A NEW APPROACH TO INVENTORY CONTROL IN A TWO-LEVEL SUPPLY CHAIN SYSTEM WITH POISSON DEMAND

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ABSTRACT

In this paper we consider a two-level supply chain system consisting of one warehouse and one retailer. The retailer faces Poisson demand. We introduce a new approach to inventory control in the supply chain management which is different from the classical policies used in the literature of inventory and production control systems. In this system, the retailer constantly orders a fixed amount of product to the warehouse in a predetermined time interval; i.e., the ordering size and the time interval between any two consecutive orders from the retailer to the warehouse are fixed numbers. The advantage of this policy is that the warehouse is facing a uniform and deterministic demand which simplifies the management of the supply chain. Using queuing theory concepts, we derive the expected total cost per time unit for this system and propose a search algorithm to compute the optimal solution.

KEYWORDS

Inventory, Queueing systems, Supply chain management

1. INTRODUCTION

In this paper we consider a two-level supply chain system consisting of one warehouse and one retailer (Fig. 1). We assume that the retailer faces a Poisson demand and unsatisfied demand will be lost. The transportation time for an order to arrive at a retailer from the warehouse is assumed to be constant. The warehouse orders to an external supplier and the lead time for an order to arrive at the warehouse is assumed to be constant.

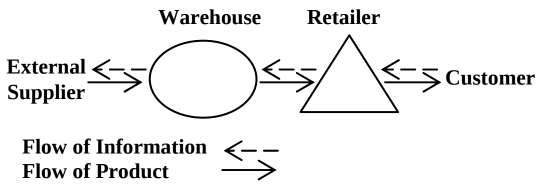

Fig. 1. A two-level supply chain

There are some papers that investigate inventory control policy in such a two-level supply chain system. Axsäter, S. (1993a) investigates a two-echelon inventory system in which the inventory policy of each echelon is (r, Q). Axsäter, S. (1993b) also considers a two-echelon inventory system based on order-up-to-S policy with periodic review. Matta, K.F. and Sinha, D. (1995) study a two-echelon inventory system consisting of a central warehouse and a number of retailers. Each retailer applies (T, S) inventory policy with an identical review interval T and different maximum inventory level S. the central warehouse applies the (T, s, S) policy, where T is the same review interval as that of retailers; s is its reorder point, and S is its desired maximum inventory level. Forsberg, R. (1996) considers an exact evaluation of (r, Q) policies for two-level inventory systems with Poisson demand. Axsäter, S. and Zhang, W.F. (1999) consider a two-level inventory system with a central warehouse and a number of identical retailers. The warehouse uses a regular installation stock batch-ordering policy, but the retailers apply a different type of policy. When the sum of the retailers' inventory positions declines to a certain "joint" reorder point, the retailer with the lowest inventory position places a batch quantity order. Seifbarghi, M. and Akbari, M.R. (2006) investigate an inventory system consisting of one central warehouse and many identical retailers controlled by continuous review policy (r, Q). The demands of retailers are independent Poisson and stockouts in the retailers amount to lost sales.

In this paper we introduce a new approach to inventory control in the supply chain management which is different from the inventory policies used in the literature of inventory and production control systems. In our model the warehouse uses continues review policy, but the retailer applies a new periodic ordering policy. In this system, the retailer orders a fixed quantity to the warehouse in every predetermined time interval; i.e., the ordering size and the time interval between any two consecutive orders from the retailer to the warehouse are fixed numbers. The most notable advantage of this policy is that the retailers' orders, which constitute warehouse demand, are deterministic. The deterministic demand for the warehouse leads to a simplified inventory control and one of whose advantages is elimination of the safety stock at the warehouse.

Using queuing theory concepts, we evaluate the expected total system cost in the steady state. The total system cost contains the holding, ordering and shortage costs at the retailer and the holding and ordering costs at the warehouse. We assume that the time interval between two consecutive orders of the retailer is known. The objective is to determine the optimal order quantity for the retailer which minimizes the total system cost.

2. COST EVALUATION

We consider a two-level supply chain system consisting of one warehouse and one retailer. In this system the warehouse uses continues review policy, but the retailer constantly orders a fixed amount of product to the warehouse in a predetermined time interval.

The assumptions of the model and the notations are as follows:

Assumptions:

- The retailer faces a Poisson demand.
- The time interval between two consecutive orders of the retailer is known.
- Unsatisfied demand by the retailer will be lost.
- Shortage is not allowed at the warehouse.
- There is no lot-splitting at the warehouse.
- The transportation time for an order to arrive at the retailer from the warehouse is constant.
- The warehouse orders to an external supplier with infinite capacity.
- The lead time for an order to arrive at the warehouse is constant.

Notation:

λ: Demand intensity at the retailer.
s_r: Cost of a lost sale at the retailer.
h_r: Holding cost rate at the retailer.
h_w: Holding cost rate at the warehouse.
A_r: Ordering cost for the retailer.
A_w: Ordering cost for the warehouse.
T_r: Time interval between any two consecutive orders of the retailer.
T_w: Time interval between any two consecutive orders of the warehouse.
Q_r: Order quantity of the retailer.
Q_w: Order quantity of the warehouse.
I_r: Average Inventory level at the retailer in the steady state.
Ch_r: Expected holding cost per time unit at the retailer in the steady state.
CS_r: Expected lost sale cost per time unit at the retailer in the steady state.
TC_r: Expected total cost per time unit at the retailer in the steady state.
TC_w: Expected total cost per time unit at the warehouse in the steady state.
TC_s: Expected total system cost per time unit in the steady state.

For this system we derive the expected total system cost in the steady state. The total system cost contains the holding, ordering and shortage costs at the retailer and the holding and ordering cost at the warehouse. In section 2.1 we investigate formulation of the retailer cost. In section 2.2 the inventory cost at the warehouse is analyzed. Finally, in section 2.3 we present the total system cost.

2.1. Formulation of the retailer cost

The retailer orders a fixed quantity (Q_r) to the warehouse in a predetermined time interval (T_r). Inventory level at the retailer changes as shown in Figure 2.

To analyze the average inventory level at the retailer we resort to some concepts of queuing theory. To do this, we consider the arrival of orders from the warehouse to the retailer as a batch arrival process to a queuing system. The inter-arrival times of batches are constant and are equal to T_r. The service time of each unit of product is the inter-arrival times of demands to the retailer which is exponential with mean λ^{-1}, and the retailer's inventory level as the number of units in the system. Hence, the inventory problem at the retailer can be interpreted as a $D^Q/M/1$ queuing system with bulk input of size Q (Gross, D. and Harris, C.M. 1998).

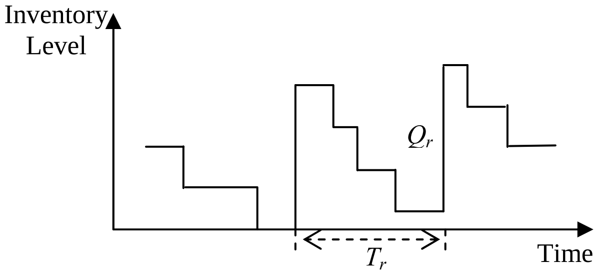

Fig. 2. Inventory level at the retailer

Let I_r represent the mean number in system at the $D^Q/M/1$ queue in the steady state; i.e., I_r is the average inventory level at the retailer in the steady state. Therefore, the expected holding cost per time unit at the retailer in the steady state would be:

$$Ch_r = h_r I_r \qquad (1)$$

We approximately calculate I_r by the formula developed by Yao, D.D.W. et al. (1984). According to this approach the mean number in system at the $D^Q/M/1$ queue in the steady state is:

$$I_r = \frac{Q_r}{2\lambda T_r}\left[\frac{\exp\left[\frac{2}{3}(Q_r - \lambda T_r)\right]Q_r}{\lambda T_r - Q_r} + Q_r + 1\right] \qquad (2)$$

The inventory level at the retailer is stable whenever the ratio of the arrival rate to the demand rate is less than 1. Thus, we consider this constraint in the model:

$$\frac{Q_r}{T_r\lambda} < 1 \Rightarrow \frac{Q_r}{T_r} < \lambda \qquad (3)$$

Let π_0 stand for the ratio of time that the retailer is out of stock. Thus, the amount of demand per time unit that is lost in the steady state is $\lambda\pi_0$. The expected lost sale cost per time unite at the retailer in the steady state is:

$$CS_r = s_r \lambda \pi_0 \qquad (4)$$

The arrival rate of product to the retailer is $Q_r T_r^{-1}$ and the rate of demand is λ, so the ratio of time that the retailer is out of stock is:

$$\pi_0 = 1 - \frac{Q_r}{\lambda T_r} \qquad (5)$$

The retailer incurs an ordering cost, A_r, whenever it makes an order; i.e., every Tr time unit. So, the average of ordering cost per time unit is $A_r T_r^{-1}$. Therefore, the expected total cost per time unit at the retailer in the steady state which contains the ordering, holding and shortage costs is as follows:

$$TC_r = \frac{A_r}{T_r} + h_r I_r + s_r \lambda \pi_0 \qquad (6)$$

$$s.t$$

$$\frac{Q_r}{T_r} < \lambda \qquad (6.1)$$

$$Q_r \ is \ \mathrm{int}$$

2.2. Formulation of the warehouse cost

The retailer's orders which constitute warehouse demands change the inventory level at the warehouse as shown in Figure 3. It is assumed that there is no lot-splitting at the warehouse. Furthermore, shortage is not allowed at the warehouse so that the order quantity of the warehouse is an integer multiple (n) of the order quantity of the retailer. Clearly, for the optimal solution the arrival of an order to the warehouse must correspond to the delivery of an order to the retailer. Thus, the maximum inventory level at the warehouse is Q_w-Q_r or $(n-1)Q_r$ and $T_w = nT_r$.

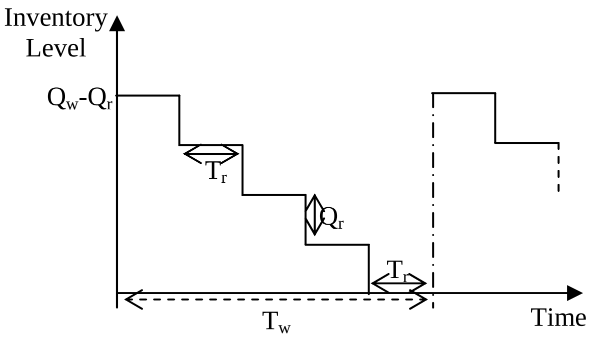

Fig 3. Inventory level at the warehouse

The expected total cost per time unit at the warehouse in the steady state is the sum of the ordering and holding costs which is formulated as follows:

$$TC_w = \frac{A_w}{T_w} + \frac{h_w(Q_w - Q_r)}{2} \qquad (7)$$

$$Q_w = nQ_r \qquad (7.1)$$

$$T_w = nT_r \qquad (7.2)$$

$$n \ is \ a \ positive \ \mathrm{int}$$

Substituting nQ_r for Q_w and nT_r for T_w in (7) we have:

$$TC_w = \frac{A_w}{nT_r} + \frac{h_w(n-1)Q_r}{2} \qquad (8)$$
$$n \text{ is a positive int}$$

2.3. The total system cost

The total system cost per time unit is the sum of the total cost per time unit at the retailer and the total cost per time unit at the warehouse. So, the expected total system cost per time unit in the steady state is:

$$TC_s = \frac{A_r}{T_r} + h_r I_r + s_r \lambda \pi_0 + \frac{A_w}{nT_r} + \frac{h_w(n-1)Q_r}{2} \qquad (9)$$

$$s.t$$

$$\frac{Q_r}{T_r} < \lambda \qquad (9.1)$$

$$Q_r, n \text{ are positive int}$$

3. OPTIMIZATION APPROACH

To obtain the optimal solution we develop a search algorithm. Regarding constraint (9.1), for a predetermined value of T_r the retailer order quantity, Q_r, can vary from 1 to λT_r. Q_r is a positive integer whose upper bound is $\lfloor \lambda T_r \rfloor$, (Let $\lfloor X \rfloor$ and $\lceil X \rceil$ represent the largest integer less than or equal to X and the smallest integer greater than or equal to X respectively) . Thus, we iterate the algorithm $\lfloor \lambda T_r \rfloor$ times and for each value of 1,2,…, $\lfloor \lambda T_r \rfloor$ for Q_r we analytically find the optimal value of n. Assuming n is continuous, we take the derivative of TC_s with respect to n and equal it to zero to obtain the optimal value of n as:

$$n^* = \sqrt{\frac{2A_w}{T_r Q_r h_w}} \qquad (10)$$

Since n is a positive integer we set $n=\lceil n^* \rceil$ and $n=\lfloor n^* \rfloor$ (if $\lfloor n^* \rfloor=0$ set n=1) then compute corresponding value of TC_s; each value for n which leads to a lower value for TC_s will be designated as n^*. The pair Q_r and n^* which give the minimum value for TC_s form, in effect, the optimal values and the corresponding TC_s is the optimal solution. The steps of the algorithm can be written as follows:

Step 0. Set Q_r=1.

Step 1. Compute I_r from Eq. (2) and Π_0 from Eq. (5).

Step 2. Compute n^* from Eq. (10).

Step 2.1. Let $n=\lfloor n^* \rfloor$, if $\lfloor n^* \rfloor=0$ then n=1, compute the corresponding value of TC_s from Eq. (9) and let TC_s-*Step*2.1= TC_s.

Step 2.2. Let $n=\lceil n^* \rceil$, compute the corresponding value of TC_s from Eq. (9) and let TC_s-*Step*2.2= TC_s.

Step 3. Set $TC_s(Q_r)$=min(TC_s-Step2.1, TC_s-Step2.2).

Step 4. Set $Q_r=Q_r+1$,

Step 5. If $Q_r \le \lfloor \lambda T_r \rfloor$ then go to step 1.

Step 6. *Op*-TC_s=min($TC_s(Q_r)$; Q_r =1,.., $\lfloor \lambda T_r \rfloor$).

Step 7. *Op*-TC_s is the optimal solution.

Step 8. End.

4. CONCLUSIONS

In this paper we introduced a new ordering policy and analyzed its application in a two-level supply chain system. The most important advantage of this policy is that the warehouse is facing a uniform and deterministic demand. This advantage facilitates the inventory planning and leads to elimination of safety stock at the warehouse.

We resorted to some concepts of queuing theory to analyze the inventory problem at the retailer. We used a $D^Q/M/1$ queuing system to compute the average of inventory level and the average of lost sales at the retailer. Since the warehouse demand which is originated by the retailer is deterministic, one can easily compute the total cost of the warehouse. We derived the expected total system cost in the steady state. Finally, we proposed a search algorithm to obtain the optimal solution.

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