is to find the economic order quantities for the retailers and the warehouse which minimize the total cost. The total cost is the sum of the holding and ordering costs at the warehouse and retailers as well as the transportation cost from the warehouse to retailers.

3. TRANSPORTATION SCHEME

In this model, we suppose that there are three types of vehicles and delivery of each order from warehouse to a retailer is made by a single vehicle without splitting. It is a common transportation scheme in most practical cases. We define these types as small (S), medium (M), and large (L). Each type has its own fixed cost, variable cost and the capacity size. Table 1 shows the context of transportation scheme.

It is assumed that $F_1<F_2<F_3$, $v_1>v_2>v_3$, $q_1<q_2<q_3$, $F_2=F_1+q_1(v_1-v_2)$, and $F_3=F_2+q_2(v_2-v_3)$. These equations are supposed to avoid any over declaration. Hence, the transportation cost varies according to the order quantity as shown in Figure 2.
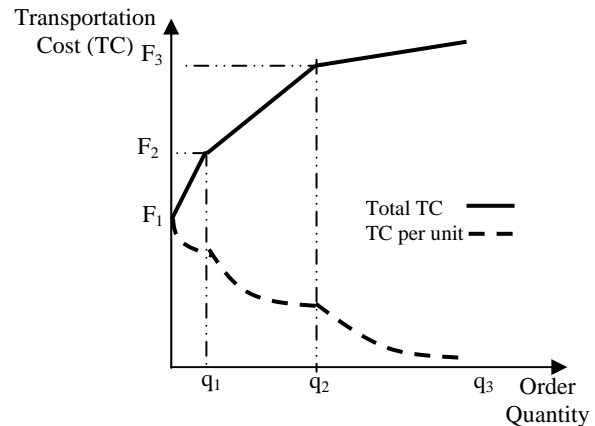
4. FORMULATION OF THE TOTAL COST

In this section, we intend to obtain the total cost. The total cost is the sum of the holding and ordering costs at the warehouse and retailers as well as the transportation cost from the warehouse to retailers. The notations used in formulation are as follows:

D_r: Demand rate at a retailer.
A_r: Ordering cost for a retailer.
A_w: Ordering cost for the warehouse.
h_r: Rate of holding cost at a retailer.
h_w: Rate of holding cost at the warehouse.
Q_r: Order quantity at a retailer.
Q_w: Order quantity at the warehouse.
m: Number of retailers.

We suppose that the demand rate at the retailers and the transportation time to the retailers are constant and shortage is not allowed at the retailers. Hence, the inventory level at the retailers is a simple EOQ model.

It is assumed that there is no lot-splitting at the warehouse. Furthermore, shortage is not allowed at the warehouse so the order quantity of the warehouse includes an integer multiple (n) of the order quantity of each retailer. Since there are m identical retailers therefore the order quantity of the warehouse is

$Q_w = mnQ_r$. For optimal solution the arrival of an order to the warehouse corresponds to the delivery of an order to each retailer. Thus, the maximum inventory level at the warehouse is $Q_w - mQ_r$.

The total cost is the sum of the holding and ordering costs at the retailers and the warehouse plus the transportation cost from the warehouse to retailers. Thus, the total cost can be written as:

$$C_{Ti}(Q_w, Q_r) = \frac{D_w A_w}{Q_w} + \frac{h_w(Q_w - mQ_r)}{2} +$$
$$m(\frac{D_r A_r}{Q_r} + \frac{h_r Q_r}{2} + \frac{D_r F_i}{Q_r} + D_r v_i) , i = 1,2,3 \quad (1)$$

To minimize the above cost we must consider the following constraints:

$$q_{(i-1)} < Q_r \le q_i \qquad\qquad (1.1)$$
$$Q_w = mnQ_r \qquad\qquad (1.2)$$
$$n \text{ is a positive int and } q_0 = 0 \qquad (1.3)$$

Index i in (1) denotes the vehicle types; 1, 2 and 3 respectively for S, M and L. D_w is the demand rate at the warehouse which is sum of the demand rates at the retailers, $D_w = mD_r$.

Substituting mnQ_r for Q_w and mD_r for D_w in (1) then our mathematical problem can be defined as:

$$Min\, C_{Ti}(n, Q_r) = \frac{D_r A_w}{nQ_r} + \frac{h_w mQ_r(n-1)}{2} +$$
$$m(\frac{D_r A_r}{Q_r} + \frac{h_r Q_r}{2} + \frac{D_r F_i}{Q_r} + Dv_i), i = 1,2,3 \quad (2)$$

s.t.

$$q_{(i-1)} < Q_r \le q_i \qquad\qquad (2.1)$$
$$n \text{ is a positive int and } q_0 = 0 \quad (2.2)$$

5. ALGORITHM TO FIND THE OPTIMAL SOLUTION

According to the assumption that we consider for the transportation scheme (Section 3), the total cost function (2) has the piece-wise convex property for a given value of n. Figure 3 graphically shows the total cost for a given value of n.

The transportation cost has an incremental discount structure. Hence, for a given value of n the method of obtaining the optimal value of Q_r is the same as the incremental quantity discount model described by Hadley, G. and Whitin, T.M. (1963).

We develop a search algorithm to obtain the optimal value of n and Q_r. As mentioned above, we apply the incremental quantity discount method for a given value of n. To create our search algorithm we need a lower bound and an upper bound for n.
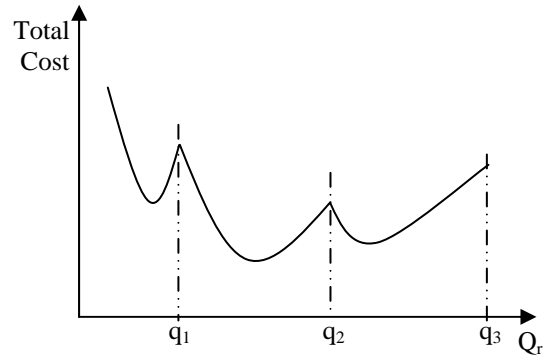

Fig. 3. Total cost for a given value of n

Clearly 1 is a lower bound for n. The following proposition generates the upper bound for n.

Proposition: The upper bound of n is:

$$n_u = \left\lfloor \sqrt{\frac{A_w(h_r - h_w)}{mh_w(A_r + F_1)}} \right\rfloor,$$

If $\left\lfloor \sqrt{\frac{A_w(h_r - h_w)}{mh_w(A_r + F_1)}} \right\rfloor = 0$ then $n_u = 1$.

($\lfloor X \rfloor$ represents the largest integer less than or equal to X).

Proof: In the first interval of Q_r, the total cost function is:

$$C_{T1}(n, Q_r) = \frac{D_r A_w}{nQ_r} + \frac{h_w mQ_r(n-1)}{2} +$$
$$m(\frac{D_r A_r}{Q_r} + \frac{h_r Q_r}{2} + \frac{D_r F_1}{Q_r} + D_r v_1), i = 1,2,3 \quad (3)$$

If we set the derivatives of C_{T1} with respect to Q_r and n equal to zero we obtain:

$$Q^*_r = \sqrt{\frac{2D_r(\frac{A_w}{n} + mA_r + mF_1)}{m(h_w(n-1) + h_r)}} \qquad (4)$$

$$n^* = \frac{1}{Q_r}\sqrt{\frac{2D_r A_w}{mh_w}} \qquad (5)$$

Substituting Q^*_r in (3) we have:

$$C_{T1}(n) = \sqrt{2D_r m(\frac{A_w}{n} + mA_r + mF_1)(h_w(n-1) + h_r)}$$
$$+ Dv_1 \qquad (6)$$

The value of n which optimizes $C_{T1}(n)$ is obtained as:

$$n^* = \sqrt{\frac{A_w(h_r - h_w)}{mh_w(A_r + F_1)}} \qquad (7)$$

From (5), it is clear that n and Q_r have an inverse relation. The value of Q_r obtained from (4) is a lower bound on Q_r, because there is no gain to decrease Q_r less than Q^*_r. Hence, the n^* in (7) would be an upper bound on n.

In summary, the algorithm to obtain the optimal values of n and Q_r is as follows:

Algorithm:

1- Set $n_u = \left\lfloor \sqrt{\dfrac{A_w(h_r - h_w)}{mh_w(A_r + F_1)}} \right\rfloor$

if $\left\lfloor \sqrt{\dfrac{A_w(h_r - h_w)}{mh_w(A_r + F_1)}} \right\rfloor = 0$ then $n_u = 1$.

2- For $n=1, 2,\dots,n_u$ find the corresponding optimal value of Q_r by incremental quantity discount method to minimize the total cost.

3- For $n=1, 2,\dots,n_u$ and the corresponding optimal value of Q_r calculate the minimum total cost.

4- The solution which has the minimum total cost among the solutions in step 3 is the overall optimal solution.

6. CONCLUSIONS

In this paper, we considered a two-level supply chain consisting of one warehouse and a number of identical retailers. Unlike the common practice which determines the optimal ordering policy according to inventory costs only, in this model we incorporate transportation costs into inventory replenishment decisions. We derived the total cost which is the sum of the holding and ordering cost at the warehouse and retailers as well as the transportation cost from the warehouse to retailers. The total cost function is a piece-wise convex function. Based on this property, we proposed a search algorithm to obtain the optimal solution.

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