A New replenishment Policy in a Two-echelon Inventory System with Stochastic Demand

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ABSTRACT

In this paper we consider a two-echelon inventory system consisting of one central warehouse and a number of non-identical retailers. The warehouse uses a regular one-for-one policy to replenish its inventory, but the retailers apply a new policy that is each retailer orders one unit to central warehouse in a pre-determined time interval. The most advantage of this policy is that the retailers' orders, which constitute warehouse demand, are deterministic. For this system we show how the inventory costs can be evaluated. By the numerical examples we compare our policy with one-for-one policy.

Keywords: inventory control, replenishment policy, two-echelon, stochastic demand

1- INTRODUCTION

We consider a two-echelon inventory system consisting of one central warehouse and a number of non-identical retailers. We assume the retailers face independent Poisson demand and unsatisfied demand will be lost. The transportation time for an order to arrive at a retailer from the warehouse is assumed to be constant. The warehouse orders to an external supplier. The lead time for an order to arrive at the warehouse is assumed to be constant.

The central warehouse uses a regular one-for-one policy, but the retailers apply a new policy which is different from the inventory policies used in the literature of inventory and production control systems. In this system, each retailer orders a demand to central warehouse in a pre-determined time interval. In other word, the time interval between any two consecutive orders of each retailer is fixed and the quantity of each order is one.

The most advantage of this policy is that the retailers' orders, which constitute warehouse demand, are deterministic. The deterministic demand for the warehouse leads to inventory control in warehouse is simplified for instance, safety stock in warehouse is eliminated.

We evaluate the total inventory system cost in steady state consisting of holding and shortage costs of retailers and holding cost of the warehouse. The purpose of this study is to determine the optimal time interval between two orders of each retailer which minimizes the total inventory system cost.

By the numerical examples we compare our policy with one-for-one policy. The numerical results indicate that there are some cases which make savings by using our policy instead of a one-for-one policy.

This paper is organized as follow. Section 2 we provide a literature review of inventory control policy for the tow-echelon inventory system. Section 3 presents cost evaluation in the steady stats. Section 4 gives some numerical examples and section 5 presents the conclusions and suggestions for further research in this area.

2- INVENTORY CONTROL POLICY FOR THE TWO-ECHELON INVENTORY SYSTEM

Two main policies applied in two-echelon inventory models are continues review and periodic review. Most researches consider continues review policy. Graves [1] considers a multi-echelon inventory model for a repairable item with one-for-one policy. He presents an exact model for finding the steady state distribution of the net inventory level. Axsäter [2] investigates a two-echelon inventory system in which the inventory policy of each echelon is *(r, Q)*. Axsäter et al. [3] generate an approximate method for inventory costs in a two-echelon inventory system with compound Poisson demand, in which the inventory policy of each echelon is order up to S. Forsberg [4] considers an exact evaluation of (r, Q) policies for two-level inventory systems with Poisson demand. Axsäter and Zhang [5] consider a two-level inventory system with a central warehouse and a number of identical retailers. The warehouse uses a regular installation stock batch-ordering policy, but the retailers apply a different type of policy. When the sum of the retailers' inventory positions declines to a certain "joint" reorder point, the retailer with the lowest inventory position places a batch quantity order. Andersson and Melchiors [6] propose an approximate method to evaluate inventory costs in a two-echelon inventory system with one warehouse and multiple retailers. All installations use (S-1, S) policy. Seo et al. [7] develop an optimal reorder policy for a two-echelon distribution system with one central warehouse and multiple retailers. Each facility uses continuous review batch ordering policy. Axsäter [8] in a two-echelon distribution inventory system presents a simple technique for approximate optimization of the reorder points. The system is controlled by continuous review installation stock (r, Q) policies. Seifbarghi and Akbari [9] investigate the inventory system consisting of one central warehouse and many identical retailers controlled by continuous review policy (r, Q). The demands of retailers are independent Poisson and stockouts in retailers are lost sales. There are a few models in two-echelon inventory which consider periodic policy. Axsäter [10] investigates a two-echelon inventory system applied order-up-to-*S* policy with periodic review. Matta and Sinha [11] investigate a two-echelon inventory system consists of a central warehouse and a number of retailers. Each retailer applies (T, S) inventory policy with an identical review interval *T* and different maximum inventory level *S*. the central warehouse applies (T, s, S) policy, where *T* is the same review interval as that of retailers; *s* is its reorder points, and *S* is its desired maximum inventory level. Kanchanasuntron and Techanitisawad [12] investigate a periodic inventory-distribution model base on Matta and Sinha model [11] to deal with the case of fixed-life perishable product and lost sales at retailers.

3- COST EVALUATION

We consider a two-echelon inventory system consisting of one central warehouse and a number of non-identical retailers. The warehouse uses a regular one-for-one policy to replenish its inventory, but the retailers apply a new policy that is each retailer orders one unit to central warehouse in a pre-determined time interval. We evaluate the total inventory system cost in steady state consisting of holding and shortage costs of retailers and holding cost of the warehouse.

In section 4.1, we investigate formulation of retailer costs. In section 4.2 we study warehouse cost accordingly retailer orders. Finally, in section 4.3, we demonstrate total inventory system cost.

Assumptions:

1- The retailers face independent Poisson demand.
2- Unsatisfied demand in retailers will be lost.
3- The transportation time for an order to arrive at a retailer from the warehouse is constant.
4- The warehouse orders to an external supplier with infinite capacity.
5- The lead time for an order to arrive at the warehouse is constant.
6- At warehouse and retailers, the replenishment cost is assumed to be zero or negligible.

Notation:

- N: number of retailers.
- λ_i: demand intensity at retailer i , i=1,2,…,N.
- s_i: penalty cost for a lost sale at retailer i, i=1,2,…,N.
- h_i: holding cost rate at retailer i, i=1,2,…,N.
- h_w: holding cost rate at central warehouse.
- T_i: time interval between any two consecutive orders of retailer i, i=1,2,…,N.
- S_0: order-up-to level at warehouse,
- I_i: inventory level at retailer i in steady state, i=1,2,…,N.
- $\overline{H}_i$: average of inventory level at retailer i in steady state, i=1,2,…,N.
- Ch_i: holding cost at retailer i in steady state, i=1,2,…,N.
- *CSi: shortage cost at retailer i in steady state,* i=1,2,…,N.
- $C_i(T_i)$: total cost at retailer i in steady state, ($C_i(T_i)$= Ch_i+ CS_i), i=1, 2,…, N.
- *TISC*: total inventory system cost in steady state.

3.1- FORMULATION OF RETAILER COSTS

The retailers are independent so that we evaluate holding and lost shortage costs for a retailer (i.e., retailer *i*) and the cost evaluation for other retailers is similar to this one. Retailer *i* orders one unit to central warehouse in a pre-determined time interval. In section 3.2 we describe applying this policy amounts to a deterministic demand for the central warehouse so that warehouse could plane its inventory in which it doesn't face any shortages. Considering this point, the lead time for retailer from warehouse is equivalent to the transportation time.

Thus, we obtain a probability distribution of inventory level in steady state with intention to evaluate the average inventory level at retailer. We define notation $\pi_n^{\,i}$, probability distribution of inventory level at retailer i in steady state, such as:

$$\pi^i{}_n = P(I_i = n) \qquad , n \geq 0$$

To evaluate $\pi_n^{\,i}$ we use a queuing system. We suppose the inventory of retailer such as the customer in a queuing system. In this concept, service to a costumer means waiting to arrive a demand. In the other word, a demand to retailer is the same as the end of service in a queuing system. Arrival an order to retailer from warehouse can be considered such as arrival a customer to the queue. The queuing system of interest is *D/M/1* queue. The arrival rate an order to retailer is equal $\frac{1}{T_i}$ and the demand rate to retailer is equal λ_i.

Probability distribution of inventory level at retailer i in steady state is [13]:

$$\pi^i{}_0 = 1 - \rho_i$$

$$\pi^i{}_n = \rho_i (1 - x_{i0}) x_{i0}{}^{n-1} \quad ; \; n \geq 1$$

ρ_i is ratio the arrival rate to the demand rate so,

$$\rho_i = \frac{1}{T_i \lambda_i}$$

x_{i0} is a number between 0 to 1 that satisfies this equation:

$$x_{i0} = e^{-T_i \lambda_i (1 - x_{i0})}$$

The average inventory level at retailer *i* in steady state is:

$$\overline{H}_i = \sum_{n=1}^{\infty} n.\pi^i{}_n = \frac{\rho}{1 - x_{i0}}$$

Therefore the holding cost at retailer i in steady state is:

$$Ch_i = \overline{H}_i . h_i = \frac{\rho_i h_i}{1 - x_{i0}} \Rightarrow Ch_i = \frac{h_i}{T_i \lambda_i (1 - x_{i0})}$$

Inventory level of retailer i is stable whenever $\rho_i < 1$, so we consider this constraint in the model:

$$\rho_i < 1 \Rightarrow T_i > \frac{1}{\lambda_i}$$

$\pi^i{}_0$ is the probability of stock out at retailer *i*. The amount of demand that is lost in steady state is $\lambda_i \pi^i{}_0$. Lost sale cost at retailer i in steady state is:

$$CS_i = s_i \lambda_i \pi^i{}_0 \Rightarrow CS_i = s_i (\lambda_i - \frac{1}{T_i})$$

The total cost at retailer *i* in steady state is:

$$C_i(T_i) = Ch_i + CS_i$$

$$\Rightarrow C_i(T_i) = \frac{h_i}{T_i \lambda_i (1 - x_{i0})} + s_i (\lambda_i - \frac{1}{T_i})$$

$$s.t$$

$$x_{i0} = e^{-T_i \lambda_i (1 - x_{i0})}$$

$$0 < x_{i0} < 1$$

$$T_i > \frac{1}{\lambda_i}$$

3.2. WAREHOUSE INVENTORY ANALYSIS

We assumed that the inventory policy of warehouse is *(S_0-1, S_0)*. It means warehouse order to the external supplier as soon as the reception of a retailer orders. The retailers' orders have deterministic time and quantity. Moreover, the lead time to warehouse from the external supplier is constant, so that warehouse could plane the arrival of an order from the external supplier and delivery it to the retailer simultaneously. Thus, retailer *i* orders a unit to the warehouse and warehouse to the external supplier every T_i unit of time. In the other word, the optimal order up-to-level is zero, $S_0=0$, .Therefore, we transform the warehouse into depot and its holding cost is zero.

3.3. TOTAL INVENTORY SYSTEM COST

The total inventory system cost in steady state consists of holding and shortage costs of retailers and holding cost of the warehouse. It is described in section 3.2; the holding cost of the warehouse is zero so the total inventory system cost includes just the holding and the shortage costs of retailers. Basing our formulation of retailer costs (section 3.1), the total inventory system cost can be calculated as follow:

$$TISC = \sum_{i=1}^{N} C_i(T_i)$$

The objective is to determine the optimal value of T_i (i=1,..,N) which minimize the total inventory system cost, so the objective function is as follow:

$$TICS = \min \sum_{i=1}^{N} \left[\frac{h_i}{T_i \lambda_i (1 - x_{i0})} + s_i (\lambda_i - \frac{1}{T_i}) \right] \qquad (2)$$

$$s.t$$

$$x_{i0} = e^{-T_i \lambda_i (1 - x_{i0})}, i = 1,.., N$$

$$0 < x_{i0} < 1, \qquad i = 1,..., N$$

$$T_i > \frac{1}{\lambda_i}, \qquad i = 1,.., N$$

4- NUMERICAL RESULTS

In this section we compare our policy with common policy one-for-one by some numerical examples. The results of numerical examples for one-for-one policy are calculated according to Andersson and Melchiors method [6]. We obtain the optimal value of T_i in function (2) by MATLAB software. In our examples, we suppose that all retailers are identical.

Table 1 contains the results of sensitivity analysis on the rate of lost sale cost at retailers. Figure 1 shows the comparison between the total system costs of our policy with one-for-one policy. In this numerical example, if the rate of lost sale cost is less than 5, applying our policy is better. Number 5 is equivalent to the rate of holding cost at retailers. Regards this result, we propose to use our policy when the rate of lost sale cost is less than the rate of holding cost at retailers. We test this hypothesis by another numerical example, table 2 and figure 2 show the results of this numerical example. The result of this numerical example confirms the hypothesis.

Table1. Sensitivity analysis on the rate of lost sale cost

$N=5, \lambda=1, L_0=1, h_0=1, h_r=5$		
s_r	**(s-1,s) Policy**	**Our Policy**
1	11.67	6.98
2	15.00	11.53
3	18.33	16.03
4	21.67	20.49
5	25.00	25.00
6	28.13	28.82
7	31.06	32.15
8	33.91	35.19
9	36.66	38.01
10	39.42	40.67

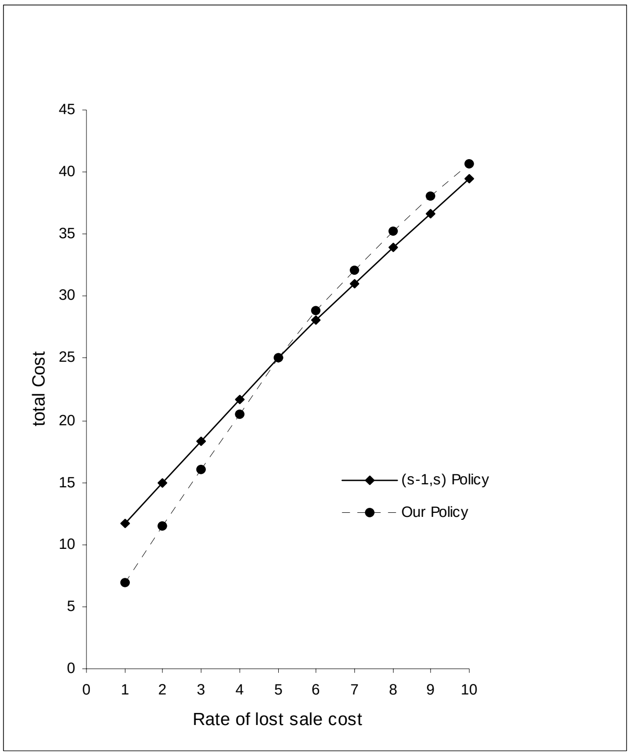

Figure1. Sensitivity analysis on the rate of lost sale cost

Table2. Sensitivity analysis on rate of holding cost at retailers

$N=5, \lambda=2, L_0=1, h_0=1, L_r=1, s_r=25$		
h_r	(s-1,s) Policy	Our Policy
20	183.76	186.88
21	187.59	190.44
22	191.36	193.85
23	195.14	197.14
24	198.92	200.29
25	202.61	203.33
26	206.28	206.25
27	209.90	209.07
28	213.43	211.78
29	216.89	214.39
30	220.22	216.91

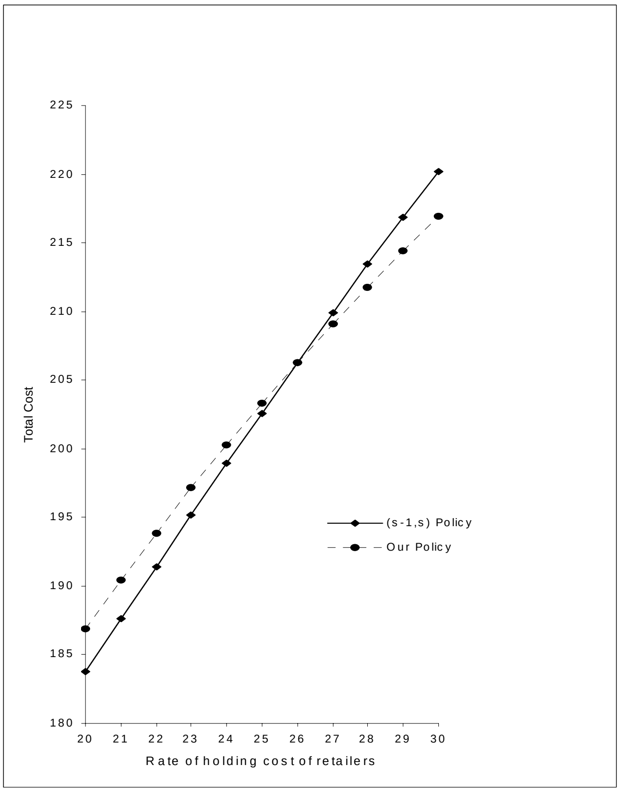

Figure2. Sensitivity analysis on rate of holding cost at retailers

5- CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

In this paper we introduced a new inventory control policy and analyzed the application of this policy in a tow-echelon inventory system. The most advantage of our policy is to change retailers' orders in one-for-one policy into the deterministic orders. This advantage facilitates the inventory planning and lead to eliminate holing inventory at the warehouse. The numerical results illustrate that in some cases our policy is better than one-for-one policy.

For future research we can consider some other criteria such as price discount or ordering cost discount into the tow-echelon systems. This model can be completed by including transportation costs. We guess that our policy reduces complexity of logistic planning and decreases transportation costs. This policy can be generalized by consideration the batch size ordering.

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