



# Analytical Chip Load Prediction for Rough End Mills

B. Moetakef Imani<sup>1</sup>, H. El-Mounayri<sup>2</sup> and S.A. Hosseini<sup>3</sup>

*Mechanical Engineering Department, Ferdowsi University, Mashhad, Iran &  
Advanced Virtual Manufacturing Lab, Mechanical Engineering Department  
Purdue School of Engineering and Technology, IUPUI, Indianapolis, USA  
Corresponding Author E-mail: imani@ferdowsi.um.ac.ir*

## Abstract

Rough end mills with serrated cutting edges are effectively used for suppressing chatter vibrations encounter during milling operations. In this study, serrated cutting edges are analytically defined and geometrically modelled by a NURBS curve. The chip load on the serrated cutting edges is computed by newly proposed recursive algorithm. It is shown that the proposed model can be used in estimating the chip load of rough end mills during operation. Validity and accuracy of the recursive algorithm is geometrically verified with ASIC 3D solid modeller. The developed algorithm can be effectively implemented for static and dynamic simulation of cutting process.

**Keywords:** Rough end mill, Serrated cutting edge, Chip load, ACIS solid modeller

## 1 Introduction

Rough end mills have serrated profiles along the helical flutes, and they are mainly used in roughing operations. Serrated edges result in variation of cutter radius along the flute, which in turn leads to irregular distribution of chip load along the flutes [1]. Generally, the serrations are periodic and smooth, in order to prevent stress concentration which leads to chipping of cutting edges. The irregular distribution of chip load leads to reduce apparent axial depth of cut or axial contact and disturb regenerative phase angle among the successive teeth, hence increasing the chatter stability of the serrated cutters. The surface finish generated by these cutters is wavy; hence these cutters are mainly used for roughing operations. Although significant work has been reported in modeling the mechanics and dynamics of milling [2], there has not been any study on serrated cutters except researches presented by Merdol and Altintas [1] and Tlustý, Ismail and Zaton [2]. Altintas presented the geometric model for serrated end mill and serrated tapered end mill using spline curves. Tlustý mainly studied the effect of undulated profile on chatter stability. The cutting edge may not be fully in contact with the work-piece for this reason the chip thickness is smaller than the radial amplitude of serration on the edge. Tlustý argued that the chatter vibration free depths of cut are increased with the serrated cutters [1]. Furthermore, since the serration profiles have phase shift from one flute to the next, they disturb the regenerative phase shift at each revolution and prevent the build up of chatter vibration energy in the system. In parallel to this research, Campomanes [3] presented modelling of serrated cutters with sinusoidal profiles using average

---

<sup>1</sup> - Assistant Professor of Mechanical Engineering

<sup>2</sup> - Associate Professor of Mechanical Engineering

<sup>3</sup> - M.Sc. Student of Mechanical Engineering

cutting force coefficients and approximate chip thickness model as presented by Altintas [4].

A comprehensive simulation of rough end milling requires geometric and physical simulation of the cutting process. This study presents a geometric modelling of rough end milling with models of the cutting edge and the chip thickness removed by the cutter along with the cutting forces model. Hence, this research presents a geometric modelling of rough end mills with sinusoidal cutting edge using NURBS relations and recursive algorithm for estimating of chip thickness during cutting process. Henceforth, the paper is organized as follows. The geometric modelling of serrated cutters is presented in the next section. In the following section chip load calculation using newly proposed algorithm is introduced. Section 4 presents simulation of rough end milling operations using ACIS 3D solid modeller followed by geometric verification of the developed algorithm

## 2 Geometric modelling of rough end mill cutting edge

The prediction of the cutting force requires identification of chip thickness and local cutting edge geometry along the serrated cutting. Consider a cylindrical serrated cutter with an outside diameter ( $D$ ) and helix angle ( $i_0$ ) as shown in figure 1. A point ( $P$ ) along the flute can be defined by the vector ( $R$ ) starting from the origin ( $O$ ) [1]:

$$R = R_x \vec{i} + R_y \vec{j} + R_z \vec{k} \quad (1)$$

$$R_x = R(z) \sin \phi_i(z) \quad , \quad R_y = R(z) \cos \phi_i(z) \quad , \quad R_z = z \quad (2)$$

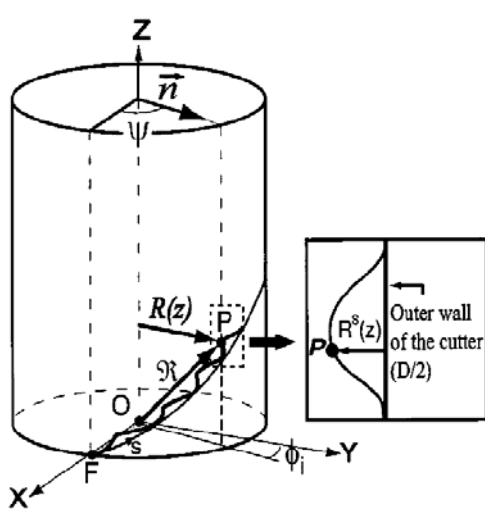


Figure 1: Cutting edge geometry [1]

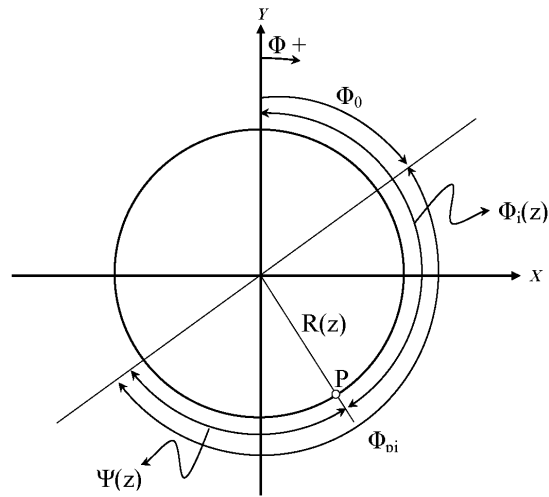


Figure 2: Angle definitions

Where  $R(z)$  is the radius of the cutter at height ( $z$ ). The angular position of the point on the cutting edge  $\phi_i(z)$  measured clockwise ( $CW$ ) from the normal axis ( $y$ ) and shown in figure 2:

$$\phi_i(z) = \phi_0 + \phi_{pi} - \psi(z) \quad \text{where} \quad \phi_{pi} = \left(\frac{2\pi}{N}\right)i \quad \text{and} \quad \psi(z) = \frac{tg(i_0)}{(D/2)} z = k_\mu z \quad (3)$$

( $\phi_{pi}$ ) is the pitch angle between successive flutes and ( $\phi_0$ ) is the angular position of the first flute ( $i=0$ ) at elevation ( $z=0$ ). The pitch angle ( $\phi_{pi}$ ) is constant for constant-



pitch end mills where  $N$  is the number of flutes on the cutter, and  $i$  is the flute index ( $i=0, 1, \dots, N-1$ ). The helix creates a lag angle for the point at the  $z$  level on the cutting edge defined by  $\psi(z)$  [1],[5].

The cutter radius  $R(z)$  is variable and represent the outer envelope of the serrated cutter. It can be expressed as bellow:

$$R(z) = \frac{D}{2} - R^s(z) \quad (4)$$

Where  $R^s(z)$  is a periodic function which is subtracted from outer radius[1]. In order to construct a serrated cutting edge along the helix, it is necessary to derive a relation between  $z$ -axis and  $s$ -axis which is locally defined along the cutting edge. Hence, the relation between  $z$ -axis and  $s$ -axis can be expressed as [1]:

$$S = \frac{Z}{\cos(i_0)} \quad (5)$$

A cutting edge with sinusoidal profile  $R^s(s)$  can be shown with equation (6). By substituting equation (5) into equation (6),  $R^s(s)$  is changed to  $R^s(z)$  which is used to construct a sinusoidal serrated cutting edge along the helix:

$$R^s(s) = A \sin(\omega s) \quad (6)$$

$$R^s(z) = A \sin\left(\omega \frac{z}{\cos(i_0)}\right) \quad (7)$$

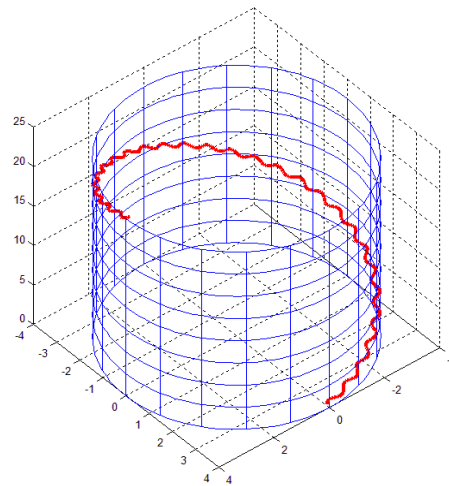
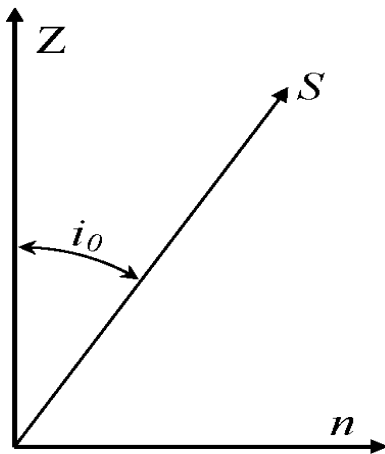


Figure 3: Definition of s-axis      Figure 4: Mathematically modelled serrated edge

In addition to analytical presentation of cutting edge, the NURBS parametric representation, which is the standard form of curve representation in the all CAD/CAM systems, is chosen to model the serrated cutting edge. A NURBS curve of degree  $p$  which is defined by control points  $P_0, P_1, \dots, P_n$ , knot vector  $U = \{u_0, u_1, \dots, u_m\}$ , and weights  $w_0, w_1, \dots, w_n$  is expressed by [6]:

$$C(u) = \frac{1}{\sum_{i=0}^n N_{i,p}(u)w_i} \sum_{i=0}^n N_{i,p}(u)w_i P_i \quad (8)$$



The number of knots, the degree of the basis functions, and the number of control points are related by  $m = n + p + 1$ .  $N_{i,p}(u)$  is the  $i$ -th basis function of degree  $p$  is defined recursively as follows:

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \leq u \leq u_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

Points along the serrated cutting edge can be obtained by deviding the edge into  $N$  equal arc-length segments:

$$r_i = x_i \vec{i} + y_i \vec{j} + z_i \vec{k} \quad i = 0, 1, 2, \dots, N \quad (10)$$

$N+1$  points on the cutting edge are interpolated by a NURBS curve in ACIS solid modeller environment by the following Scheme-interface command [7]:

*(edge:spline (list of  $N+1$  points on the cutting edge))*

### 3 Analytical modelling of chip load

The mechanics of cutting and modelling the cutting forces require the identification of chip load (chip thickness) perpendicular to the serrated flute. In common helical milling tools such as flat or ball-end mills, the geometry of all cutting edges is identical. Thus, the chip load variations is a function of angular position of the cutting edge ( $\varphi$ ) and lag angle  $\psi(z)$  of the point on the edge due to helical shape of cutting edge. Figure 5, 6 shows the axial and angular position of a point on the cutting edge at  $z$ -level along with the chip thickness.

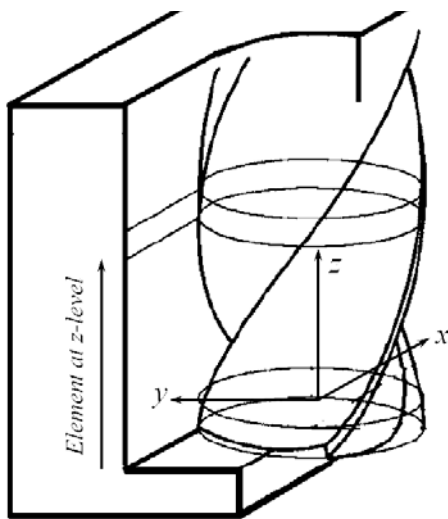


Figure 5: Axial position of element

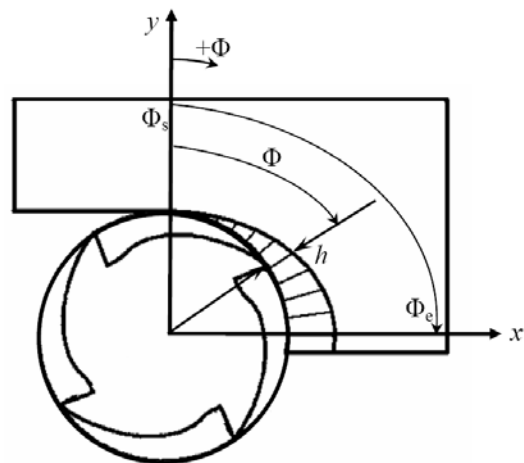


Figure 6: Angular position of element

Where

$$h = c \sin(\varphi)$$

Where  $c$  is feed(mm) / tooth

(11)

There is no difference between the geometry of cutting edges in flat or ball-end mill tools. In contrast there is a phase shift between successive serrated cutting edge along z-axis in rough end mill tools. As a result, the chip load is a function of the following parameters:

- Angular position of cutting edge ( $\varphi$ ).
- z-level of the point on the cutting edge (helix effect).
- The surface generated by the prior cutting edge. This undulated surface and the serrated profile of the current cutting edge will determine the variation of chip load along cutting edge.

Figure 7 shows the variation of chip load along z-axis from first engagement of tool with workpiece to steady state cutting after several revolution.

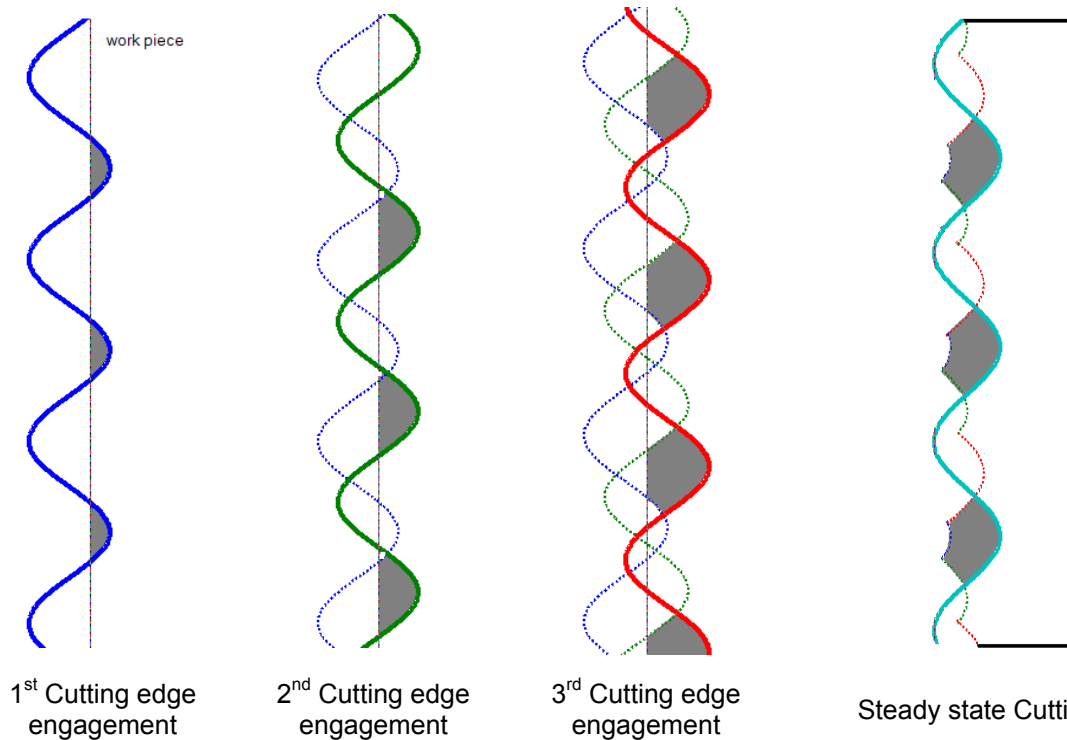


Figure 7: variation of chip load along z-axis from first engagement of tool with workpiece to steady cutting after several revolution

After several revolutions cutting process becomes stable with period of one revolution. The following recursive relationship is proposed for the calculation of chip load. Using this relationship, it is possible to predict chip load for the first engagement to final steady state engagement.

$$h_n = nc \sin(\varphi) - \left[ \frac{D}{2} - R_n^s(z) \right] - \sum_{i=0}^{n-1} h_i \quad \text{If } h_n < 0 \Rightarrow h_n = 0 \text{ and } h_0 = 0 \quad (12)$$

Where  $n$  is the number of revolution,  $c$  is feed(mm)/tooth,  $D$  is diameter of the tool,  $R_n^s$  is the equation of engaged cutting edge,  $h_n$  is chip load in the  $n^{th}$  revolution and  $h_i$  is chip load in the  $i^{th}$  revolution of the tool. Figure 8 illustrates the 3D chip geometry simulated by ACIS solid modeller.

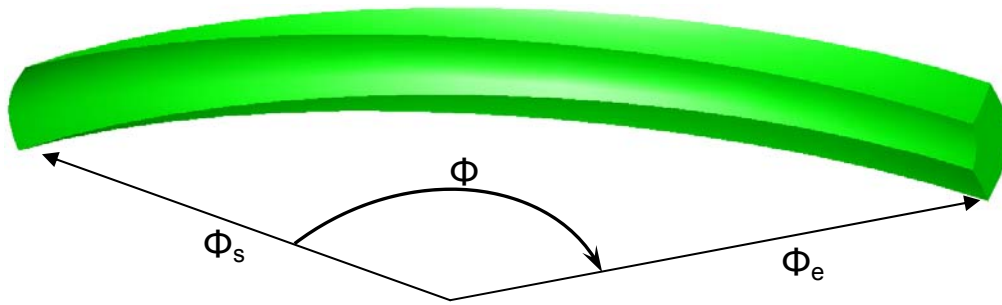


Figure 8: 3D Chip geometry

#### 4 Part Updating Using ACIS Solid Modeller

In order to compute and verify chip load for serrated end mill, ACIS solid modeller is used as geometric engine. The work piece is modelled by a solid block (20\*20\*4mm). The constructed cutting edge is used to define swept surface which simulate the cutting action of the cutter. The swept surface is constructed as follows:

- Cutting edge transfer to initial cutting position.
- Copies of cutting edge are rotated by an angular increment about z-axis and at the same time transfer along feed direction.
- Swept surface is constructed by the following command:  
(sheet:skin-wires (list of cutting edge))

In the next step part updating is performed by boolean subtract operation between part and the swept surface by the following Scheme-interface command:  
(bool:subtract part1 sheet1 )

Figure 9 shows these steps .

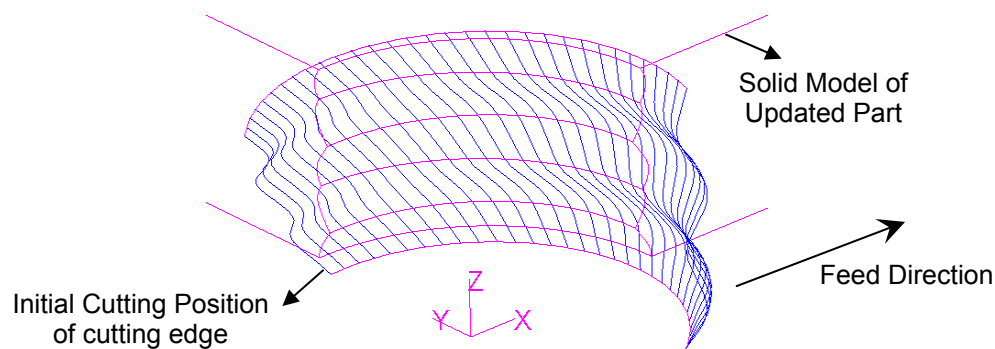


Figure 9: Swept surface generated by rotation of cutting edge in ACIS

During primary contacts of the generated swept surface with part, the chip geometry is varying at each updating stage. But, after several subtraction the chip geometry become stable with period of one revolution. It should be noted that there is a phase shift between successive cutting edges. Hence, the chip load is varying along z-axis. Figure 10 shows the feed marks and varying chip load along z-axis.

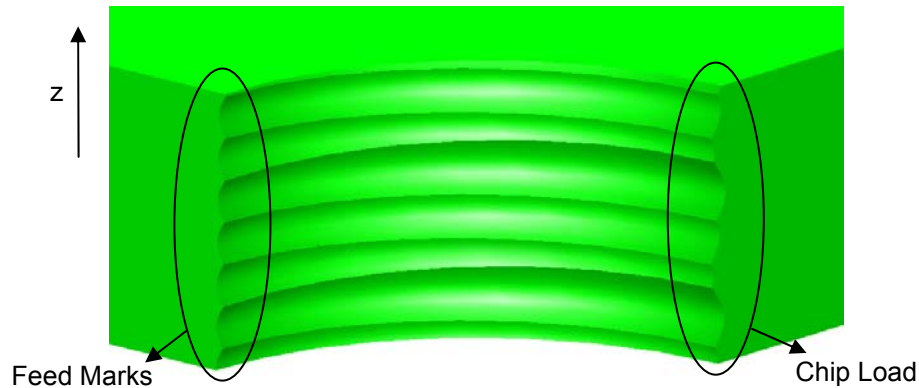


Figure 10: Variation of chip load along z-axis

## 5 Geometric chip load verification

In the previous section, the analytical chip load computation was presented. The recursive algorithm is fast and it can be used for static and dynamic simulation of cutting process. In this section the proposed algorithm is verified by the geometric simulation. The updated part model which was built in section 4 is used for geometric chip load verification. The steps of chip load computation can be summarized as:

- Cutting edge is rotated and transferred for the next engagement.
- Radial lines are created from a point on z-axis to the point of cutting edge at z-level parallel to the  $xy$  plane.
- The intersection of the radial line with updated part is found by following ACIS command:  
(*bool:intersect updated-part radial-line "keep-blank"*)

The length of intersection line is reported as chip load for the cutting edge rotation angle ( $\varphi$ ) and z-level. Figure 11 illustrates computed chip load on the cutting edge for each rotation angle.

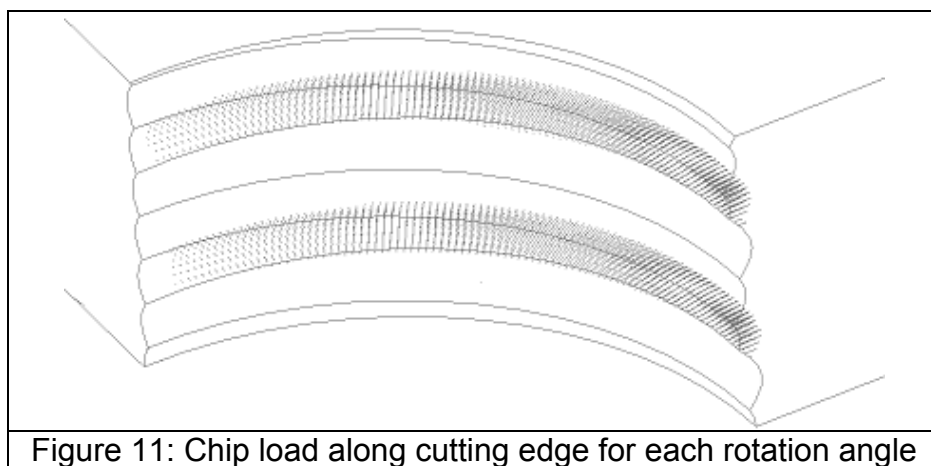


Figure 11: Chip load along cutting edge for each rotation angle

Figure 12 compares the results of analytical computation of chip load using MATLAB R2006a with the results of geometric estimation of chip load using ACIS solid modeller. Specification of the rough end mill tool is reported in Table 1. Cutting condition for the current computation and simulation are presented in Table 2.





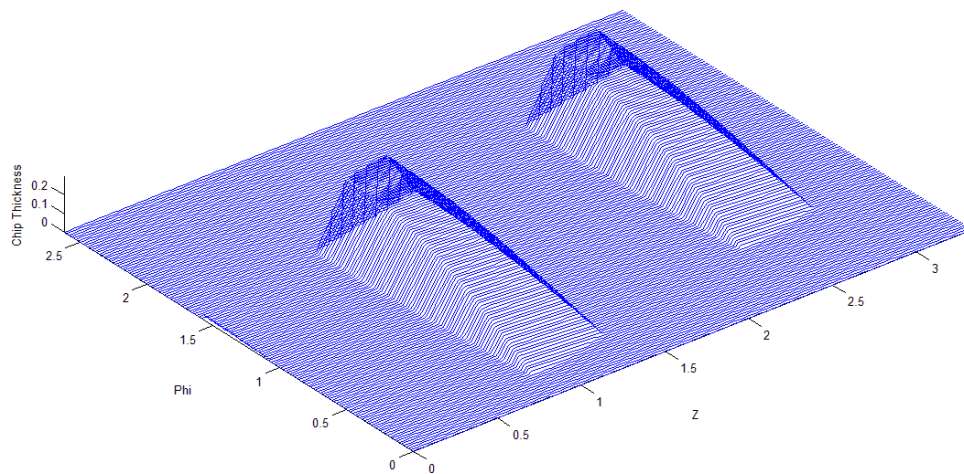
Table 1: Specification of the rough end mill tool

Cutter type	serration			No. of flute	Diameter (mm)	Helix angle (deg)
	Height (mm)	A (mm)	W (rad/mm)			
Kennametal MDRHEC8MS3 KC 625	20	0.2	3.21	3	8	40

Table 2: Cutting Condition

Axial Depth of Cut	C feed(mm)/tooth	$\phi_s$ (deg)	$\phi_e$ (deg)
20 mm	0.1 mm	0	90

MATLAB R2006a Results



ACIS Solid Modeller Results

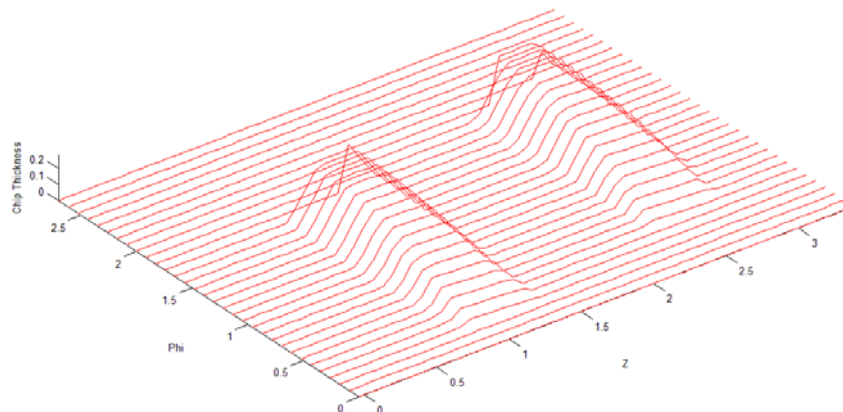


Figure 12: Comparison of the analytical and geometrical results

As it can be seen, there is excellent agreement between analytical and geometrical verification. Thus, the proposed recursive algorithm which can compute chip load much faster than geometric simulation can be efficiently implemented for static and dynamic simulation of cutting process.





## Conclusion

The optimal design of the serrated end mills which are mainly used for suppressing chatter vibration requires effective computation of the chip load exerted on each cutting edge. This research developed a new recursive relationship for computation of chip load. The proposed recursive algorithm can effectively compute the chip load for the first engagement to the final steady state engagement. ACIS 3D solid modeller is used for simulation of the process and verification of the developed algorithm. The geometric simulation confirms the accuracy of the proposed analytical chip load prediction algorithm.

## References

- [1] S.D. Merdol and Y. Altintas, "Mechanics and Dynamics of Serrated Cylindrical and Tapered End Mills", *Journal of Manufacturing Science and Engineering, Transactions of the ASME*, Vol. 126, MAY 2004, pp 317-326.
- [2] Tlusty, J., Ismail, F., and Zaton, W., "Milling Cutters with Irregular Pitch" Technical Report, McMaster Engineering, 1982.
- [3] Campomanes, M. L., "Kinematics and Dynamics of Milling With Roughing Endmills," *Metal Cutting and High Speed Machining*, Kluwer Academic/Plenum Publishers, 2002.
- [4] Altintas, Y., "Manufacturing Automation", Cambridge University Press, 2000.
- [5] Tlusty, G., "Manufacturing Processes and Equipment", Prentice Hall, 1999.
- [6] Piegl, L., and Tiller, W., "The NURBS Book", Springer, 1998.
- [7] ACIS Scheme help, ACIS 3D Solid Modeler, Schemers Inc. & Spatial Technology Inc., 1996.