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# The Effect of Self-Gravity on the Equilibrium Structure of a Non-Rotating Thick Disk

**Jamshid Ghanbari[*] [a], Shahram Abbassi [b] and Sareh Ataiee [a]**
*[a] Department of Physics, School of Sciences, Ferdowsi University of Mashhad, Mashhad, Iran*
*[b] School of Physics, Damghan University of Basic Sciences, Damghan, Iran*

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**Abstract:** We investigate the effect of self-gravity on the equilibrium structure of a thick non-rotating disk around a central object by the self-similar method. We introduce three dimensionless variables $C_{sg}$, $C_t$ and $C_k$ that indicate the relative importance of self-gravity, thermal energy and kinetic energy, respectively. We study the effect of each of them on the structure of the disk. Our self-similar solutions show that the self-gravity modifies the structure of the disk. We find out by increasing the ratio of disk mass to the central object mass, the disk becomes thinner. Our results show that increasing kinetic and thermal energies have similar effects on the structure of the disk and make it thicker.

**Keywords:** accretion disc, magnetic field, MHD.

## Introduction

The dynamics of accretion disks in the context of high energy astrophysics have been studied by several authors [1]. The plasma processes in the vicinity of the compact objects are believed to the main mechanism for the generation of the energy in such objects [1]. If the central compact object has also an intrinsic magnetic field, then the plasma flow is governed by the structure of the magnetic field which arises due to the distribution of the seed field by currents flowing through the disks.

The first analytical equilibrium solution including the conductivity of the plasma for a case of non-rotating magnetized star accreting matter from a disk was obtained by Kaburaki [2], [3].

Geometrically, accretion disks are classified into two categories: thin disks and thick disks [4]. For thin disks, a generally accepted model was proposed by Shakura & Sunyaev (1973), while for thick disks in spite of many efforts [5], [6] no standard model yet exists and many theoretical uncertainties remain about the nature, structure and stability of thick disks [7]. The study of thick disks is important since not only these structures could be formed in some systems like proto-stars or AGNs, but also they are theoretically so important because of their ability for providing a better understanding of thin disks and intermediate cases [4].

Self-gravity in accretion disks is effective when the mass of the disk is comparable to the mass of the central object or where the self-gravitating acceleration is not much less than the central object acceleration. Then the self-gravity cannot be neglected for the massive disks. Therefore, it is appropriate to consider the self-gravity when we are going to study the thick disks. Nevertheless, as the self-gravity enters the equations, they become

[*] Corresponding author.
Email address: ghanbari@ferdowsi.um.ac.ir
Fax: +98-511-8796983

non-linear. To solve such equations, the self-similar method could be useful and help us to study the problems with acceptable details especially in equilibrium states [8]. Pen presented a general classification of self-similar solutions for self-gravitating fluids [9]. Bodo & Curir computed the equilibrium structure of a self-gravitating thick accretion disk by an iterative procedure, which produced a final density distribution in equilibrium with the potential coming from it [10]. They showed that the size and geometrical shape of the disk were affected by the self-gravity.

Today, it is completely clear that the magnetic fields have an important role on the accretion disks. Magnetic fields could be involved in generating jets and bipolar outflows from disks [11]. Thick disks are also successful to describe rapid changes in radiation from X-ray sources [12]. Tripathy et al. investigated the dynamics of a thick disk of accreting magnetofluid. They considered a thick disk with finite conductivity around a compact object [12]. They found that the azimuthal current produced due to the motion of the magnetofluid modifies the magnetic field structure inside the disk. Banerjee et al. examined rotating thick disk equilibria in the presence of an external gravity and a dipolar magnetic field. Their solution showed that the pressure and the density profiles were strongly modified by a generated toroidal magnetic field [13]. However, they had not considered the self-gravity, which is so important in thick disks. Ghanbari & Abbassi studied the effect of self-gravity in rotating thick disk equilibria with using self-similar method [14]. Their solutions showed that the structure of the disk is modified by the self-gravity of the disk, the magnetic field of the central object, and the azimuthal velocity of the gas in the disk. They found that self-gravity and magnetic field from the central object could change the thickness and the shape of the disk.

In this work, we investigate the equilibrium configuration of a non-rotating thick disk that is affected by its self-gravity. We consider a non-rotating, non-accreting, stationary, axi-symmetric thick disk around a compact object owned a bipolar magnetic field. We write our equations in a non-relativistic domain neglecting viscosity. Our aim in this work is generally to understand the effect of self-gravity in thick disk's equilibrium structures. In the following section we bring and talk about equations governing a thick disk around a compact object and at the presence of a bipolar magnetic field. In the second section the final equations are solved by the self-similar method and the effects of some parameters are discussed. We present our results and some suggestions for future works in the Discussion and Conclusion.

## Basic Equations

As we stated in the introduction, we are interested in analyzing the role of self-gravity in a non-rotating thick disk equilibrium in the presence of dipolar magnetic field of the central accretor. We assume the disk as a non-rotating, non-accreting, flow around a compact object and at the presence of a bipolar magnetic field. The disk is considered stationary and axi-symmetric. We use a spherical polar coordinate system fixed on the central object. For simplicity, we ignore the influence of dissipative processes such as viscosity or radiation. Thus, our basic equations including the equation of continuity are

$$\nabla.(\rho \mathbf{V}) = 0\,, \tag{1}$$

the equation of motion(or Euler's equation in this case)

$$\rho(\mathbf{V}.\nabla)\mathbf{V} = -\nabla p - \rho\nabla(\Psi + \Psi_{ext}) + \frac{1}{c}\mathbf{J}\times\mathbf{B}\,, \tag{2}$$

the Maxwell's equations

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J}\,, \tag{3}$$

$$\mathbf{E} + \frac{1}{c}\mathbf{V}\times\mathbf{B} = 0\,, \tag{4}$$

$$\nabla\times\mathbf{E} = 0\,, \tag{5}$$

and the Poisson's equation

$$\nabla^2(\Psi + \Psi_{ext}) = 4\pi G(\rho + \rho_{ext})\,, \tag{6}$$

where **V** is the gas velocity, $\rho$ and P are the gas density and pressure respectively. $\psi_{ext}$ denotes the gravitational potential of the external object and $\psi$ is the self-gravitational potential of the disk. **E** and **B** are electric and magnetic fields, **J** is the electromagnetic current density and $\rho_{ext}$ denotes the central object density. In this configuration we chose dipolar configuration for the magnetic field of central stars which is confirmed with observations and also we ignore the magnetic field of the accreting materials compare to the magnetic field of the central compact object. We propose that these accretion disks are bathed on the poloidal magnetic field of the central star which is a good approximation for neutron stars and white dwarfs.

It is difficult to find the solution of the set of equations in the most general form. Therefore, we need some simplifications. In this model we have ignored the effect of viscosity and any heating and cooling mechanism.

In order to do some simplifications we start with Maxwell equations. We assumed that the magnetic field of central stars is nearly dipole which it has a good observational background at least for neutron stars and white dwarfs.

We take the components of magnetic field like Banerjee et al. [13]

$$B_r = 2B_0\left(\frac{R}{r}\right)^3 \cos\theta, \tag{7}$$

$$B_\theta = B_0\left(\frac{R}{r}\right)^3 \sin\theta, \tag{8}$$

where $B_0$ is the magnetic field strength on the surface of the compact object and R is the radius of the compact object.

Combining the electric field components from (4) and the magnetic field from (7) and (8), we could find a relation between radial and polar components of the velocity, that is

$$V_r = 2V_\theta \cot\theta, \tag{9}$$

Using relation (1) and relation (9), we could find a relation for $\rho v_\theta$

$$\rho V_\theta = \rho_0 V_0 a^{\frac{5}{2}-n}\left(\frac{R_*}{r}\right)^{5/2-n} \sin^{-2n}\theta, \tag{10}$$

where $\rho_0$ and $v_0$ are the density and the velocity of the gas at the radius of the disk ($r = R_d$) and equatorial plane ($\theta=\frac{\pi}{2}$). When $n$ is a real constant chosen so that $\rho v_\theta$ vanishes as r approaches infinity and $a$ is the ratio of the disk radius to the central object radius.

If we derive the components of the current density from equation (3), we could see that the azimuthal component of the current density in the disk is zero ($J_\varphi=0$) because of the bipolar magnetic field.

Using above relations, Euler's equation (2) gives

$$\rho(4V_\theta\frac{\partial V_\theta}{\partial r}\cot^2\theta + 2\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial\theta}\cot\theta - 3\frac{V_\theta^2}{r} - 2\frac{V_\theta^2}{r}\cot^2\theta - \frac{V_\varphi^2}{r} + \frac{\partial\Psi}{\partial r} + \frac{GM}{r^2}) + \frac{\partial\hat{p}}{\partial r} = -\frac{B_\varphi^2}{4\pi r}, \tag{11}$$

$$\rho(2\frac{V_\theta^2}{r}\cot\theta + 2V_\theta\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial\theta} - \frac{V_\varphi^2}{r}\cot\theta + \frac{1}{r}\frac{\partial\Psi}{\partial r}) + \frac{1}{r}\frac{\partial\hat{p}}{\partial\theta} = -\frac{B_\varphi^2}{4\pi r}\cot\theta, \tag{12}$$

$$\rho(3\frac{V_\theta V_\varphi}{r}\cot\theta + 2V_\theta\frac{\partial V_\varphi}{\partial r}\cot\theta + \frac{V_\theta}{r}\frac{\partial V_\varphi}{\partial\theta}) = \frac{B_0}{4\pi}\frac{R^3}{r^4}\left[\frac{\partial}{\partial\theta}(B_\varphi\sin\theta) + 2r\frac{\partial B_\varphi}{\partial r}\cos\theta\right], \tag{13}$$

where $\hat{p}$ is the effective pressure which is defined by

$$\hat{p} = p_{gas} + \frac{B_\varphi^2}{8\pi}. \tag{14}$$

We have also Poisson's equation that in spherical polar coordinates is as follow

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) = 4\pi G\rho. \tag{15}$$

Now, we introduce some dimensionless variables for simplifying our equations.

$$x = \frac{r}{R_d}, \tag{16}$$

$$\tilde{V}_\theta(x,\theta) = \frac{V_\theta(r,\theta)}{V_0}, \tag{17}$$

$$\tilde{V}_\varphi(x,\theta) = \frac{V_\varphi(r,\theta)}{V_0}, \tag{18}$$

$$\tilde{p}(x,\theta) = \frac{\hat{p}(r,\theta)}{p_0}, \tag{19}$$

$$\tilde{\rho}(x,\theta) = \frac{\rho(r,\theta)}{\rho_0}, \tag{20}$$

$$\tilde{\Psi}(x,\theta) = \frac{\Psi(r,\theta)}{\Psi_0}, \tag{21}$$

$$\tilde{B}_\varphi(x,\theta) = \frac{B_\varphi(r,\theta)}{B_0}. \tag{22}$$

where $R_d$ is the radius of the disk and $M_d$ represents the mass of the disk. $p_0$, $\psi_0$ and $M_d$ are defined as

$$p_0 = \hat{p}(r,\theta)\Big|_{r=R_{d*},\theta=\frac{\pi}{2}} = p_{gas}\Big|_{r=R_{d*},\theta=\frac{\pi}{2}} + \frac{B_\varphi^2}{8\pi}\Big|_{r=R_d,\theta=\frac{\pi}{2}}, \tag{23}$$

$$\Psi_0 = \frac{GM}{R_d}, \tag{24}$$

$$M_d = \frac{4\pi}{3}R_d^3\rho_0. \tag{25}$$

By inserting these dimensionless parameters into equations (11), (12), (13), (15) and after doing some manipulations, we obtain three equations to be solved

$$\frac{\partial\tilde{\Psi}}{\partial x} = -C_t\tilde{V}_\theta x^{\frac{5}{2}-n}\sin^{2n}\theta\frac{\partial\tilde{p}}{\partial x} - \frac{1}{x^2} + C_k(2\frac{\tilde{V}_\theta^2}{x}\cot^2\theta + 3\frac{\tilde{V}_\theta^2}{x} - 2\frac{\tilde{V}_\theta}{x}\frac{\partial\tilde{V}_\theta}{\partial\theta}\cot\theta - 4\tilde{V}_\theta\frac{\partial\tilde{V}_\theta}{\partial x}\cot^2\theta), \tag{26}$$

$$\frac{\partial\tilde{\Psi}}{\partial\theta} = -C_t\tilde{V}_\theta x^{\frac{5}{2}-n}\sin^{2n}\theta\frac{\partial\tilde{p}}{\partial\theta} - C_k(\tilde{V}_\theta\frac{\partial\tilde{V}_\theta}{\partial\theta} + 2x\tilde{V}_\theta\frac{\partial\tilde{V}_\theta}{\partial x}\cot\theta + 2\tilde{V}_\theta^2\cot\theta), \tag{27}$$

$$\frac{\tilde{V}_\theta}{x^2}\frac{\partial}{\partial x}\left(x^2\frac{\partial\tilde{\Psi}}{\partial x}\right)+\frac{\tilde{V}_\theta}{x^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\tilde{\Psi}}{\partial\theta}\right)= \qquad (28)$$

$$C_{sg}x^{n-\frac{5}{2}}\sin^{-2n}\theta,$$

where $C_t$, $C_k$ and $C_{sg}$ are defined by:

$$C_t=\frac{p_0/\rho_0}{\Psi_0}, \qquad (29)$$

$$C_k=\frac{V_0^2}{\Psi_0}, \qquad (30)$$

$$C_{sg}=\frac{3M_d}{M}. \qquad (31)$$

These dimensionless parameters indicating the relative importance of the thermal energy ($C_t$) and the kinetic energy ($C_k$) respect to the gravitational potential energy of the central object. $C_{sg}$ is the ratio of the disk mass to the central object mass and gives the importance of the self-gravity. We can study the effect of each of them separately.

## Self-Similar Solutions

Our basic equations are non-linear and we cannot solve them analytically. Then, it is useful to employ a method to investigate the solutions as it could eliminate the difficulties of solving the non-linear equations. One of them is the self-similar method. In this method, we assume when one of variables changes, the functions behave similar. This method could be used when we are not interested in boundaries or where sudden changes happen.

Since we are interested to study the equilibrium state, the self-similar method is a suitable one. We assume our functions have similar behaviors in every radius and we will investigate the solutions with respect to the polar angle. Thus, we consider the self-similar variables like below

$$\tilde{V}_\theta(x,\theta)=\tilde{V}_\theta(\theta)x^{-1/2}, \qquad (32)$$

$$\tilde{\Psi}(x,\theta)=\tilde{\Psi}(\theta)x^{-1}, \qquad (33)$$

$$\tilde{p}(x,\theta)=\tilde{p}(\theta)x^{-4}. \qquad (34)$$

To simplify, we define w(θ) as

$$w(\theta)=\frac{d\tilde{\Psi}(\theta)}{d\theta}. \qquad (35)$$

Inserting these variables into our basic equations gives a system of ordinary non-linear differential equations for the four self-similar variables $\tilde{V}_\theta$, $\tilde{\Psi}_\theta$ $\tilde{p}_\theta$ and w.

$$\frac{d\tilde{V}_\theta(\theta)}{d\theta}=\frac{\tilde{\Psi}(\theta)\tan\theta}{2C_k\tilde{V}_\theta(\theta)}+\frac{2C_t\tilde{p}(\theta)}{C_k\cos\theta\sin\theta}-\frac{2\tilde{V}_\theta(\theta)}{\tan\theta}+ \qquad (36)$$

$$\frac{3}{2}\tilde{V}_\theta(\theta)\tan\theta,$$

$$\frac{d\tilde{\Psi}(\theta)}{d\theta}=w(\theta), \qquad (37)$$

$$\frac{dw(\theta)}{d\theta}=\frac{C_{sg}\sin^2\theta}{\tilde{V}_\theta(\theta)}-\frac{w(\theta)}{\tan\theta}, \qquad (38)$$

$$\frac{d\tilde{p}(\theta)}{d\theta}=-\frac{w(\theta)\sin^2\theta}{C_t\tilde{V}_\theta(\theta)}-\frac{3C_k\tilde{V}_\theta(\theta)\sin\theta\cos\theta}{C_t}-$$

$$\frac{\tilde{\Psi}(\theta)\sin^2\theta\tan\theta}{2C_t\tilde{V}_\theta(\theta)}+\frac{\sin^2\theta\tan\theta}{2C_t\tilde{V}_\theta(\theta)}- \qquad (39)$$

$$\frac{3C_k\tilde{V}_\theta(\theta)\sin^2\theta\tan\theta}{2C_t}-2\tilde{p}(\theta)\tan\theta.$$

Indeed, we need four boundary conditions to solve the equations numerically. All of the boundary conditions could find at the equatorial plane ($\theta=\frac{\pi}{2}$). As we defined $\tilde{V}_\theta$ in (32), the boundary condition for this variable is:

$$\tilde{V}_\theta\left(\frac{\pi}{2}\right)=1 \qquad (40)$$

For other variables, we need to use the conditions that were presented by Narayan et al. [15]. These conditions indicate that the smooth changes of pressure and polar velocity at the equatorial plane are:

$$\left.\frac{d\tilde{p}(\theta)}{d\theta}\right|_{\theta=\frac{\pi}{2}}=0, \qquad (41)$$

$$\left.\frac{d\tilde{V}_\theta(\theta)}{d\theta}\right|_{\theta=\frac{\pi}{2}}=0. \qquad (42)$$

Using these relations in the basic equations at $\theta=\frac{\pi}{2}$, we obtain the other boundary conditions as

$$\tilde{p}\left(\frac{\pi}{2}\right)=1, \qquad (43)$$

$$\tilde{\Psi}\left(\frac{\pi}{2}\right)=1-4C_t-3C_k, \qquad (44)$$

$$w\left(\frac{\pi}{2}\right)=0. \qquad (45)$$

We have a system of four coupled ordinary differential equations with the four boundary conditions all in one point ($\theta=\frac{\pi}{2}$). It is suitable to use the ODIENT method [16], which is so efficient and popular to solve theses kind of problems. Now, we are able to solve the equations and study the effects of the self-gravity, the thermal energy and the kinetic energy by giving different values to $C_{sg}$, $C_t$ and $C_k$. During solving the equations for different values of $C_{sg}$, we noticed that the values of $\frac{M_d}{M}$ are limited by values of $C_t$ and $C_k$. Increasing these two parameters leads to increasing $\frac{M_d}{M}$. The ratio of the disk mass to the central object mass could at most increases up to 0.9. In Fig. 1, we plot the pressure, the density and the radial velocity profiles for different values of $C_{sg}$. We can see that these parameters are sensitive to the influence of the self-gravity. Fig. 1 shows that the pressure and the density decrease as the self-gravity increases.

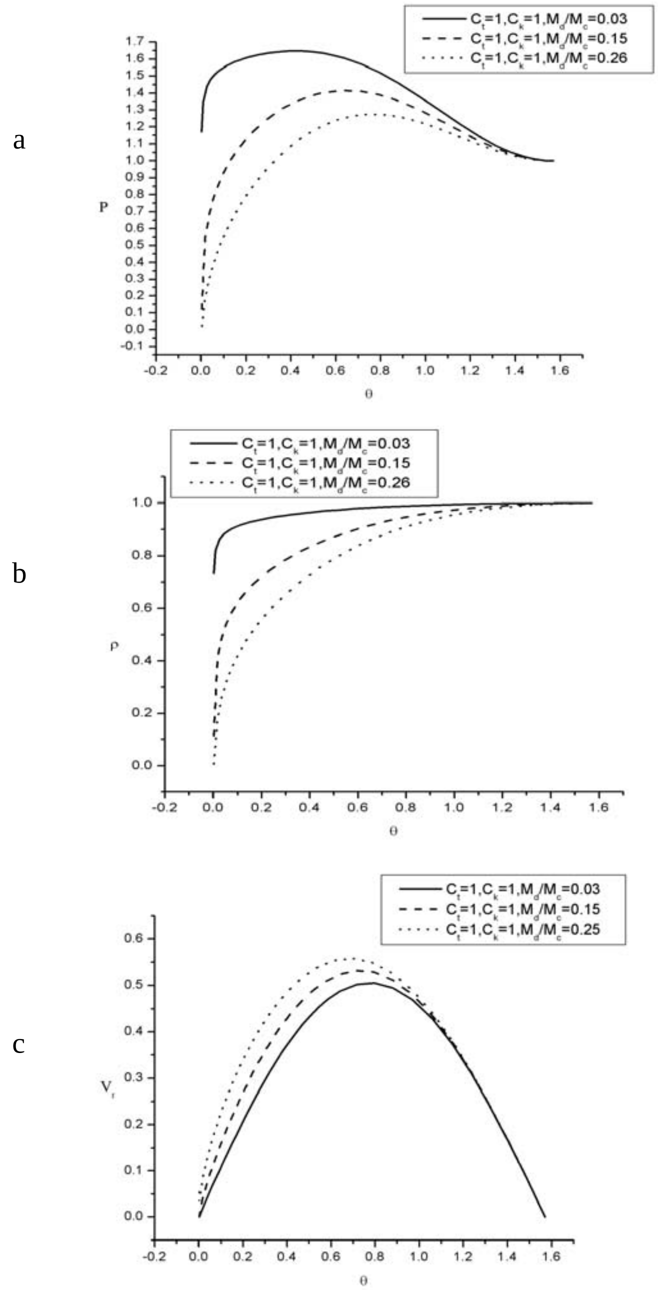


**Figure 1.** Variation of the pressure (a) , density (b) and radial velocity (c) for different values of $C_{sg}$ that represents the importance of self-gravity. θ indicates the polar radius in radiant.

a

b

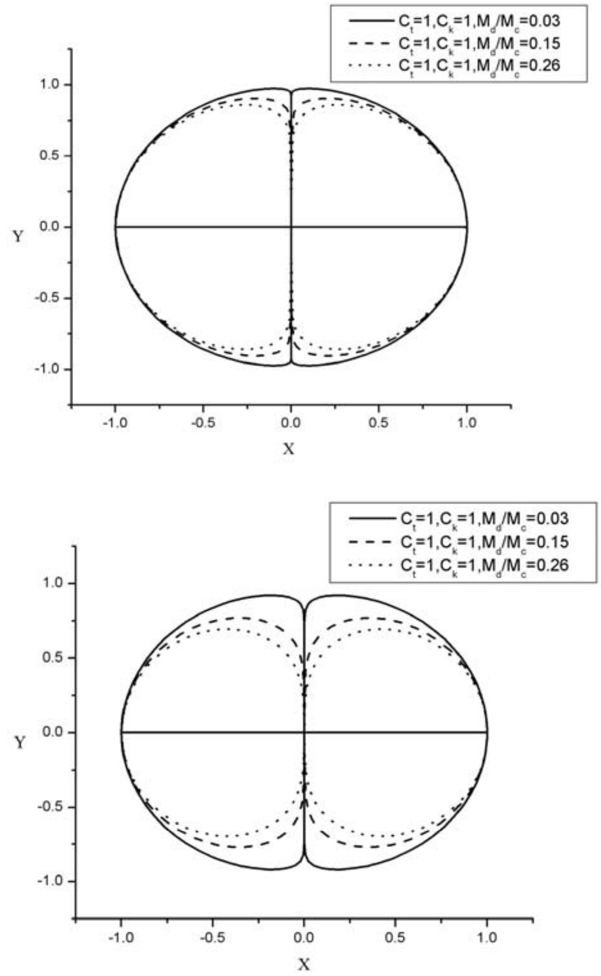


**Figure 2.** Iso-pressure (a) and iso-potential (b) counters of the self-gravitating disk.

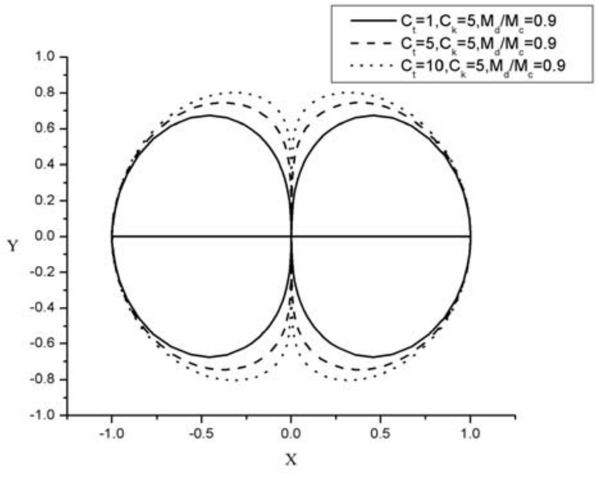


**Figure 3.** Iso-pressure counters for different values of $C_t$ that represent the effect of thermal energy.

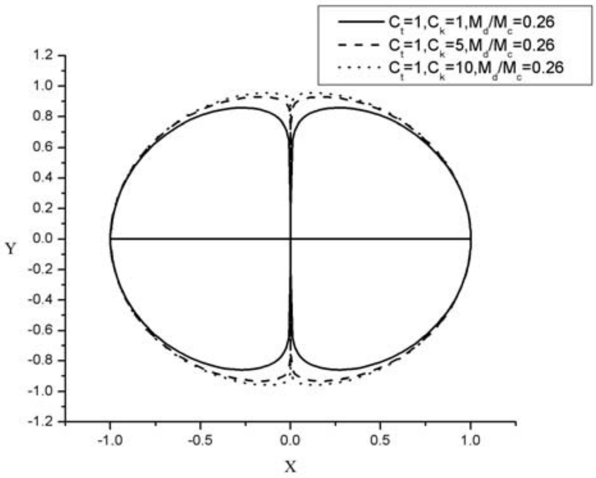


**Figure 4.** Iso-pressure counters for different values of $C_k$ that represent the effect of kinetic energy.

These changes are more noticeable in the polar regions. As it could be seen in Fig. 1(c), increasing the self-gravity increases the radial velocity. In Fig. 2, we plot the iso-pressure and iso-potential contours. We can see that the geometrical shape of the disk tends to become thinner for greater values of $C_{sg}$.

In Fig. 3, the effect of the thermal energy is shown. It indicates that the thickness of the disk increases by increasing the thermal energy (larger $C_t$). The same effect can be seen for larger values of kinetic energy or $C_k$ in Fig. 4.

## Discussion and Conclusion

In this paper, we study the effect of self-gravity on a non-rotating thick disk around a magnetized compact object. We ignored the dissipative processes such as viscosity or radiation. Since the considered disk is non-rotating, the radial and polar components of velocity are important and we cannot neglect them. The magnetic field has no effect on the disk because the toroidal component of the magnetic field is not created in the disk. We find self-similar solutions for such disks that show the effect of the self-gravity on the disk physical parameters and its shape.

We find that the self-gravity can change the thickness of the disk especially near the poles. Fig. 2 shows when the self-gravity plays an important role in the disk (by increasing $C_{sg}$) and the disk becomes thinner. Our results agree well with Ghanbari & Abbassi [14]; however, they considered the disk is rotating and neglect the radial velocity.

We also find that by increasing the thermal energy, the disk becomes thicker Fig. 3.

Since the rotation has an important role on the structure of the disks [14], it is better to study the equilibrium structure of the disks by including all components of the velocity. By this way, the effect of mass flow is included and it could provide comparing the results with the observations.

In this work, we ignore the dissipative processes for simplicity. Finding more detailed results for physical parameters encourages one to consider radiation or viscosity.

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