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A novel technique for finding the 3-Degree of freedom orientation by an ultrasound tracker

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***Abstract*— The ability to determine the 3-Degree of freedom (DOF) orientation of a device is of fundamental importance in many applications including virtual reality, augmented reality and context aware applications. There are some conventional approaches to determine the orientation, each with their advantages and disadvantages. Ultrasound trackers are cheap and small and are not influenced by magnetic fields. In this work we present a novel approach to add the ability of finding a 3-DOF orientation of a device, to an ultrasound tracker.**

Keywords: ultrasound tracker, planar orientation, 3-DOF orientation, quaternion, roll, pitch and yaw.

I. INTRODUCTION

Finding the 3-DOF orientation of a device relative to a coordinate reference system is an important problem in virtual reality, augmented reality and context aware applications [1], [2], [3].

There are some conventional methods to find the orientation, each having its own advantages and disadvantages [2], [4]. For instance, the magnetic orientation tracker has a good accuracy but exhibits enormous errors when near magnetic or time-varying fields, both rather common in most modern buildings with computers and other electrical equipments [2]. The inertial orientation tracker (gyro) has a drift problem and needs another tracker to calibrate it periodically [4]. In [1] a novel method is proposed to find a 1-DOF orientation in a horizontal plane with respect to a beacon using the time of flight of ultrasound waves. Ultrasound trackers are cheap and small and are not influenced by magnetic fields [2]. The ultrasound trackers are divided into two architectures, passive mobile and active mobile architecture. In the passive mobile architecture, the transmitters are fixed and mounted on the ceiling or wall and the receivers are on the device, while in the active mobile architecture, the receivers are fixed and mounted on the ceiling or wall and the transmitters are on the device [7].

In this paper, we expand the ultrasound tracker to find a 3-DOF orientation information. The paper is organized as the following: In section II, a method to calculate the planar orientation which is mentioned in [1] is described and in section III, we expand the method to find the 3-DOF orientation. Section IV contains the simulation results.

II. CALCULATING THE PLANAR ORIENTATION

Cricket is an ultrasound tracker, designed at MIT University [5]. It measures the time of flight of ultrasound waves between receivers and transmitters with known coordinates, multiplies it by the speed of the ultrasound wave to obtain the distance and then uses trilateration [6] to find the position of a device in a passive mobile architecture.

The method mentioned in [1] provides the cricket the ability of finding the planer orientation with respect to a beacon. In this method, which is shown in Fig. 1 two receivers with the separation L, are mounted on a horizontal plane on the device. The angle of rotation of the line passing through two receivers with respect to the transmitter (beacon), θ (planar orientation), is found by measuring the difference of the distances of the two receivers to the transmitter. It can be shown that [1]

$$\sin\theta = \frac{d_2 - d_1}{L\sqrt{1-(\frac{z}{\bar{d}})^2}} \qquad (1)$$

where z is the beacon height over the horizontal plane and $\bar{d}$ is the distance of the beacon from the center of the two receivers.

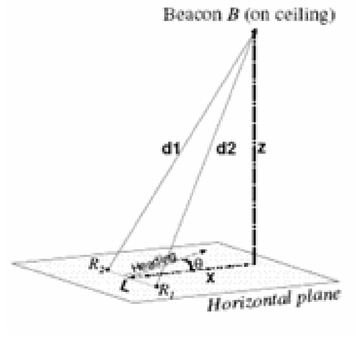

Figure 1. Determining the angle of orientation along the horizontal plane. [1]

The aim of this work is to find the 3-DOF orientation of a device based on the method mentioned above.

III. Theory of Operation

To find the 3-DOF orientation of a device, we must find the rotation of a local coordinate system, attached to the device, relative to a reference (global) coordinate system.

To define the local coordinate system, we put three tags, n_1, n_2 and n_3 (on the object), with a separation L as shown in Fig. 2. The angle $n_2 n_1 n_3$ is the right angle. These tags can be ultrasound receivers in passive mobile architecture or transmitters in active mobile architecture. The tag n_1 is the one whose coordinates are determined in the global coordinate system by the tracker. We define the local x axis to run through n_1, n_2 and the local y axis to run through n_1, n_3 with the origin at n_1. Hence, the line perpendicular to the x-y plane at the origin is the local z axis.

To find the 3-DOF orientation of a coordinate system, relative to a coordinate reference system, the coordinates of at least three nodes in one frame and their corresponding coordinates in the other frame are necessary [8]. One of them is the coordinate of tag n_1 whose coordinates in both coordinate frames are known. The global coordinates of beacons are also known. Hence, if we determine the local coordinates of at least two beacons, then we have enough information to determine the orientation of the local coordinate system relative to the global coordinate system.

A. *Finding the local coordinates of beacon*

In this section, we find the local coordinates of a beacon r_{local,B_1} by measuring the differential distance of the beacon to tags n_1, n_2, (i.e. $d_{n_1,B_1} - d_{n_2,B_1}$) and to tags n_1, n_3 (i.e. $d_{n_1 B_1} - d_{n_3,B_1}$). We draw the perpendicular line from the node n_2 to the line passing n_1, B_1, that intersects it at p.(see Fig. 2)

$$d_{1,B}, d_{2,B} >> L \Rightarrow \cos(p\hat{n}_1 n_2) = \frac{n_1 p}{L} \approx \frac{d_{n_1,B_1} - d_{n_2,B_1}}{L} \qquad (2)$$

And the x coordinate of the vector $\overrightarrow{n_1 B_1}$ is equal to

$$x_1 = \left\|\overrightarrow{n_1 B_1}\right\| \cos(p\hat{n}_1 n_2) \approx \left\|\overrightarrow{n_1 B_1}\right\| \frac{d_{n_1,B_1} - d_{n_2,B_2}}{L} \qquad (3)$$

The y coordinate of a vector $\overrightarrow{n_1 B_1}$ (y_1) can be obtained in a similar way, and the z coordinate of the vector is equal to:

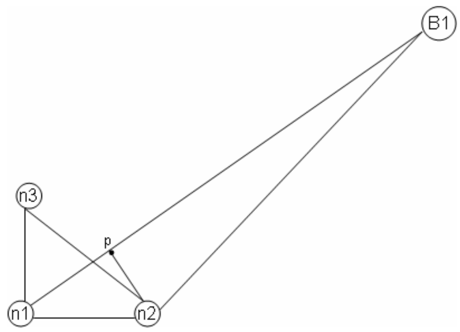

Figure 2. The arrangement of three tags, which are attached to the device

$$z_1 = \sqrt{\left\|\overrightarrow{n_1 B_1}\right\|^2 - (x_1^2 + y_1^2)} \qquad (4)$$

The local coordinates of the second beacon r_{local,B_2} can be obtained in a similar way.

IV. Representation of Rotation

There are many ways to represent rotation, including the following: Euler angles, direction cosine matrix (DCM), Gibbs vector, Cayley-Klein parameters, Rodrigues parameters, orthonormal matrices, and Hamilton's quaternions [9], [10]. The quaternion representation has many advantages over the other ones [9]. Therefore we use the quaternion representation.

A. *Quatrenions*

A quaternion can be thought of as a vector with four components, as a composite of a scalar and an ordinary vector, or as a complex number with three different imaginary parts [10]. Quaternions will be denoted here by using symbols with circles above them. Thus, using complex number notation, we have

$$\dot{q} = q_0 + iq_x + jq_y + kq_z \tag{5}$$

a quaternion with real part q_0 and three imaginary parts q_x, q_y, q_z. Multiplication of quaternions can be defined in terms of the products of their components. Suppose that we let:

$$\begin{aligned} &k^2 = -1, \quad j^2 = -1, \quad i^2 = -1, \\ &ij = k, \quad jk = i, \quad ki = j \\ &ik = -j, \quad kj = -i, \quad ji = -k \end{aligned} \tag{6}$$

Then, if

$$\dot{r} = r_0 + ir_x + jr_y + kr_z$$

We get

$$\begin{aligned} \dot{r}\dot{q} = &(r_0 q_0 - r_x q_x - r_y q_y - r_z q_z) \\ &+ i(r_0 q_x + r_x q_0 + r_y q_z - r_z q_y) \\ &+ j(r_0 q_y - r_x q_z + r_y q_0 + r_z q_z) \\ &+ k(r_0 q_z + r_x q_y - r_y q_x + r_z q_0) \end{aligned} \tag{7}$$

The dot product of two quaternions is the sum of products of corresponding components:

$$\dot{p} \cdot \dot{q} = p_0 q_0 + p_x q_x + p_y q_y + p_z q_z \tag{8}$$

The square of the magnitude of a quaternion is the dot product of the quaternion with itself:

$$\|\dot{q}\|^2 = \dot{q} \cdot \dot{q} \tag{9}$$

A unit quaternion is a quaternion whose magnitude equals 1. Taking the conjugate of a quaternion negates its imaginary:

$$\dot{q}^* = q_0 - iq_x - jq_y - kq_a \tag{10}$$

B. Unit quatrenions and rotation

If we consider the vector $\vec{r} = (q_x, q_y, q_z)$,as a purely imaginary quaternion, $\dot{r} = 0 + iq_x + jq_y + kq_z$, then a rotated version of $\vec{r}$, by an angle θ, about the axis defined by the unit vector $w = (w_x, w_y, w_z)$, $\vec{r}\,'$, can be obtained by: [10]

$$\dot{r}' = \dot{q}\dot{r}\dot{q}^* \tag{11}$$

Where

$$\dot{q} = \cos(\theta/2) + \sin(\theta/2)(iw_x + jw_y + kw_z) \tag{12}$$

Composition of rotations corresponds to multiplication of quaternions [10].

V. FINDING THE ORIENTATION

We apply the approach proposed in [10] to find the best rotation between tow coordinate systems that minimizes the least square of errors. It turns out to be useful to refer all measurements to the centroids defined by:

$$\bar{\vec{r}}_{local} = \frac{1}{3}\sum_{i=1}^{3} \vec{r}_{local,i} \qquad \bar{\vec{r}}_{global} = \frac{1}{3}\sum_{i=1}^{3} \vec{r}_{global,i} \tag{13}$$

Let us denote the new coordinates by

$$\vec{r}_{local,i} = \vec{r}_{local,i} - \bar{\vec{r}}_{local} \tag{14}$$

$$\vec{r}\,'_{global,i} = \vec{r}_{global,i} - \bar{\vec{r}}_{global} \tag{15}$$

First, the plane containing the global measurements has to be rotated to bring it into coincidence with the plane containing the local measurements (Fig. 3). We do this by rotating the global measurements about the line of intersection of the two planes. The direction of the line of intersection is given by the cross product of the normals of the two planes. The angle of rotation is that required to bring the global normal to coincide with the local normal, that is, the angle between the two normals

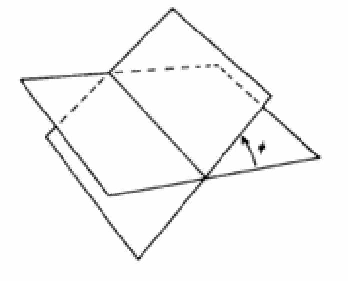

Figure 3. The rotation decomposes conveniently into rotation about the line of intersection of the two planes and rotation about a normal of one of the planes. (In this figure the two coordinate systems have been aligned and superimposed.)

.At this point, the remaining task is the solution of a least squares problem in a plane. We have to find the rotation that minimizes the sum of squares of distances between corresponding local and (rotated) global measurements, Fig4. This second rotation is about the normal to the plane. The angle is determined by the solution of the least-squares problem.

The overall rotation is just the composition of the two rotations found above. It can be found by multiplication of unit quaternions. We start by finding normals to the two planes. We can use the cross products of any pair of nonparallel vectors in the plane:

$$\vec{n}_{local} = \vec{r}\,'_{local,B_1} \times \vec{r}\,'_{local,B_2} \tag{16}$$

$$\vec{n}_{global} = \vec{r}\,'_{global,B_1} \times \vec{r}\,'_{global,B_2} \tag{17}$$

We next construct unit normals, $\hat{\vec{n}}_{local}$ and $\hat{\vec{n}}_{global}$ by dividing $\vec{n}_{local}$ and $\vec{n}_{global}$ by their magnitudes. The line of intersection of the two planes lies in both planes, so it is perpendicular to both normals. It is thus parallel to the cross product of the two normals. Let

$$\vec{a} = \vec{n}_{local} \times \vec{n}_{global} \tag{18}$$

We find a unit vector $\hat{\vec{a}}$ in the direction of the line of intersection by dividing $\vec{a}$ by its magnitude.
The angle ϕ through which we have to rotate, is the angle between the two normals. So

$$\cos\phi = \hat{\vec{n}}_{local} . \hat{\vec{n}}_{global} \tag{19}$$

$$\sin\phi = \left\| \hat{\vec{n}}_{local} \times \hat{\vec{n}}_{global} \right\| \tag{20}$$

We now rotate the global measurements into the plane containing the local measurements. Let $\vec{r}''_{global}$ be the rotated version of $\vec{r}'_{global}$. The rotation can be accomplished by:

$$\dot{q}_a = \cos(\phi/2) + \sin(\phi/2)(i\hat{a}_x + j\hat{a}_y + k\hat{a}_z) \tag{21}$$

We now have to find the rotation in the plane of the global measurements that minimizes the sum of squares of distances between corresponding measurements(fig. 4). That is, we wish to minimize:

$$\sum_{i=1}^{3} \left\| \vec{r}'_{local,i} - \vec{r}''_{global,i} \right\|^2 \tag{22}$$

It can be shown that is equal to:

$$\sum_{i=1}^{3} \left\| \vec{r}'_{local,i} \right\|^2 + \left\| \vec{r}''_{global,i} \right\|^2 - 2\left\| \vec{r}''_{global,i} \right\| \left\| \vec{r}'_{local,i} \right\| \cos(\alpha_i) \tag{23}$$

which α_i is the angle between corresponding measurements. When the global measurements are rotated in the plane through an angle θ, the angles α_i are reduced by θ. So to minimize the sum of squares of distances we need to maximize:

$$\sum_{i=1}^{3} \left\| \vec{r}''_{global,i} \right\| \left\| \vec{r}'_{local,i} \right\| \cos(\alpha_i - \theta) \tag{24}$$

That has the maxima at

$$\theta_{\max} = \arcsin\left(\frac{S}{\sqrt{S^2 + C^2}}\right) \quad 0 < \theta_{\max} < \pi/2 \tag{25}$$

where

$$C = \sum_{i=1}^{3} \vec{r}'_{local,i} . \vec{r}''_{global,i} \tag{26}$$

and

$$S = \left(\sum_{i=1}^{3} \vec{r}'_{local,i} \times \vec{r}''_{global,i}\right) . \hat{\vec{n}}_{local} \tag{27}$$

The second rotation is then about the axis $\hat{\vec{n}}_{local}$ by an angle $\theta_{\max}$.
It can be represented by the unit quaternion

$$\dot{q}_p = \cos(\theta/2) + \sin(\theta/2)(i\hat{n}_x + j\hat{n}_y + k\hat{n}_z) \tag{28}$$

The overall rotation of the global coordinate system into local coordinate system , $\dot{q}$, is the composition of the two rotations:

$$\dot{q} = \dot{q}_p \dot{q}_a \tag{29}$$

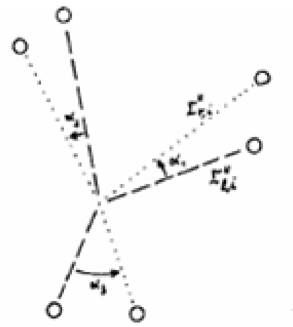

Figure 4. The second rotation, is about the normal of one of the planes

Finally we convert the resultant quaternion into angles roll, yaw and pitch [9] that have a better intuitive view for reader.

VI. SIMULATION RESULTS

We have applied the above-mentioned method to the comprehensive raw cricket data available at [15]. We perform our simulation for three different position errors, 2.2cm, 3.4cm, and 4.5cm (the greater error corresponds to greater velocity of mobile device), each for ten samples. Figures 5, 6, and 7 show the average error of roll, yaw and pitch angles as a function of position error. As we can see, the orientation estimation error increases when the accuracy of position estimation decreases. The average error in roll, pitch and yaw angles are 2.69, 3.07 and 2.81 degrees, respectively.

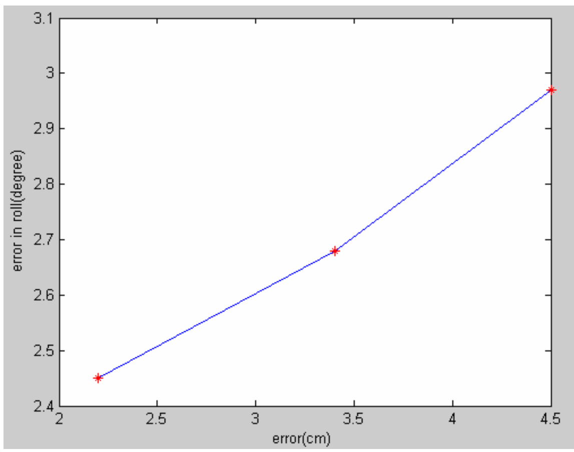

Figure 5. Error in roll (degree) as a function of position error(cm).

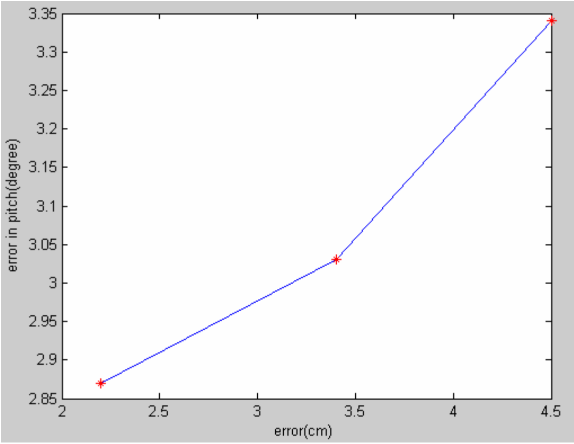

Figure 6. Error in pitch (degree) as a function of position error (cm).

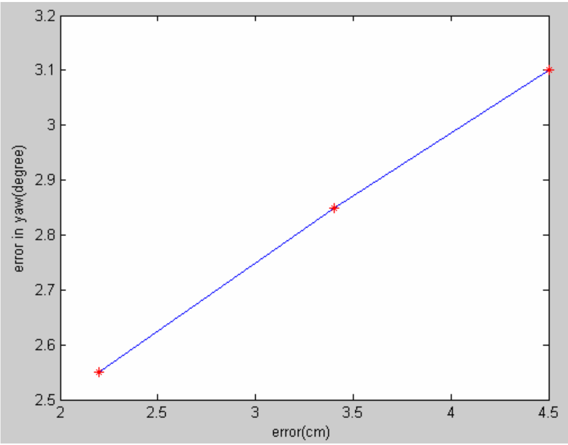

Figure 7. Error in yaw (degree) as a function of position error (cm)

VII. CONCLUSION

In this paper, we presented a novel approach for finding the 3-DOF orientation based on the time of flight of ultrasound waves. Simulations show that this method has a few degrees error. and Its accuracy is dependent on the accuracy of the position estimation. We can improve the accuracy of orientation estimation by improving the accuracy of position estimation.

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