

## An Investigation into Optimization of Flapping Gait of Snake Robot

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### Abstract

One of the main advantages of snake-like robot over wheeled vehicles is its capability to move with different modes of locomotion, also called gaits. Because of inherent power inefficiency of snake robots, finding parameters that allow efficient motion for different gaits is of great importance. One of these gaits is called "flapping gait". This gait is usually used for obstacle avoidance in the path of motion.

In this study, parameters effecting efficient flapping gait locomotion are investigated. We first drive dynamic equations of motion for an n-link two dimensional snake robot. We study the effect of flapping gait locomotion parameters on speed and input power. We then use Genetic Algorithm (GA) to find parameters for optimal motion. We also investigate the effect of numbers of robot links on maximum achievable speed and obtained optimal gait. We will show that the obtained optimal parameters, generate a similar gait for robot with any number of links. Effect of different environmental conditions (friction effects) on optimal parameter are also investigated. Next we consider input power and robot speed as optimization criterion and find pareto-optimal curve. By extracting points on this curve we find relations that are useful for designing an efficient robot moving with different speed.

**Keywords:** snake robot, flapping gait, genetic algorithm, dynamic equation, optimal motion.

### Introduction

Snake robots are serially connected, multilink articulated mechanisms, which propel themselves by body shape undulations. Despite having challenges in the area of control and inefficiency in locomotion due to high friction snake-like robots have attracted the attention of researchers for applications not suitable for wheeled and legged robots. Applications such as ruins of collapsed buildings or narrow passages in search and rescue operations are good examples where snake robot may be used.

Snake robots are advantageous over wheeled vehicles due to their terrainability, high adaptability to environment (by using different gaits) and increasing reliability when made modular. However, the two main challenges of snake robots over wheeled mechanisms are difficulty in dynamic analysis of snake-like locomotion as well as their poor power efficiency for surface locomotion. Former has been addressed by many researchers [2]. In this study we address the power efficiency in the case when robot is having a

flapping gait motion. This gait is similar to a back stroke used by human swimmers.

Snake robot gaits can be divided into two main classes: Snake-like and non snake-like gaits. Serpentine, concertina and sidewinding are three common Snake-like gaits which are inspired from real snakes. Non snake-like gaits do not exist in nature but are useful in snake robot motion. However, these gaits are less addressed in literature. Spinning gait and flapping gait [3] are examples of such gaits.

The first snake robot was built by Hirose [1]. He studied kinematics of serpentine gait, and proposed a 'Serpentoid curve' as a means to generate serpentine locomotion. McIsaac and et al [3] studied snake robot locomotion theory based on Geometric Mechanics. They introduced other gaits such as flapping and spinning gaits for eel-like locomotion. Saito et al [4] made a snake robot without wheels and analyzed the optimally efficient serpentine locomotion. And more recently L. Chen and et al [5] studied dynamics of snake robot moving with concertina gait. Recently, the control methodology based on central pattern generator (CPG) is attracting a great deal of attention as a methodology to realize quick and adaptive motion generation of robots having large degrees of freedom ([6] and [7]). K. Inoue [8] optimized network of central pattern generators in order to maximize robot speed. I. Tanev [9] used genetic programming in order to automatically design 3-dimensional snake robot locomotion gait with maximum speed.

The rest of the paper is organized as follows. Section 2 describes dynamic equation of motion of snake robot. Flapping gait is introduced in section 3. Section 4 describes effect of flapping gait parameters on speed and input power. Section 5 describes how to find optimal and pareto optimal flapping gait parameters using GA. Finally section 6 concludes the results and proposed future works

### Dynamic Equations of Motion of Snake Robot

Consider the planar snake robot depicted in Figure 1, which consists of n links connected through n-1 joints. Each link is rigid with uniformly distributed mass and is equipped with a torque actuator (motor). Each link is of mass  $m_i$ , length  $2l_i$ , and moment of inertia  $J_i=m_i l_i^2/3$ . Let  $(x_i, y_i)$  and  $\theta_i$  define the center of gravity and the angle between the link and the x-axis, respectively. Free-body diagram for the i-th link is depicted in Figure 2, where  $\theta_i$  is angle between i-link and x-axis.  $f_i$  and  $\tau_i$  are the force and the torque due to the friction between the links and

the horizontal surface.  $g_i$  and  $g_{i-1}$  are the constraint forces from the adjacent links. Lastly,  $u_i$  and  $u_{i-1}$  are the joint torques from the actuators.

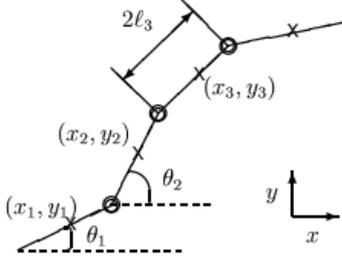


Figure 1: n-link snake robot

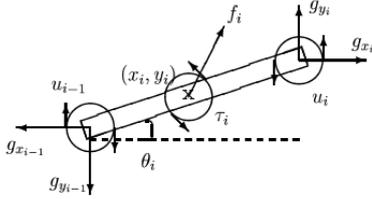


Figure 2: Free body diagram of  $i$ -th link

We consider a simple viscous friction model. Friction force is modeled by the following equations

$$f_t = -c_t m_i v_t \quad (1)$$

$$f_n = -c_n m_i v_n \quad (2)$$

Where  $c_t$  and  $c_n$  are normal and tangential viscous friction coefficients. Suffix  $i$  indicates correspondence to  $i$ -th link,  $f_t$  and  $f_n$  are friction forces in tangential and normal direction respectively,  $v_t$  and  $v_n$  are velocities in two directions.

Applying the Newton second principle to the free-body diagram of the  $i$ -th link in Figure 2 and assembling into the  $n$ -link snake robot, we obtain the equations of translational and rotational motion as

$$\sum F_x = ma_x \rightarrow M\ddot{x} = f_x + D^t g_x \quad (3)$$

$$\sum F_y = ma_y \rightarrow M\ddot{y} = f_y + D^t g_y \quad (4)$$

$$\sum M = J\ddot{\theta} \rightarrow J\ddot{\theta} = -S_\theta LA^t g_x + C_\theta LA^t g_y + D^t u \quad (5)$$

Where  $g_x, g_y, u \in \mathbb{R}^{n-1}$  and  $f_x, f_y, u, x, y \in \mathbb{R}^n$  and  $(f_x, f_y)$  are the component of the friction force vector  $f_i$ . Utilizing kinematics constraints we can derive at relations between  $x_i, y_i$  (center position of individual links) and  $\theta_i$  (angle between the link and the  $x$ -axis) and  $\omega_x, \omega_y$  which are position of center of mass (CM) of the snake robot. Substituting  $\theta$  and  $\omega$  in terms of  $x$  and  $y$  into (3) and (4) will define  $g_x$  and  $g_y$  which are then substituted into (5). This will arrive at,

$$P\ddot{\theta} + C\dot{\theta}^2 = W^t f + D^t u \quad (6)$$

Newton law for whole robot body (equations of translational motion):

$$m\ddot{\omega} = E^t f \quad (7)$$

Where:

$$A = \begin{bmatrix} 1 & 1 & & \\ & \ddots & \ddots & \\ & & 1 & 1 \end{bmatrix} \in \mathbb{R}^{(n-1) \times n}$$

$$D = \begin{bmatrix} 1 & -1 & & \\ & \ddots & \ddots & \\ & & 1 & -1 \end{bmatrix} \in \mathbb{R}^{(n-1) \times n}$$

$$E = \begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}, e = [1 \ \cdots \ 1] \in \mathbb{R}^n, m = \sum_{i=1}^n m_i$$

$$S_\theta = \text{diag}(\sin \theta_1, \dots, \sin \theta_n), C_\theta = \text{diag}(\cos \theta_1, \dots, \cos \theta_n),$$

$$J = \text{diag}(J_1, \dots, J_n), M = \text{diag}(m_1, \dots, m_n),$$

$$L = \text{diag}(l_1, \dots, l_n), H = LA^t (DM^{-1}D^t)AL,$$

$$N = M^{-1}D^t (DM^{-1}D^t)AL, P = J + S_\theta HS_\theta + C_\theta HC_\theta,$$

$$C = S_\theta HC_\theta - C_\theta HS_\theta, W = [S_\theta N^t, -C_\theta N^t],$$

$$B = DP^{-1}D^t, K = P^{-1}D^t B^{-1}, \rho = 1/(e^t P e), r = \rho e,$$

By defining two new variables in (8), dynamic equation of motion can be decoupled into two parts: the shape motion (joint torques  $\rightarrow$  joint angles) and inertial locomotion (joint angles  $\rightarrow$  position CM and overall orientation of the robot)

$$\dot{\theta} = K \dot{\phi} + e \dot{\psi} \quad (8)$$

Where  $\dot{\phi}$  is vector of relative angles between adjacent bodies and  $\dot{\psi}$  can be thought of as an average angular momentum. Substituting (8) into (6) and (7) we have,

$$\begin{bmatrix} \rho & 0 \\ 0 & mI \end{bmatrix} \begin{bmatrix} \ddot{\psi} \\ \ddot{\omega} \end{bmatrix} + \begin{bmatrix} r^t R r & r^t S \\ S^t r & Q \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\omega} \end{bmatrix} + \begin{bmatrix} r^t R \\ S^t \end{bmatrix} K \dot{\phi} = 0 \quad (9)$$

$$\ddot{\phi} + DP^{-1}(C\dot{\theta}^2 + R\dot{\theta}^2 + S\dot{\omega}) = Bu \quad (10)$$

Therefore, the dynamic equation is decoupled into two parts. Torque input,  $u_i$  which drives the shape  $\phi$  of the snake through equation (10). The rate of shape change,  $\dot{\phi}$ , when coupled with the inertial motion will generate the friction force and torque. The friction force and torque thus drive the snake robot with respect to the inertial frame through the equation (9).

## Generation of Flapping Gait

In a serpentine gait, a snake robot moves in the forward direction. Flapping gait allows motion to either side directions which is useful when robot encounters an obstacle. As introduced in [4], changing the relative angles of the eel-like robot using (4) results in flapping gait locomotion.

$$\varphi_i(t) = \begin{cases} \alpha \sin(\omega t + (i-m)\beta) + \gamma, & 1 \leq i \leq m \\ \alpha \sin(\omega t + (m+1-i)\beta) + \gamma, & m < i < n \end{cases} \quad (11)$$

Where  $(i=1, \dots, n)$  is the index of links and  $m=(n-1)/2$  for odd number of links. Minimum number of links for this gait is  $n=5$ .

We show that the same flapping gait motion can be achieved for a snake robot using viscous friction model. Figure 3 illustrates result of solving differential equations (1) for  $\psi$  and  $\omega$  while  $\phi$  is given by equations (4) for a 5-link snake robot using viscous friction model.

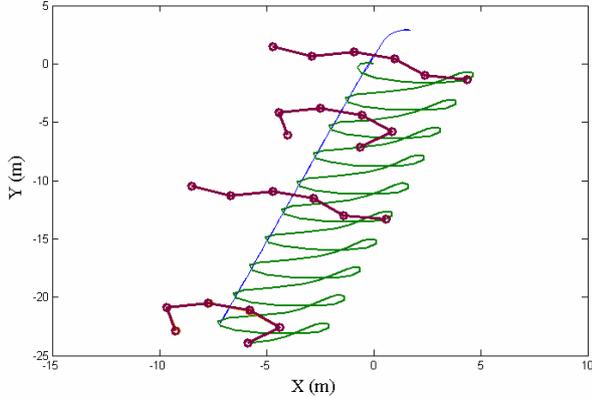


Figure 3: Robot configuration in flapping gait.

### Effect of Flapping Gait Parameters on Efficiency of Motion

In this section, we study the effect of flapping gait parameters ( $\alpha$ ,  $\beta$ ,  $\omega$ ,  $\gamma$ ) on velocity and input power. Efficient motion is defined as motion with minimum power loss and maximum velocity.

Instantaneous input power for each link is obtained by multiplying the input torque by angular velocity of corresponding joint. Average power is calculated by dividing sum of these values over duration of motion. By dividing the parameter space as  $\alpha=0,\pi/20,\dots, \pi/2$ ,  $\beta=0,\pi/20,\dots, \pi$ ,  $\gamma=0,\pi/20,\dots, \pi/2$  and  $\omega=0,1,\dots,20$  rad/sec, the input power is calculated via simulation for each parameter set of ( $\alpha$ ,  $\beta$ ,  $\omega$ ,  $\gamma$ ). In order to show how flapping gait parameters affect input power, different surfaces are plotted. Due to similarities between these surfaces, we will only show two of them in Figure 4 and Figure 5. Figure 4 shows surface of input power for different parameter set of ( $\alpha,\gamma$ ) while  $\beta$  is constant. Figure 5 shows surface of input power for different parameter set of ( $\alpha,\beta$ ) while  $\gamma$  is constant. We also repeat simulations for different value of  $c_n/c_t$  ranges from 0 to 1. We obtain the following conclusions by observing simulation results:

1. Increasing  $\alpha$  and  $\omega$  increases input power.(Figure 5 and Figure 6)
2. Increasing  $\beta$  decreases input power.(Figure 5)
3.  $\gamma$  does not have a considerable effect on power. (Figure 4)
4. Input power increases by increasing friction coefficient ratio of  $c_n/c_t$ . (Figure 7)
5. Flapping gait parameters ( $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\omega$ ) independently affect input power. (similarity of the plotted surfaces).

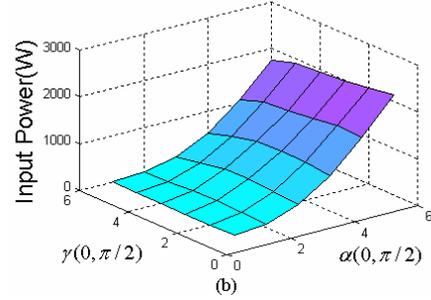
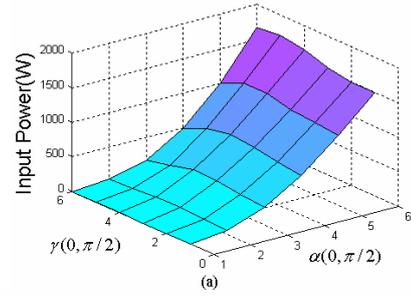


Figure 4: Input power vs. ( $\alpha,\gamma$ ), for (a)  $\beta=4\pi/10$  (b)  $\beta=3\pi/10$ .

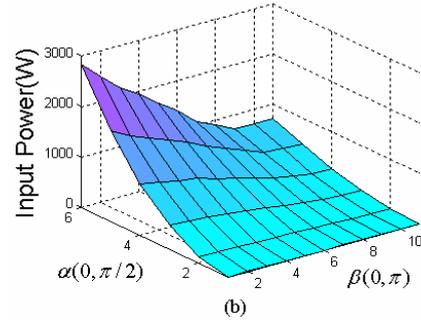
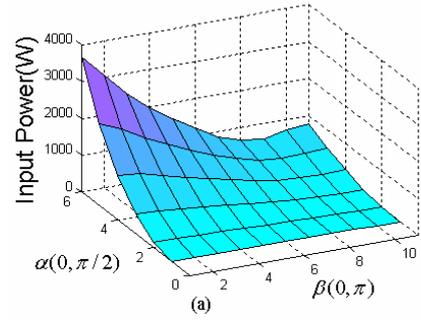


Figure 5; Input power vs. ( $\alpha,\beta$ ). for (a)  $\gamma=2\pi/10$ , (b)  $\gamma=3\pi/10$ .

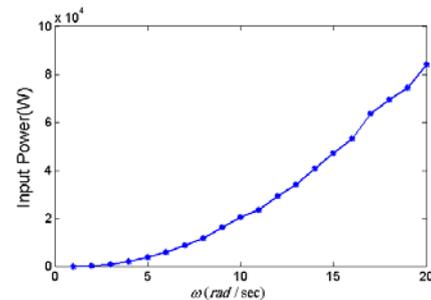
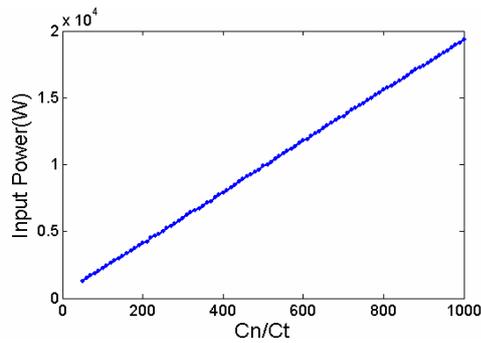


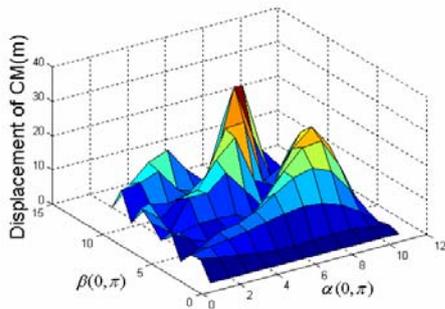
Figure 6: Input power vs.  $\omega$  ( $\alpha=9, \pi/10, \beta=6\pi/10$ ,  $\gamma=2\pi/10$ ).



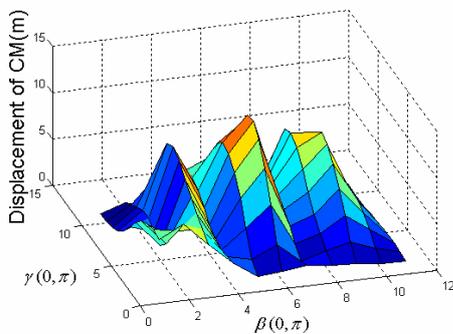
**Figure 7: Input power vs.  $c_n/c_t$  ( $\alpha=9\pi/10$ ,  $\beta=6\pi/10$ ,  $\gamma=2\pi/10$ ).**

Average velocity is simply determined by calculation of center of mass displacement during the complete motion. By dividing the parameter space as  $\alpha=0,\pi/20,\dots,\pi$ ,  $\beta=0,\pi/20,\dots,\pi$ ,  $\gamma=0,\pi/20,\dots,\pi$  and  $\omega=0,1,\dots,20$  rad/sec, average speed is calculated via simulation for each parameter set of  $(\alpha, \beta, \omega, \gamma)$  and plotted. Due to similarities between these surfaces, we will only show two of them in Figure 8 and Figure 9. We also repeat simulations for different values of  $c_n/c_t$ , ranges from 0 to 1. We obtain the following conclusions by observing simulation results:

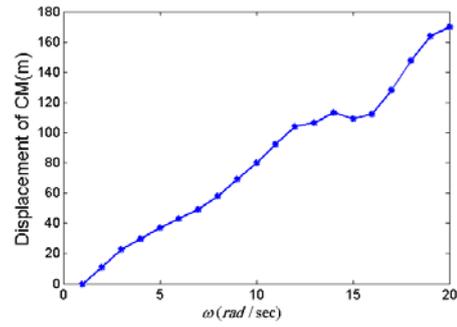
- 1- Increasing  $\omega$  results in increasing velocity. (Figure 10)
- 2- Velocity increases by increasing friction coefficient ratio of  $c_n/c_t$  (Figure 11)
- 3-  $\beta$ ,  $\gamma$  and  $\alpha$  effect CM displacement in a coarse sinusoidal manner as illustrated in Figure 8 and Figure 9.



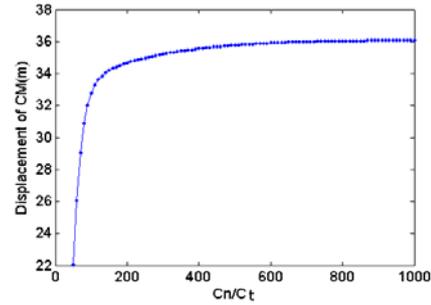
**Figure 8: Displacement of CM vs.  $(\alpha, \beta)$ ,  $\gamma=2\pi/10$ .**



**Figure 9: Displacement of CM vs.  $(\gamma, \beta)$ ,  $\alpha=2\pi/10$ .**



**Figure 10: Displacement of CM vs.  $\omega$  ( $\alpha=9\pi/10$ ,  $\beta=6\pi/10$ ,  $\gamma=2\pi/10$ ).**



**Figure 11: Displacement of CM vs.  $c_n/c_t$  ( $\alpha=9\pi/10$ ,  $\beta=6\pi/10$ ,  $\gamma=2\pi/10$ ).**

### Optimization using Genetic Algorithms

GA can deal with optimization of complicated systems by simply simulating system behavior and applying an evaluation index. Therefore we use GA in order to find flapping gait parameters that optimize speed of motion. GA is an optimization method imitating biological evolution. The target of optimization (flapping gait parameters  $\alpha$ ,  $\beta$  and  $\gamma$ ) is coded as chromosome and called genotype. Phenotype is the result of decoding of genotype (resultant robot locomotion, in this study) and is evaluated by the use of computer simulations. Genotypes with lower evaluation in gene pool will be deleted from the pool and remaining superior genotypes will be succeeded to next generation after being processed by genetic operators, i.e. mutation and crossover. By repeating this process, finally chromosomes with high quality can be derived.

The following is GA setting in this study for each step. Coding genotype: We code chromosomes by serially connecting parameters discretized into 16-bit integers chromosomes become 48-bit (3 parameters). At the first stage of GA, new chromosomes are created by randomly selecting parameters within ranges defined for each parameter. Constraint test: Because of mechanical limit on motors rotation angle, parameter  $(\alpha+\gamma)$  is constrained to a maximum value of  $\theta_{max}$ . Chromosomes not satisfying these constraints will be deleted and new random chromosome will be created. Evaluating fitness: Fitness function that should be minimized is the inverse of center of mass displacement for the total simulation time. Selection: Chromosomes are selected using roulette rule based on fitness value. we applied elite preservation method for fast convergence. Genetic operations: Between remaining chromosomes, crossover

and mutation operation is applied. The cross method used is one-point crossover with a given probability  $P_c$ . Mutation is done by randomly reversing bits with a given probability  $P_m$ . Other GA parameters are set as follows: Population  $N=10$ , Crossover probability  $P_c=0.5$ , Mutation probability  $P_m=0.01$ , Number of generations  $G=200$ .

### Results of GA

In this example we assume  $\omega=\pi$  rad/s,  $\theta_{max}=7\pi/8$  and environment condition,  $c_t/c_n=0.01$ . We next find the optimized flapping gait parameters  $\alpha$ ,  $\beta$  and  $\gamma$  that maximizes the speed of robot. Development of fitness value for one of these optimizations is shown in Figure 12. In this figure, "best" is the best fitness in the population and "mean" is mean value. As shown, after 200 generations, best fitness value converges to 38.24. Best individuals corresponding to the best fitness value are ( $\alpha=1.373$ rad,  $\beta=3.854$ rad  $\gamma=0.590$ rad). Utilizing these values, snake can achieve a maximum speed of 1.32 m/sec.

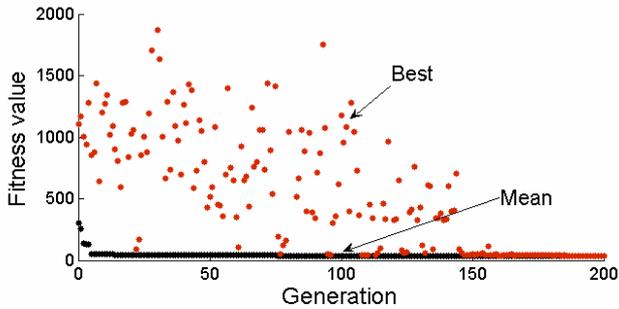


Figure 12: Development of the best and mean value of fitness (1/speed) for every generation.

Figure 13 shows snake robot moving with the optimized flapping gait. Configurations of snake robot illustrated in this figure are numbered based on time. These configurations are plotted in every 2 seconds for one complete cycle of motion. Path followed by center of mass and the end of the robot are also illustrated. As shown in Figure 13, the quickest flapping gait which is result of GA optimization run is roughly similar to butterfly swimming.

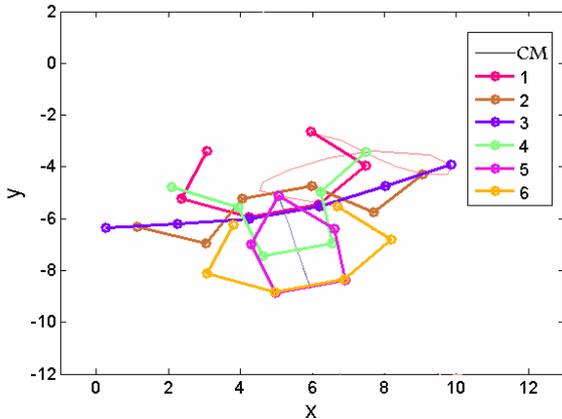


Figure 13: Snake robot configurations moving with optimized flapping gait.

### Friction Effect

In real world application of snake robot, environment condition is not constant therefore robot should be adaptive to different environments. Or at least if it is optimally designed for a specific environment, it should be less sensitive to change in environment conditions. In our case we calculated optimal flapping gait parameters for a specific ratio of friction coefficients. Therefore, we should also investigate the effect of friction changes on optimal parameters. In order to do this, we run GA optimization for different friction coefficient environment. GA settings are similar to section 2. Figure 14 shows optimal flapping gait parameters for different ratio of friction coefficient. As illustrated in this figure the optimal parameters are approximately constant. Another words, robot continues to move efficiently although friction coefficient of the environment slightly changes.

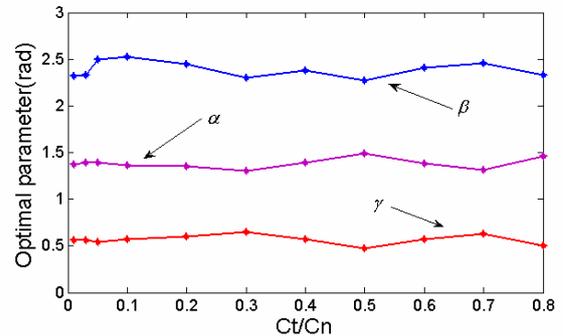


Figure 14: Effect of friction on optimal flapping gait parameters.

### Number of links Effect

In order to investigate the effect of number of links on flapping gait characteristics, we run GA optimization for robot with 5, 7 and 9 links. Results are outlined in Table 1. Results indicate that maximum velocity of flapping gait increases with increase in numbers of links.

TABLE I  
EFFECT OF NUMBER OF LINKS ON OPTIMAL FLAPPIN GAIT

Number of Links.	$\alpha$ (rad)	$\beta$ (rad)	$\gamma$ (rad)	velocity (m/sec)
5	1.373	3.854	0.590	1.31
7	1.351	1.760	0.442	2.45
9	1.050	1.356	0.328	3.44

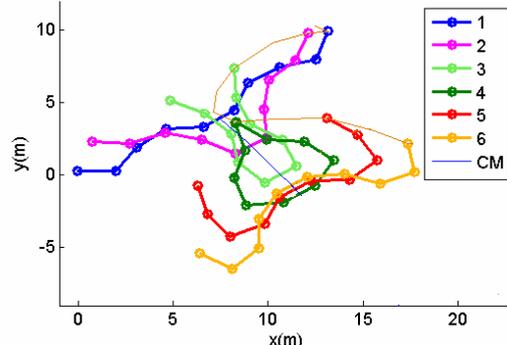


Figure 15: 9 link snake robot configurations moving with optimized flapping gait.

Figure 15 shows a 9-link snake robot moving with optimized flapping gait. These configurations are plotted in every 2 seconds for one complete cycle of motion. Path followed by CM and the end of the robot are also illustrated. As shown in this figure, the quickest flapping gait which is result of GA optimization is roughly similar to butterfly swimming and is similar to the previous results for 5-link snake robot.

### Multi objective optimization

Locomotion in different environment may require robot to move with different speeds. Minimization of power is essential regardless of the necessary speed. In this section we identify the flapping gait parameters ( $\alpha, \beta, \gamma$ ) for different average speeds that minimizes the average power loss. In order to do so we run GA optimizations to minimize input power for a specific velocity. Fitness function for these GA optimizations is input power. When velocity is more than the specified value, a large number for fitness function is assigned. This will penalizes the fitness function and forces the GA to optimize parameters around the given value for the speed. This method will therefore find flapping gait parameters with minimum input power for any specific velocity.

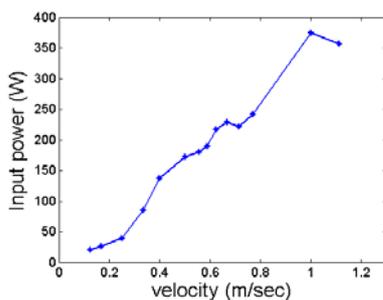


Figure 16: Pareto-optimal curve.

Figure 16 clearly shows the pareto-optimal curve corresponding to the minimum power with fixed values of the speed. The corresponding  $\alpha$ ,  $\beta$  and  $\gamma$  can now be extracted from points on the pareto-optimal curve. These values are shown in Figure 17.

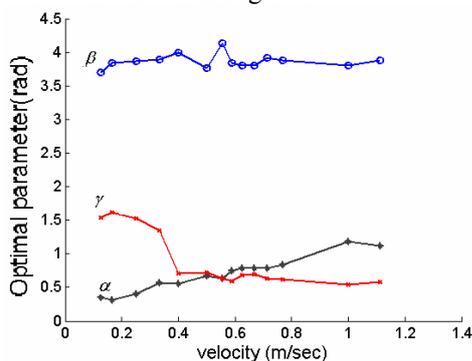


Figure 17: Pareto-optimal flapping gait parameters vs. robot speed

As illustrated in Figure 17, optimal parameter  $\beta$  is approximately constant, optimal  $\alpha$  increases and optimal  $\gamma$  decreases with increase in speed. Therefore, optimal power and desired speed may be achieved by setting  $\beta$  as 3.8 and selecting the corresponding value for  $\alpha$  and  $\gamma$ .

### Conclusion

In this paper we found optimally efficient parameters for snake robot flapping gait using GA. We showed that the optimal flapping gait parameters are roughly constant while friction coefficient between ground and the robot changes. Results of GA optimization show that maximum speed of 3.3 m/sec can be achieved by 9-link snake robot. We run GA optimization for robot with different numbers of links and showed that the optimal gait is similar for robots with different numbers of links. We observe that regardless of the numbers of link, the optimized gait parameters results in a motion that is similar to butterfly swimming. Finally we consider input power and robot speed as optimization criterion and find pareto-optimal curve. By extracting points on this curve we found relations that can be used for designing an efficient robot moving with different speeds. Results presented in this paper can be used as a bed stone for construction of an efficient snake robot with increased maneuverability. Future research will focus on identifying optimal parameters for other locomotion gaits, developing additional autonomy and intelligence, as well as constructing an experimental test bed to verify the theoretical results.

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