Economic Order Quantity in a Centralized Two-Level Supply Chain with Transportation Cost

Armand Baboli[1], Mohammadali Pirayesh Neghab[1,2], Rasoul Haji[2]

[1]Lab. PRISMa, INSA de Lyon, 19 avenue Jean Capelle, 69621Villeurbanne Cedex, France
(arman.baboli@insa-lyon.fr) Tel: +33472436201, Fax: +33437479113
[2]Industrial Engineering Dept, Sharif University of Technology, Tehran, Iran

ABSTRACT

This paper considers a two-level supply chain consisting of one warehouse and one retailer. Unlike the most similar models which determine the optimal ordering policy according to inventory cost only, in this model, we also consider the transportation cost. We assume that the demand rate at the retailer is known and the demand is confined to a single item. Shortages are allowed neither at the retailer nor at the warehouse. The objective is to find the economic order quantities for both the retailer and the warehouse which minimize the total cost. That is, the sum of the holding and ordering cost at the retailer and warehouse as well as the transportation cost from warehouse to retailer. Numerical results show savings can be made by this model in comparison to the model of optimal ordering policy in which the transportation cost is not considered.

Keywords: Economic Order Quantity, Supply chain, Inventory control, Transportation

1- INTRODUCTION

The goal of many research efforts related to the supply chain management is to present models to reduce operational costs. Inventory holding cost and the transportation cost are regarded as the most important operational costs in a supply chain. There are many research undertakings in supply chain that consider the transportation cost as a part of the ordering cost and thus assume that it is independent of the size of the shipment. Axsäter [1,2], Forsberg [3,4], Matta and Sinha[5], and Seifbarghi and Akbari [6] investigate different models in a two-level inventory system. These models include a central warehouse and a number of retailers. In these models, the total cost consists of holding cost at the warehouse and at the retailers as well as the shortage cost at the retailers. Silver et al. [7] investigate a supply chain consisting of one warehouse and one retailer with external demand rate known with certainty. In this model, the total cost consists of fixed replenishment costs of warehouse and the retailer as well as the holding costs of warehouse and retailer.*

In practical cases, transportation cost is affected by the shipment size and vice versa. So, it is important to determine the economic order quantity to minimize the overall logistics costs. Ghanshan [8] introduces a three-level supply chain consisting of a number of identical retailers, one central warehouse, and a number of identical suppliers. In this model, the objective function consists of ordering, holding and transportation costs. This model considers the transportation cost as a function of order quantity but ignores the capacity of vehicle. Ertogral et al. [9] consider a vendor-buyer supply chain model and incorporate the transportation cost. The transportation cost depends on shipment size. All-unit-discount transportation cost structures with and without over declaration have been considered in their work. Huang et al. [10] consider a two-level supply chain in which a warehouse delivers its products to many retailers. Each retailer faces a constant and deterministic demand. The objective is to determine an optimal stationary ZIO (Zero Inventory Ordering) policy for both warehouse and retailers in which the average transportation and inventory cost is minimized. Transportation cost consists of a fixed cost and a variable cost which is linearly proportional to the amount of quantity ordered.

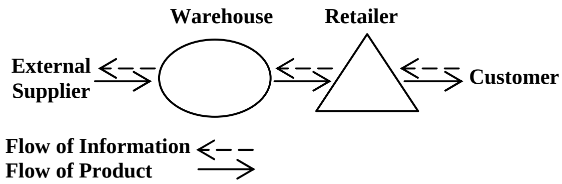

Fig 1. A Two-Level Supply Chain

In this paper we consider a two-level supply chain consisting of one warehouse and one retailer (Fig1).
We assume that the retailer faces deterministic demand with a constant demand rate and the demand is confined to a single item. Shortage is allowed neither at the retailer nor at the warehouse. The transportation time for an order to arrive at a retailer from the warehouse is assumed to be constant. The warehouse orders to an external supplier. The lead time for an order to arrive at the warehouse is assumed to be constant. The objective is to find the economic order

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quantity (EOQ) for the retailer and the warehouse. Unlike the most similar models which determine EOQ just according to inventory costs, in this model, we consider the transportation cost as well. Thus, the total cost is the sum of holding and ordering costs at retailer and at warehouse plus the transportation cost from warehouse to retailer.

2- TRANSPORTATION SCHEME

In this model, we suppose that there are three types of vehicle and delivery of each order from warehouse to retailer is made by a single vehicle without splitting. It is a common transportation scheme in most practical cases. We define these types as small (S), medium (M) and large (L). Each type has its own fixed cost, variable cost and the capacity size. Table 1 shows the context of transportation scheme.

Table 1. Transportation Scheme

Vehicle Type	Capacity	Fixed Cost	Variable Cost
S	q_1	F_1	v_1
M	q_2	F_2	v_2
L	q_3	F_3	v_3

It is assumed that $F_1<F_2<F_3$, $v_1>v_2>v_3$,, $q_1<q_2<q_3$, and
$F_2=F_1+q_1(v_1-v_2)$, $F_3=F_2+q_2(v_2-v_3)$. These equations are supposed to avoid any over declaration.

So, the transportation cost (TC) according to the order quantity varies as shown in Figure 2.

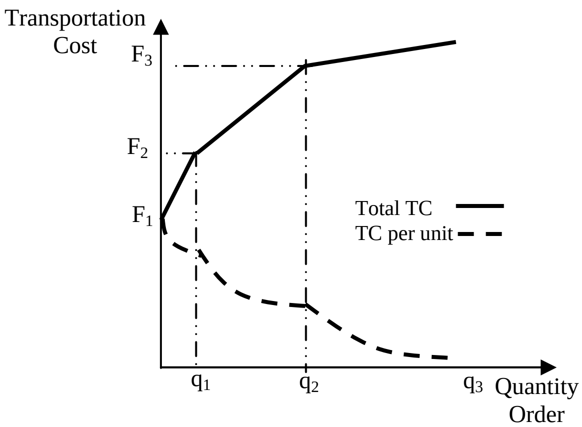

Fig 2. Variation of Transportation Cost

3- FORMULATION OF THE TOTAL COST

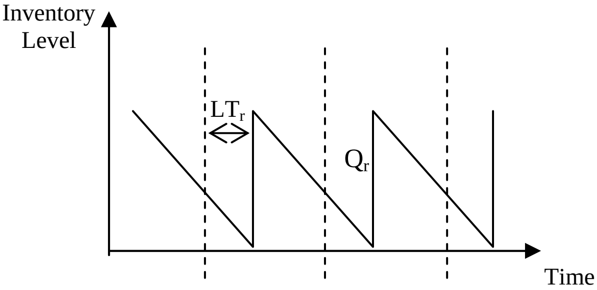

Fig 3. Inventory Level at Retailer

We suppose that the demand rate at the retailer and the transportation time to the retailer are constant. Shortage is not allowed at retailer, so, inventory level at retailer is a simple EOQ model and behaves as depicted in Figure 3.

Q_r: Batch size of the retailer,
LT_r: Lead Time (Transportation Time) from the warehouse to the retailer,

It is assumed that there is no lot-splitting at the warehouse. On the other hand, shortage is not allowed at the warehouse so that the order quantity of the warehouse is an integer multiple (n) of the order quantity of the retailer. Changes in the inventory level at the warehouse are shown in Figure 4.

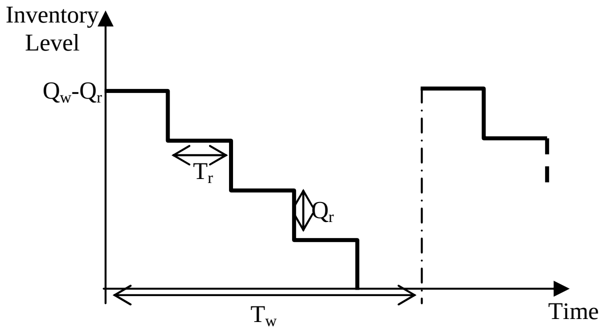

Fig 4. Inventory Level at Warehouse

Q_w: Batch size of the warehouse,
T_r: The time interval between any two consecutive orders of the retailer,
T_w: The time interval between any two consecutive orders of the warehouse

The total cost is the sum of holding and ordering costs at the retailer and at the warehouse plus the transportation cost from warehouse to the retailer. The objective is to find the economic order quantity (EOQ) for the retailer and for the warehouse in order to minimize the total cost:

$$\min C_{Ti}(Q_w,Q_r) = \frac{DA_w}{Q_w} + \frac{h_w(n-1)Q_w}{2n} + \frac{DA_r}{Q_r} + \frac{h_rQ_r}{2} + \frac{DF_i}{Q_r} + Dv_i \qquad (1)$$

,

$$q_{(i-1)} < Q_r \le q_i,\ i = 1,2,3$$
$$Q_w = nQ_r$$
$$n \ \ is \ \ \text{int}$$

D: Demand rate at the retailer,
A_r: Ordering cost for the retailer,
A_w: Ordering cost for the warehouse,
h_r: Rate of holding cost at the retailer,
h_w: Rate of holding cost at the warehouse,
n: Integer multiple of the order quantity of the retailer,
$q_0 = 0$,

Index i denotes the vehicle type; 1, 2 and 3 respectively for S, M and L. For shipment of retailer's order, according to order quantity, one of these types must be chosen. So, the ordering-size is restricted by the vehicle capacity.

We substitute nQ_r for Q_w in (1). So, the objective function in terms of Q_r, n is:

$$\min C_{Ti}(n,Q_r) = \frac{DA_w}{nQ_r} + \frac{h_w(n-1)Q_r}{2} + \frac{DA_r}{Q_r} + \frac{h_rQ_r}{2} + \frac{DF_i}{Q_r} + Dv_i \qquad (2)$$

,

$$q_{(i-1)} < Q_r \le q_i,\ i = 1,2,3$$
$$n \ is \ \mathrm{int}$$

4- ALGORITHM TO FIND THE OPTIMAL SOLUTION

Figure 5 shows graphically the total cost for a pre-determined n. We analyze the properties of this piece-wise convex function with respect to the order quantity of retailer. First, we present propositions 1,2 and 3 to show that for a pre-determined n the total cost function is a piece-wise convex function.

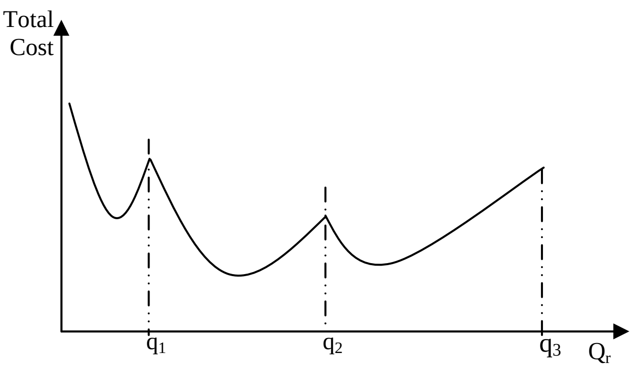

Fig 5. Total Cost for a Pre-determined n

Proposition 1: For a pre-determined n the total cost function, C_{Ti}, is convex, $i=1, 2, 3$.

Proof: For i=1, the first interval of Q_r, the total cost function is:

$$C_{T1}(n,Q_r) = \frac{DA_w}{nQ_r} + \frac{h_w(n-1)Q_r}{2} + \frac{DA_r}{Q_r} + \frac{h_rQ_r}{2} + \frac{DF_1}{Q_r} + Dv_1$$

We have:

$$\frac{\partial^2 C_{T1}}{\partial^2 Q_r} = \frac{2DA_w}{nQ_r^{\ 3}} + \frac{2DA_r}{Q_r^{\ 3}} + \frac{DF_1}{Q_r^{\ 3}}$$

It is clear that $\frac{\partial^2 C_{T1}}{\partial^2 Q_r} > 0$, thus C_{T1} is convex. Similarly For $i=2, 3$ we can prove that C_{Ti} is convex.

Proposition 2: For a pre-determined n, *$C_{T1}(q_1)=C_{T2}(q_1)$ and $C_{T2}(q_2)=C_{T3}(q_2)$.*

Proof: C_{T1} at point q_1 is as follow:

$$C_{T1}(q_1) = \frac{DA_w}{nq_1} + \frac{h_w(n-1)q_1}{2} + \frac{DA_r}{q_1} + \frac{h_rq_1}{2} + \frac{DF_1}{q_1} + Dv_1$$

Based on the chosen transportation scheme we know that $F_2=F_1+q_1(v_1-v_2)$. By substituting F_2 for F_1 we obtain:

$$C_{T1}(q_1) = \frac{DA_w}{nq_1} + \frac{h_w(n-1)q_1}{2} + \frac{DA_r}{q_1} + \frac{h_rq_1}{2} + \frac{DF_2}{q_1} + Dv_2$$

The left side of the above equation is $C_{T2}(q_1)$. Similarly we can prove $C_{T2}(q_2)=C_{T3}(q_2)$.
Proposition 2 indicates that the total cost function is continuous.

Proposition 3: For a pre-determined n, the slope of C_{T1} at q_1 is greater than the slope of C_{T2} at q_1, also the slope of C_{T2} at q_2 is greater than the slope of C_{T3} at q_2.

Proof. The derivative of C_{T1} minus the derivative of C_{T2} at q_1 is:

$$\frac{\partial C_{T1}(q_1)}{\partial Q_r} - \frac{\partial C_{T1}(q_2)}{\partial Q_r} = \frac{D}{q_1^{\ 2}}(F_2 - F_1)$$

According to the assumptions of the transportation scheme we know that F2>F1, which means the slope of C_{T1} at q_1 is greater than the slope of C_{T2} at q_1. Similarly, it can be shown that the slope of C_{T2} at q_2 is greater than the slope of C_{T3} at q_2.

Proposition 3 indicates that the minimum of the total cost function cannot occur at the break points.

According to propositions 1, 2 and 3, we can conclude that for a pre-determined n the optimal value of Q_r can be obtained the same as an EOQ model with incremental quantity discounts, as described by Hadley and Whitin [11].

We develop a search algorithm based on the model presented by Ertogral et al. [9] to obtain the optimal value of n, and Q. As mentioned above, we apply the EOQ model. Thus, we need a lower bound and an upper bound for n to create our search algorithm.

Set 1 as a lower bound for *n*. The following proposition generates the upper bound for *n*.

Proposition 4: The upper bound of *n* is:

$$n_{up} = \left\lfloor \sqrt{\frac{A_w(h_r - h_w)}{h_w(A_r + F_1)}} \right\rfloor$$

Proof: In the first interval of Q_r, the total cost function is:

$$C_{T1}(n, Q_r) = \frac{DA_w}{nQ_r} + \frac{h_w(n-1)Q_r}{2} + \frac{DA_r}{Q_r} + \frac{h_r Q_r}{2} + \frac{DF_1}{Q_r} + Dv_1$$

If we set the derivatives of C_{T1} with respect to Q_r and *n* equal to zero we obtain

$$Q^*_r = \sqrt{\frac{2D(\frac{A_w}{n} + A_r + F_1)}{h_w(n-1) + h_r}} \qquad (3)$$

$$n^* = \frac{1}{Q_r}\sqrt{\frac{2DA_w}{h_w}} \qquad (4)$$

Substituting Q^*_r in

$$C_{T1}(n) = \sqrt{2D(\tfrac{A_w}{n} + A_r + F_1)(h_w(n_I - 1) + h_r)} + Dv_1$$

the value of *n* which optimizes $C_{T1}(n)$ is obtained as:

$$n^* = \sqrt{\frac{A_w(h_r - h_w)}{h_w(A_r + F_1)}} \qquad (5)$$

From Eq. (4) we can conclude that *n* and Q_r have an inverse relation. The value of Q_r obtained from Eq. (3) is a lower bound on Q_r, because there is no gain to decrease Q_r less than Q^*_r. Hence, the n^* in Eq. (5) would be an upper bound on *n*.
In summary, the procedure to obtain the values of *n* and Q_r is as follow:

Algorithm:

1- Set $n_{up} = \left\lfloor \sqrt{\frac{A_w(h_r - h_w)}{h_w(A_r + F_1)}} \right\rfloor$.

2- For each n=1, 2,…,n_{up} find the optimal value of Q_r and the corresponding minimum total cost (C_T).

3- The solution which has the minimum total cost among the solutions in step 2 is the overall optimal solution.

5- NUMERICAL RESULTS

In this section we apply this model to a real case. We present a pharmaceutical downstream supply chain of a public hospital in which, the hospital pharmacy (H.P) is considered as retailer (because it delivers the pharmaceutical products to several care units). The hospital pharmacy orders these products to the central pharmacy (C.P), considered here as the warehouse that delivers to several hospitals [12]. We target a part of pharmaceutical products, ordered regularly with fixed quantities and at fixed periods. The objective is to minimize the total cost of system consisting of the sum of holding and ordering costs at the central pharmacy and at hospital pharmacy as well as the transportation cost.

Tables 2 and 3 give the transportation data and four types of pharmacy product respectively. The capacity and variable costs of vehicles are according to volume. In order to apply our algorithm we need to transform these parameters (capacity and variable cost of vehicles) in unit of product. Table 4 presents the capacity and variable costs of vehicles in term of product unit.

Table 2. Transportation Data

Vehicle Type	Capacity (m^3)	Fixed Cost (€)	Variable Cost (€/m^3)
S	8	169	5
M	14	179	3.75
L	20	196	2.5

Table 3. Data of Pharmacy Products

Product No.	Demand (unit/year)	Price (€/unit)	Volume (m^3)
1	19518	0.07	0.011
2	15106	0.11	0.017
3	1685	1.74	0.073
4	1864	6.26	0.045

Table 4.Transportation Data According to Product Unit

Product No.	Vehicle Type	Capacity (unit)	Variable Cost (€/unit)
1	S	727	0.06
	M	1273	0.04
	L	1818	0.03
2	S	471	0.09
	M	824	0.06
	L	1176	0.04
3	S	110	0.037
	M	192	0.27
	L	274	0.18
4	S	178	0.23
	M	311	0.17
	L	444	0.11

The ordering cost of hospital pharmacy (retailer) and central pharmacy (warehouse) are 1 and 3 respectively, thus, A_r=1 and A_w=3. The rate of holding cost per unit time at hospital pharmacy is 30% of the unit price and at central pharmacy is 20%. Table 5 contains the rate of holding cost.

Table 5. Rate of Holding Cost

Product No.	h_r (€/unit/year)	h_w (€/unit/year)
1	0.02	0.01
2	0.03	0.02
3	0.52	0.35
4	1.88	1.25

Table 6 illustrates the effect of transportation cost on the ordering-size. In these examples, we obtained EOQ for hospital pharmacy and for central pharmacy by two models. The results obtained from our model are denoted by *In.Tr* in Table 6, and the results obtained from the model which ignores the transportation cost are denoted by *In* in the same table. In the first model (*In.Tr.*) the transportation cost is integrated directly in the total cost. In the second model (*In.*), in calculating the EOQ, we ignore the transportation cost but in obtaining the total cost we must consider the sum of the inventory and the transportation costs. We compared the total costs of these models and found the savings obtained by our model. The results show that the transportation cost affects the ordering-size and the total cost. Note that the saving for product 2 is 0%, because EOQ of hospital pharmacy obtained by *In.* model is 2007 unit which is more than the capacity of large vehicle. Therefore, we have to restrict it to 1176 unit which is the large vehicle capacity. Thus, the total cost, in this situation, is the same for both models.

EOQ of hospital pharmacy which defines vehicle type, depends on many parameters such as the rate of holding cost, transportation cost *etc*. For product 1 we made sensitivity analysis on the rate of holding cost at retailer, on the fixed cost of small vehicle (S.V) and on the variable cost of large vehicle (L.V). Tables 7, 8, and 9 show the results of these analyses. These analyses show that whenever the rate of holding cost at hospital pharmacy increases, we should decrease the inventory level at the hospital pharmacy, i.e., decrease the order quantity. The results of Table 7 confirm this fact. According to the results shown in Tables 8 and 9, we can conclude that the economic order quantity is affected by transportation cost.

Table 6. The Results for Four Pharmacy Products

Product No.	Model	E.O.Q of H.P (unit)	E.O.Q of C.P (unit)	Total Cost (€/year)	Saving (%)
1	In.Tr	1818	3636	2743.9	0.6
	In	1804	3608	2760.7	
2	In.Tr	1176	2352	3183.4	0
	In	1176	2352	3183.4	
3	In.Tr	274	274	3424.3	10.4
	In	238	238	3820.5	
4	In.Tr	444	444	1462	64.6
	In	89	89	4133.2	

Table 7. Results of Sensitivity Analysis on Rate of Holding Cost at H.P

h_r (€/unit/year)	E.O.Q of H.P (unit)	Vehicle Type
< 1	1818	L
1	1818	L
2	1818	L
3	1605	L
4	1387	L
5	1186	M
6	1083	M
7	1003	M
8	870	M
9	834	M
10	839	M
11	800	M
12	766	M
13	735	M
14	689	S
15	665	S
> 15	< 665	S

Table 8. Results of Sensitivity Analysis on fixed cost of Small Vehicle

Fixed Cost of S.V (€)	E.O.Q of H.P (unit)	Vehicle Type
< 10	727	S
10	727	S
20	727	S
30	727	S
40	727	S
50	1818	L
60	1818	L
70	1818	L
> 70	1818	L

Table 9. Results of Sensitivity Analysis on variable cost of Large Vehicle

Variable Cost of L.V (€/unit)	E.O.Q of H.P (unit)	Vehicle Type
< 0.08	1818	L
0.08	1818	L
0.09	1818	L
1	1273	M
1.1	1273	M
>1.1	1273	M

5- CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In this paper, we study optimal ordering policy in a two-level supply chain. Unlike the most similar models which determine the economic order quantity just based on inventory costs, in this model, we incorporate transportation costs into inventory replenishment decisions. The numerical results indicate that our model leads to savings in comparison with the traditional model in which the transportation cost is not considered. For future research we intend to complete this model by including the multi-item lot-sizing problem.

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