THE DETERMINATION OF THE RESONANCE FREQUENCY OF THE TE^{Y}_{111} MODE IN A RECTANGULAR DIELECTRIC RESONATOR FOR ANTENNA APPLICATION

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Abstract

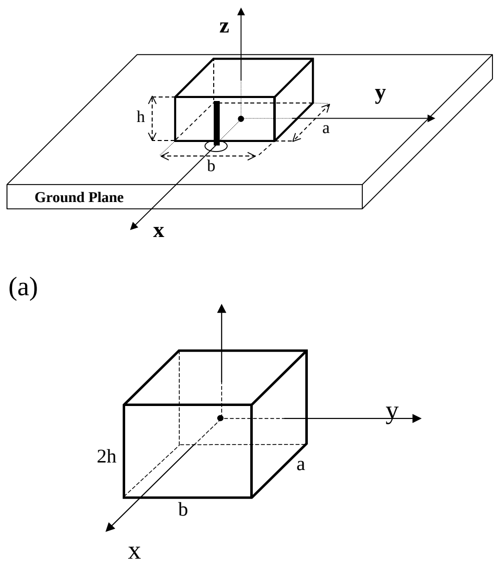
In this study the conventional dielectric waveguide model (CDWM) is used to determine the resonance frequency of a rectangular dielectric resonator operating at the TE^{y}_{111} mode for antenna application. A fitted closed form formula is obtained for the prediction of the resonance frequency. The results obtained are compared with the experimental results and that determined using the Marcatili and EDC methods. The formula provides a simple method for the estimation of the resonance frequency of the TE^{y}_{111} mode with an error ranging from –9.3% to +2.05% for ε_r<100.

Dielectric Resonators (DRs) in cylindrical, rectangular and other geometries positioned on the top of a ground plane could operate as an efficient antenna [1,2]. Several methods have been proposed to predict the resonance frequency of a dielectric resonator antenna (DRA) [3-8], including the closed cavity model [3], the effective dielectric constant (EDC) model [7,9], the modified wave guide model (MWGM) [8] and the conventional dielectric waveguide model CDWM [10].

In this paper the conventional dielectric waveguide model (CDWM) is used to determine the resonance frequency of a rectangular dielectric resonator operating at the TE^{y}_{111} mode for antenna application. A fitted closed form formula is obtained for the prediction of the resonance frequency. The results obtained are compared with the experimental results and that determined using the Marcatili and EDC methods. The formula provides a simple method for the estimation of the resonance frequency of the TE^{y}_{111} mode with an error ranging from –9.3% to +2.05% for ε_r<100.

The Conventional Dielectric Waveguide Model Approximation

The RDRA under consideration is shown in Figure 1a. The resonator is located on a large ground plane. The dimensions of the resonator are a, b, h in x, y and z directions respectively and the DR has a relative dielectric constant of ε_r. The width and height aspect ratios are defined as p=(b/a) and q=(2h/a) respectively. The DR is excited by a probe on the x-axis at x=a/2. Assuming the ground plane is infinitely large, image theory can be applied equivalently to replace the resonator by an isolated DR, as shown in b. The equivalent resonator has twice the height of the original resonator.



(a)

(b)

Figure 1. A rectangular dielectric resonator antenna a) on a ground plane excited by a probe and b) its equivalent isolated resonator

The field components of the TE^{y}_{111}mode inside the resonator can be obtained to be [10]:

$$
\begin{aligned}
E_x &= -(A/\varepsilon_d)\,k_z \cos(k_x x)\cos(k_y y)\sin(k_z z)\\
E_y &= 0\\
E_z &= (A/\varepsilon_d)\,k_x \sin(k_x x)\cos(k_y y)\cos(k_z z)\\
H_x &= (A/j\omega\mu_0\varepsilon_d)\,(k_x k_y)\sin(k_x x)\sin(k_y y)\cos(k_z z)\\
H_y &= (A/j\omega\mu_0\varepsilon_d)\,(k_x^2 + k_z^2)\cos(k_x x)\cos(k_y y)\cos(k_z z)\\
H_z &= (A/j\omega\mu_0\varepsilon_d)\,(k_y k_z)\cos(k_x x)\sin(k_y y)\sin(k_z z)
\end{aligned}
$$

where ε_d ($\varepsilon_d = \varepsilon_r \varepsilon_0$) is the dielectric constant of the resonator, and k_x, k_y and k_z are the wavenumbers in x, y and z directions respectively which can be determined from the boundary conditions. In the analysis of the CDWM, the boundary conditions consist of four perfect magnetic walls at x=±a/2, z=±h surfaces and the continuous tangential fields at y=±b/2 surfaces. The characteristic equations for the wavenumbers are [10]:

$$k_x = (\pi/a), \quad k_y \tan(k_y b/2) = \sqrt{k_x^2 + k_z^2 - k_0^2},$$
$$k_z = (\pi/2h)$$

and the wave numbers satisfy the separation equation given by:

$$k_x{}^2 + k_y{}^2 + k_z{}^2 = k_0{}^2 \varepsilon_r$$

where $k_0 = 2\pi f_0 / c$ and c=3×10^8 m/s is the speed of light in free space. Based on this approximation, the variation of fields in x or z direction is a complete half cycle, and a fraction of a half cycle in y direction with a fraction δ_{CDWM} defined as:

$$k_y = \delta_{CDWM}\, \pi/b$$

Hence, the resonance frequency can be obtained as:

$$f_0 = \frac{c}{2a\sqrt{\varepsilon_r}} \sqrt{1 + q^{-2} + p^{-2}\delta_{CDWM}^2}$$

In general, δ_{CDWM} is a function of the dielectric constant and aspect ratios p and q. For the width aspect ratio p=1, the variations of δ_{CDWM} for three typical values of relative dielectric constant ε_r are shown in

a. It can be seen that δ_{CDWM} has a weak dependence on the dielectric constant for a given value of q, especially for ε_r>30, which is used in most antenna applications. By choosing the average of the values for ε_r =38 and 100, the numerical values could be fitted to a closed form formula, which has an exponential dependence of q given by:

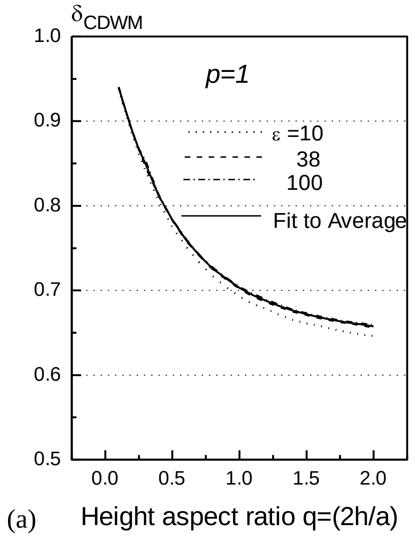
$$\delta_{CDWM} = 0.652 + 0.286\exp(-1.937q + 0.198)$$

Similarly, for other values of p, the numerical values of δ_{CDWM} can be fitted in closed form formulas with different coefficients. By fitting the coefficients as a function of p, a generalised closed form formula can be obtained for 0.5<p<2.0 and q<2.0 as,

$$\delta_{CDWM} = (0.35 + 0.38p - 0.078p^2) + (0.41 - 0.11p)$$
$$\times \exp\left(-\frac{q - 0.09 - 0.01p}{0.31 + 0.26p - 0.06p^2}\right)$$

The dependence δ_{CDWM} on q for various values of *p* is shown in

b. Upon obtaining δ_{CDWM}, the resonance frequency could be obtained using equation (4).



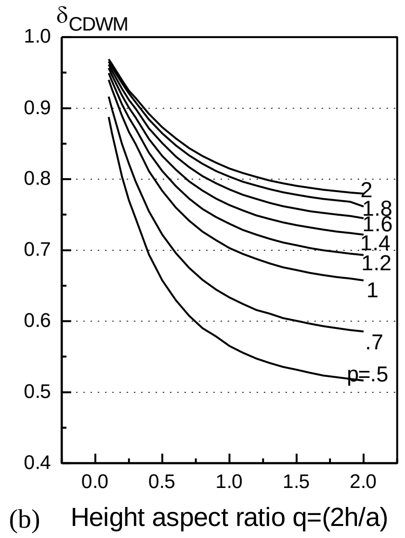


Figure 2. Variation of the cycle ratio versus height aspect ratio q: a) for three value of ε_r and *p=1*and b) variation of cycle ratio for different value of p.

Results and Discussion

For a number of DRs with different dielectric constant and physical dimensions, the determined frequencies using equation (4) are tabulated in Table 1 together with the experimental results obtained independently or from [8] or [9], and with the results determined using the Marcatili method [11], and the EDC method [12]. The errors of prediction for different methods with respect to the measured resonance frequency are also listed in the table. It can be seen that the fitted equation, i.e. equation (4), generally gives a better prediction of the resonance frequency of the TE^{y}_{111} mode in a rectangular DR, with an error of prediction ranges from –9.3% to +2.05%.

Conclusions

In this paper, it has been shown that the CDWM can provide a better prediction of the resonance frequency of the TE^{y}_{111} mode of a rectangular DRA than the Marcatili and EDC method. Based on the CDWM, a fitted closed form formula has been obtained for the calculation of the resonance frequency of a RDRA with $\varepsilon_r>30$, $0.5<p<2.0$ and $0.2<q<2$. The formula provides a simple method for the estimation of the resonance frequency of the TE^{y}_{111} mode with an error ranging from –9.3% to +2.05% for $\varepsilon_r<100$.

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Table 1. Theoretical and measured resonance frequency of the RDRAs operating at the fundamental TE^{y}_{111} mode.

ε_r	a (mm)	b (mm)	h (mm)	Measured f_0 (GHz)	CDWM F_0/error(%) (GHz)	Marcatili[11] f_0/error% (GHz)	EDC [12] f_0/error (%) (GHz)	Ref.
37.1	6	4	3	7.27	6.99/-3.8	6.89/-5.2	6.86/-5.6	[9]
37.1	6	3	3	7.91	7.56/-4.4	7.34/-7.2	7.33/-7.3	[9]
37.84	8.77	8.77	3.5	5.34	4.89/-8.4	4.80/-10.1	4.65/-13	[8]
37.84	9.31	9.31	4.6	4.59	4.16/-9.3	4.08/-11.1	4.05/-11.7	[8]
37.84	8.6	2.58	8.6	5.34	5.45/2.05	4.26/-20	4.06/-24	[8]
37.84	8.77	3.51	8.6	4.79	4.68/-2.2	4.02/-16	3.89/-18.7	[8]
37.84	9.31	4.6	9.2	4.11	4.06/-1.2	3.76/-8.5	3.64/-11.4	[8]
79.46	12.7	12.7	6.35	2.64	2.49/-5.6	2.10/-20	2.07/-21.6	[9]
100	10	10	2	4.57	4.22/-7.6	4.17/-8.8	4.15/-9.2	[8]
100	10	10	1	7.97	7.76/-2.6	7.58/-4.9	7.46/-6.4	[8]
100	12.7	12.7	1	7.72	7.67/-0.6	7.42/-3.9	7.23/-6.3	[8]
100	5	10	1	8.85	8.19/-7.4	8.08/-9.0	8.06/-9.1	[8]
100	10	5	1	8.5	8.03/-5.5	7.89/-7.1	7.65/-10	[8]
38	19	19	9.5	2.20	2.02/-8.1	1.99/-9.5	1.97/-10.4	Exp.*
37	18	18	9	2.34	2.16/-7.7	2.12/-9.4	2.11/-9.8	Exp.*

*: Experimental value obtained by the authors.