

# The effects of thermal conduction on the ADAF with a toroidal magnetic field

S. Abbassi,<sup>1★</sup> J. Ghanbari<sup>2★</sup> and S. Najjar<sup>3</sup>

<sup>1</sup>*Department of Physics, Damghan University of Basic Sciences, Damghan, Iran*

<sup>2</sup>*Department of Physics, School of Sciences, Ferdowsi University of Mashhad, Mashhad 91775-1436, Iran*

<sup>3</sup>*Department of Physics, Islamic Azad University, Mashhad Branch, Mashhad, Iran*

Accepted 2008 April 29. Received 2008 April 14; in original form 2008 February 12

## ABSTRACT

The observation of the hot gas surrounding Sgr A\* and a few other nearby galactic nuclei imply that electron and proton mean free paths are comparable to the gas capture radius. So, the hot accretion flows are likely to proceed under weak-collision conditions. Hence, thermal conduction has been suggested as a possible mechanism by which the sufficient extra heating is provided in hot advection-dominated accretion flow (ADAF) accretion discs. We consider the effects of thermal conduction in the presence of a toroidal magnetic field in an ADAF around a compact object. For a steady-state structure of such accretion flows, a set of self-similar solutions are presented. We find two types of solutions which represent high and slow accretion rate. They have different behaviours with saturated thermal conduction parameter,  $\phi$ .

**Key words:** accretion, accretion discs – black hole physics – conduction.

## 1 INTRODUCTION

An advection-dominated accretion flow (ADAF) is defined as one in which a large fraction of the viscously generated heat is advected with the accreting gas, and only small fraction of the energy is radiated. An advection-dominated accretion can occur in two different limits: (i) at very high mass accretion rates, radiation is trapped in the accreting gas because of the large optical depth and is advected with the flow. This limit of advection dominated, typically occurs for mass accretion rates  $\dot{M} > \dot{M}_{\text{Edd}}$  (the Eddington rate). (ii) At sufficiently low  $\dot{M}$ , the accreting gas can become optically thin. The cooling time of the gas is then longer than the accreting time, and once again we have an ADAF.

The thin ADAF model has been investigated extensively since the end of 1990s (Ichimaru 1977; Narayan & Yi 1994, hereafter NY1994; Abramowicz et al. 1995; Narayan & Yi 1995a,b). This model has been used to interpret the spectra of black hole X-ray binaries in their quiescent or low/hard state as an alternative to the Shapiro, Lightman & Eardley (1976, hereafter SLE) solutions. Since ADAFs have large radial velocities, and infalling matter carries the thermal energy into the black hole, advective energy transport can stabilize the thermal instability; thus, ADAF models have been widely applied to explain observations of galactic black hole candidates (e.g. Narayan, McClintock & Yi 1996; Hameury et al. 1997), the spectral transition of Cyg X-1 (Esin et al. 1996) and mul-

tiwavelength spectral properties of Sgr A (Narayan & Yi 1995b; Manmoto et al. 2000; Narayan, Kato & Honma 1997). In addition, many ADAF-like models have been proposed including outflows, convection, etc.

The theories for the structure and properties of hot accretion flows have been remained controversial. However, NY1994 derived self-similar ADAF solutions which emphasized on the importance of the stabilizing role of the radial heat advection. Subsequent analytical work on the hot accretion flow has emphasized on the possibility of outflows, motivated by a positive Bernoulli constant (Fukue 2004; NY1994). Also investigation was indicated a ADAF, where the disc is convective in the radial direction (Narayan, Igumenshchev & Abramowitz 2000; Quataret & Gruzinov 2000).

The diversity of the models indicates that modelling the hot accretion flows is a challenging and controversial problem. We think that one of the largely neglected physical phenomena is the thermal conduction while recent observations of the hot accretion flow around active galactic nuclei (AGN) indicated that it should be based on collision less regime. So, thermal conduction probably has an important role in energy transport in the accreting materials in a hot accretion disc where they are nearly completely ionized. Recently, Medvedev & Narayan (2001) discover a new type of accretion flow, a hot settling flow around a rapidly rotating neutron star. The flow is cooling dominated and energetically similar to the SLE solution. The cooling-dominated SLE solution has been shown to be thermally unstable (Piran 1978; Wandel & Liang 1991; Narayan & Yi 1995a), and hence unlikely to exist in the nature. It has been shown that any accretion flow in which the heating balances cooling is

\*E-mail: sabbassi@dubs.ac.ir (SA); ghanbari@ferdowsi.um.ac.ir (JG)

thermally unstable, if the cooling is due to the bremsstrahlung emission (Shakura & Sunyaev 1976; Piran 1978). But ADAF is known to be thermally stable (Narayan & Yi 1995a; Kato, Abramowicz & Chen 1996; Kato et al. 1997) therefore the cooling is weak and the thermal energy of the flow is not radiated but advected with the gas. Since the SLE solution for non-ADAF discs is known to be thermally unstable, one might suspect that the new solution would also be unstable. However, due to the very high temperature of the accreting gas, thermal conduction is very strong and could suppress the thermal instability. So, it should be important to consider the role of thermal conduction in a SLE solution. There is a branch of solution for the problem of a smooth transition from an outer Shakura and Sanyev disc (SSD) to an inner ADAF disc around a compact object which include an additional mechanism of energy transport, thermal conduction. A number of authors showed that the SSD–ADAF transitions were realizable if an extra heat flux caused by thermal conduction was invoked either in the radial direction (Honma 1996; Manmoto et al. 2000; Gracia et al. 2003) or in the vertical direction (Meyer & Meyer-Hofmeister 1994; Meyer, Liu & Meyer-Hofmeister 2000).

The weakly collisional nature of ADAFs has been noted previously (Mahadevan & Quataert 1997). But a few authors tried to study the role of turbulent heat transport in ADAF discs (Honma 1996; Manmoto et al. 2000). Since the thermal conduction acts to oppose the formation of the temperature gradient that causes it, which might expect that the temperature and density profile in a thermal conducting disc should be different from the case in which the disc is not under the influence of thermal conduction. Recently, Tanaka & Menou (2006) studied the effect of saturated thermal conduction on optically thin ADAFs using an extension of self-similar solution of NY1994. Their solutions suggest that the thermal conduction may have an important role in the dynamical behaviour of the hot accretion flows and probably is an important factor to understand the physics of hot accretion discs.

The aim of this work is to consider the possibility of the thermal conduction in the presence of toroidal magnetic field, which has been largely neglected ingredient before, could affect the global properties of the hot accretion flows substantially. The tangled magnetic fields in accretion flows would likely reduce the effective mean free paths of particles. The magnitude of this reduction, which will depend on the field geometry, is still unknown. In this paper, we will investigate the effect of thermal conduction on the physical structure of ADAF-like accretion flow around a black hole in the presence of a toroidal magnetic field.

## 2 THE BASIC EQUATIONS

Let us consider a gaseous disc rotating around a Schwarzschild black hole of mass  $M$ . The disc is assumed to be in an advection-dominated regime. We assume a steady-axisymmetry accretion flow ( $\frac{\partial}{\partial t} = \frac{\partial}{\partial \phi} = 0$ ) and a geometrically thin disc. In cylindrical coordinates  $(r, \phi, z)$ , we vertically integrate the flow equations. Also, we suppose that all flow variables are only a function of  $r$ . We ignored the relativistic effect and we use Newtonian gravity. The disc is supposed to be turbulent and possesses an effective turbulent viscosity. We adopt  $\alpha$ -prescription for viscosity of rotating gas. We have assumed that the generated energy due to viscosity dissipation and heat conduction into the volume are balanced by advection cooling. The magnetic field was considered with toroidal configurations.

We can describe the accretion flows by the fundamental governing equations which are written by the equation of continuity which is integrated in the vertical direction are expressed for presence

purpose as

$$\frac{1}{r} \frac{d}{dr} (r \Sigma v_r) = 2\dot{\rho}H, \quad (1)$$

where  $v_r$  is the accretion velocity,  $\dot{\rho}$  is the mass-loss rate per unit volume,  $H$  is the disc half-thickness and  $\Sigma$  is the surface density, which is defined as  $\Sigma = 2\rho H$ .

The equation of motion in the radial direction is

$$v_r \frac{dv_r}{dr} = \frac{v_\phi^2}{r} - \frac{GM}{r^2} - \frac{1}{\Sigma} \frac{d}{dr} (\Sigma c_s^2) - \frac{c_A^2}{r} - \frac{1}{2\Sigma} \frac{d}{dr} (\Sigma c_A^2), \quad (2)$$

where  $v_\phi$ ,  $c_s$  and  $c_A$ , are the rotation velocity of gas disc, sound speed and Alfvén speed, respectively. Sound speed is defined as  $c_s^2 = \frac{p_{\text{gas}}}{\rho}$ ,  $p_{\text{gas}}$  being the gas pressure and Alfvén velocity is defined as  $c_A^2 = \frac{B_\phi^2}{4\pi\rho} = \frac{2p_{\text{mag}}}{\rho}$ , where  $p_{\text{mag}}$  being the magnetic pressure. The first term in the right-hand side of this equation implies the centrifugal force while the third term implies the pressure gradient force. The fourth and fifth terms represent the magnetic force.

By integrating the  $z$ -component of the momentum equation along the  $z$ -axis, we have hydrostatic balance in the vertical direction as

$$\frac{GM}{r^3} H^2 = c_s^2 \left[ 1 + \frac{1}{2} \left( \frac{c_A}{c_s} \right)^2 \right] = (1 + \beta) c_s^2, \quad (3)$$

where  $\beta = \frac{p_{\text{mag}}}{p_{\text{gas}}} = \frac{1}{2} \left( \frac{c_A}{c_s} \right)^2$  which indicates the importance of magnetic field pressure compared to gas pressure. We assume that this value is constant through the disc, but we will show the dynamical properties of the disc for different values of  $\beta$ .

For writing the angular momentum equations, we need to choose proper mechanism for viscosity. We adopt  $\alpha$ -prescription for viscosity. The  $r\phi$ -component of the viscosity stress tensor is proportional to pressure (Shakura & Sunyaev 1973):

$$T_{r\phi} = \eta r \frac{d\Omega}{dr} = \alpha p,$$

where  $\eta = \rho\nu$  is the viscosity,  $\nu$  is the kinetic viscosity and  $\alpha$  is a viscous parameter. Here, we can see that the viscosity stress tensor is proportional to the gas pressure or to the total (gas and magnetic) pressure. So, we can choose two cases: Case 1: when the pressure is assumed to be the gas pressure (thermal pressure). Case 2: when the pressure is assumed to be the magnetic pressure plus the gas pressure. For simplicity in this investigation, we use the case 1, we adopt the form:

$$\nu = \Omega_k^{-1} \alpha \left( \frac{p}{\rho} \right) = \alpha C_s H. \quad (4)$$

This form is so-called  $\alpha$ -prescription where  $\alpha$  is a constant less than unity (Shakura & Sunyaev 1973). So, the angular momentum transfer equation is

$$r \Sigma v_r \frac{d}{dr} (r v_\phi) = \frac{d}{dr} \left( \frac{\alpha \Sigma c_s^2 r^3}{\Omega_k} \frac{d\Omega}{dr} \right), \quad (5)$$

where  $\Omega$  is angular velocity and  $\Omega_k = \sqrt{\frac{GM}{r^3}}$  is Keplerian angular velocity.

Now we can write the energy equation considering cooling and heating processes in an ADAF. The energy equation is

$$\frac{\Sigma v_r}{\gamma - 1} \frac{dc_s^2}{dr} + \frac{\Sigma c_s^2}{r} \frac{d}{dr} (r v_r) = Q_{\text{vis}} - Q_{\text{rad}} + Q_{\text{cond}}, \quad (6)$$

where  $Q_{\text{vis}}$ ,  $Q_{\text{cond}}$  and  $Q_{\text{rad}}$  are the locally released energies due to viscous dissipation, transported by the thermal conduction and radiative cooling rate, respectively.

The viscosity generates thermal energy through the differential rotation of turbulently moving gas, and the viscous heating rate is expressed by

$$Q_{\text{vis}} = r T_{\text{r}\phi} \frac{d\Omega}{dr} = \Sigma \frac{\alpha c_s^2 r^2}{\Omega_k} \left( \frac{d\Omega}{dr} \right)^2.$$

The radiated cooling rate is expressed as

$$Q_{\text{rad}} = \frac{8acT^4}{3\kappa\rho H},$$

where  $\kappa$  is the opacity of the rotating gas. We assumed that the disc to be radiation pressure dominated with opacity due to the electron scattering only. In the right-hand side of the energy equation, we have

$$Q_{\text{vis}} - Q_{\text{rad}} + Q_{\text{cond}} = Q_{\text{adv}},$$

where  $Q_{\text{adv}}$  represents the advective transport of energy and is defined as the difference between the viscous heating rate,  $Q_{\text{vis}}$ , and radiative cooling rate,  $Q_{\text{rad}}$ , plus the energy transported by conduction,  $Q_{\text{cond}}$ . We employ the parameter  $f = 1 - \frac{Q_{\text{rad}}}{Q_{\text{vis}}}$  to measure the high degree to energy is transported by radiation. When  $f \sim 1$ , radiation can be negligible so it is in advection-dominated state while for small  $f$ , disc is in radiation-dominated case. So, we can rearrange the right-hand side of equation (6) to  $fQ_{\text{vis}} + Q_{\text{cond}}$ , where  $f \leq 1$ . In general,  $f$  varies with  $r$  and depends on the details of heating and cooling processes. For simplicity, it is assumed a constant.

The energy transport by thermal conduction,  $Q_{\text{cond}}$ , is

$$Q_{\text{cond}} = -\frac{2H}{r} \frac{d}{dr}(rF_s), \quad (7)$$

adopted from the formulation of Cowie & McKee (1977) and Tanaka & Menou (2006). The saturated conduction flux is then written as  $F_s = 5\phi\rho c_s^3$ , where  $\phi$  is the saturated constant (presumably  $\leq 1$ ),  $\rho$  is the gas mass density and  $c_s$  is its sound speed. In order to implement the thermal conductivity as correctly as possible, it is essential to know whether the mean free paths are less than (or comparable to) the length-scale of the temperature gradient. For electron mean free paths which are greater than the length-scale of temperature gradient, the thermal conductivity is said to be ‘saturated’. Because it is a saturated flux, it no longer explicitly depends on the magnitude of the temperature gradient but only does on the direction of this gradient. The heat will flow outward in a hot accretion flow with a near-virial temperature profile, hence the positive sign was adopted for  $F_s$ .

Finally, since we consider the toroidal component for the global magnetic field of central stars, the induction equation with field scape can be written as

$$\frac{d}{dr}(v_r B_\phi) = \dot{B}_\phi, \quad (8)$$

where  $\dot{B}_\phi$  is the field escaping or crating rate due to the magnetic instability or dynamo effect. We can rewrite this equation as

$$v_r \frac{dc_A^2}{dr} + c_A^2 \frac{dv_r}{dr} - \frac{c_A^2 v_r}{r} = 2c_A^2 \frac{\dot{B}_\phi}{B_\phi} - c_A^2 \frac{2\rho\dot{H}}{\Sigma}. \quad (9)$$

Now, we have a set of magnetohydrodynamic equations which describe the dynamical behaviour of ADAF flows. The solution of these equations gives us the dynamical behaviour of the disc, which strongly depends on the viscosity, magnetic field strength, thermal conduction and advection rate of energy transport.

### 3 SELF-SIMILAR SOLUTIONS FOR ADAFS WITH SATURATED CONDUCTION

The self-similar solution cannot be able to describe the global behaviour of the accretion flow, because in this method there are not boundary conditions which have been taken into account. However, as long as we are not interested in the solutions near the boundaries, such solutions describe correctly the true and useful solutions asymptotically at intermediate areas.

NY1994 simplified 2D axisymmetric steady-state ADAF problem by assuming that the dynamical variables of the flow have a power-law dependence on  $r$ . This allowed them to evaluate directly all radial derivations in the equations. Further, they eliminated  $\theta$  by considering a height-integrated set of equations. The equation has an analytical self-similar solution which depends only on three parameters: the viscous parameter  $\alpha$ , the ratio of specific heats of the accreting gas  $\gamma, f$ . In our case, we have a magnetic field in the structure of the ADAF. So, we have an extra free parameter  $\beta$ , which introduces the magnetic field strength in our self-similar solution.

We assume that each physical quantity can be expressed as a power law of the radial distance,  $r^\nu$ , where power index  $\nu$  is determined for each physical quantities self-consistently. The solutions are

$$v_r = -C_1 \alpha \sqrt{\frac{GM}{r}}, \quad (10)$$

$$v_\phi = C_2 \sqrt{\frac{GM}{r}}, \quad (11)$$

$$c_s^2 = \frac{p}{\rho} = C_3 \frac{GM}{r}, \quad (12)$$

$$C_A^2 = \frac{B_\phi^2}{4\pi\rho} = 2\beta C_3 \frac{GM}{r}, \quad (13)$$

$$\Sigma = \Sigma_0 r^{-\frac{1}{2}}, \quad (14)$$

$$\dot{\rho} = \dot{\rho}_0 r^{-3}, \quad (15)$$

$$\dot{B}_\phi = \dot{B}_0 r^{-\frac{11}{4}}, \quad (16)$$

where  $C_1, C_2$  and  $C_3$  are coefficients that we will determine later.  $\dot{\rho}_0$  and  $\dot{B}_0$  are constants, which provide convenient units with which the equations can be written in the non-dimensional form.

Using these solutions from the continuity, the momentum, angular momentum, hydrostatic, energy and induction equations, we can obtain the following system of dimensionless equations, solved for  $C_1, C_2, C_3, f$  and  $\phi$ :

$$\dot{\rho} = 0, \quad (17)$$

$$\frac{1}{2}\alpha^2 C_1^2 + C_2^2 - 1 + \frac{1}{2}[3 - \beta]C_3 = 0, \quad (18)$$

$$C_1 = \frac{3C_3}{2}, \quad (19)$$

$$\frac{H}{r} = \sqrt{(1 + \beta)C_3}, \quad (20)$$

$$C_2^2 = \frac{3 - \gamma}{\gamma - 1} \frac{2}{9f} C_1 + \frac{40}{9\alpha f} \frac{-1}{\sqrt{3/2}} \phi \sqrt{C_1}, \quad (21)$$

$$\dot{B}_0 = \frac{5C_1\alpha GM}{4} \sqrt{4\pi\Sigma_0 \frac{\beta C_3}{\sqrt{(1+\beta)C_3}}}. \quad (22)$$

This solution tends to the solution presented by Akizuki & Fukue (2006) for  $\phi = 0$ . Our solution is compatible with the standard solution of ADAF, NY1994, having no mass-loss (equation 17). Actually, radial dependence of surface density, equation (14), is a free parameter. For finding a physical solution, it should be more than  $-1$ . We chose it equal to  $-\frac{1}{2}$  to have a physical solution which is compatible with the standard solution in the case there are no any wind or mass-loss effects. When it is more than  $-\frac{1}{2}$ , we have mass-loss in the discs. Fukue (2004) solved the standard disc for the case that radial dependence of surface density is equal to 2. He showed that the global behaviour of the mass-loss disc is quite similar to advective discs. In additions, we can easily see from equation (20) that the disc thickness becomes large due to magnetic pressure.

In the case of very small  $\alpha$ , we have  $\dot{B}_0 = 0$  which means that the creation or escape, so the toroidal components of magnetic field are balanced each other. In this case, the disc supports itself with rotation, gas and magnetic pressure. In the case of a finite  $\alpha$ , the effect of injection of the toroidal component of magnetic field, can moderate the dynamical behaviour of the discs.

The above equations describe self-similar behaviour of the optically thick advection-dominated accretion disc with saturated thermal conduction in the presence of toroidal magnetic field. In a finite  $\alpha$ , we will solve these equations. After some algebraic manipulations, we can find a fourth-order equation for  $C_1$  as follow.

$$D^2 C_1^4 + 2BDC_1^3 + (B^2 - 2D)C_1^2 - (A^2 + 2B)C_1 + 1 = 0, \quad (23)$$

where

$$D = \frac{1}{2}\alpha^2,$$

$$B = \left[ \frac{4}{9f} \left( \frac{1}{\gamma - 1} - \frac{1}{2} \right) - \frac{\beta}{3} + 1 \right],$$

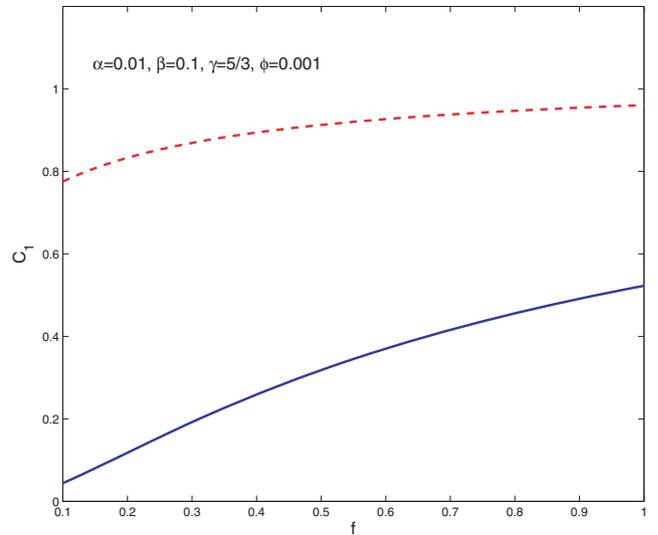
$$A = \frac{40}{9\alpha f} \sqrt{\frac{2}{3}} \phi.$$

This Algebraic equation shows that the variable  $C_1$  which determines the behaviour of radial velocity depends only on the  $\alpha$ ,  $\phi$ ,  $\beta$  and  $f$ . Other flows quantity such as  $C_2$  and  $C_3$  can be obtained easily from  $C_1$ .

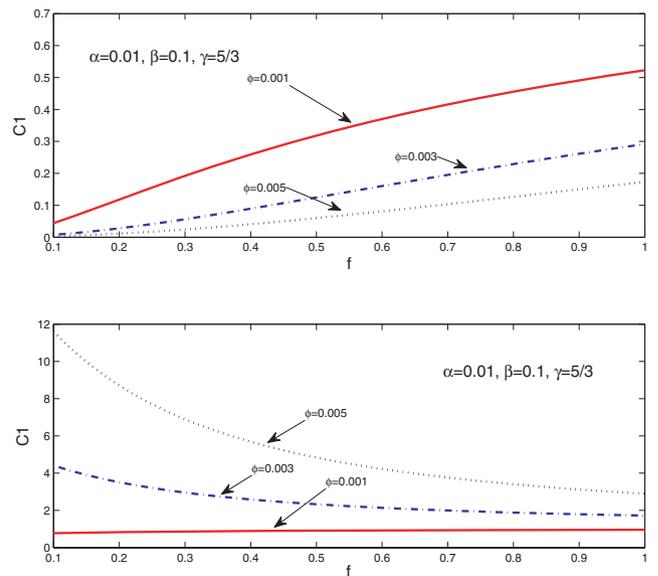
This equation is plotted in Fig. 1 for the given parameters. We have found that it has two real roots for this range of parameter space with two distinct behaviours. One of them represents low accretion rate and other one represents high accretion rate where both of them are sub-Keplerian. The parameters of the model are the ratio of specific heats  $\gamma$ , the standard viscous parameter  $\alpha$ , the radiation transport parameter  $f$  and the degree of magnetic pressure to gas pressure  $\beta$ .

Fig. 1 shows the coefficient  $C_1$ , which represents the behaviour of the radial flows of accreting materials. Although in ADAF model, the radial infall velocity is generally slower than the Keplerian speed ( $= \sqrt{\frac{GM}{r}}$ ), it becomes large with  $f$ . As the level of advection is increased (by increasing the  $f$ ), the accreting materials increase their inflow speed in both two distinct solutions. This variation is around two or three times for different range of  $f$ . This dependence is consistent with the usual ADAF solutions (Akizuki & Fukue 2006; Tanaka & Menou 2006).

In Fig. 2, we investigate the role of saturated thermal conduction on the radial flow for the two above solutions. These solutions have different behaviours as we increase the role of thermal conduction

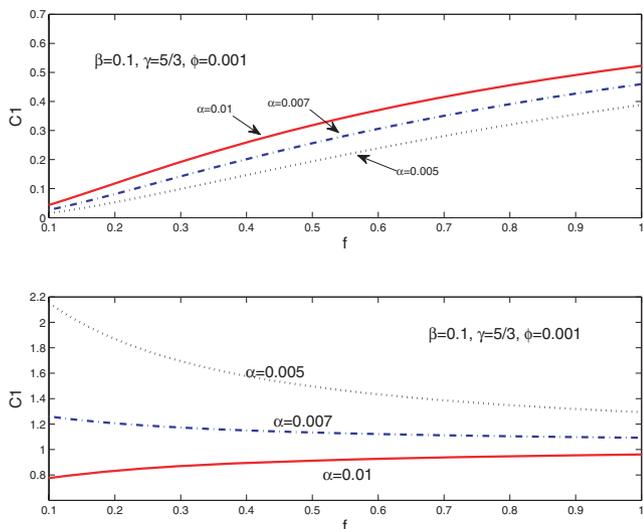


**Figure 1.** Profile of  $C_1$ ; coefficient of infall velocity as a function of  $f$  (which represents the importance of radiation transport). This profile illustrates the solutions of fourth-order equation for  $C_1$  in a given range of parameters. The solid line represents the low accretion rate solution while the dashed line represents the high accretion rate solutions.



**Figure 2.** Numerical coefficient  $C_1$  as a function  $f$  for different values of thermal conduction parameter,  $\phi$ . Upper panel: for low accretion rate solution and lower panel: for high accretion rate solution.

by adding  $\phi$ . The results for  $\phi = 0.001$  to  $0.006$  are shown in this paper. Tanaka & Menou (2006) have shown that for a very small  $\phi$  their solutions coincide the original 1D ADAF solutions; but by adding saturated conduction parameter,  $\phi$ , the effect of thermal conduction can be better seen when we approach to  $\sim 0.001$ – $0.01$ . So, we have plot our solution in this range. As it can be seen in Fig. 2 (upper panel) in the low accretion rate solution, when we increase  $\phi$  the radial flow decreased. But for the high accretion rate solution, the radial flow increases by increasing the  $\phi$  (Fig. 2, lower panel). Increasing the saturated conduction in high accretion solution will change the behaviour of the radial flow in different range of advection parameter. The effect of thermal conduction for low  $f$  is larger than compare to  $f \sim 1$ . The thermal conduction in low



**Figure 3.** Numerical coefficient  $C_1$  as a function  $f$  for different values of viscous parameter,  $\alpha$ . Upper panel: for low accretion rate solution and lower panel: for high accretion rate solution.

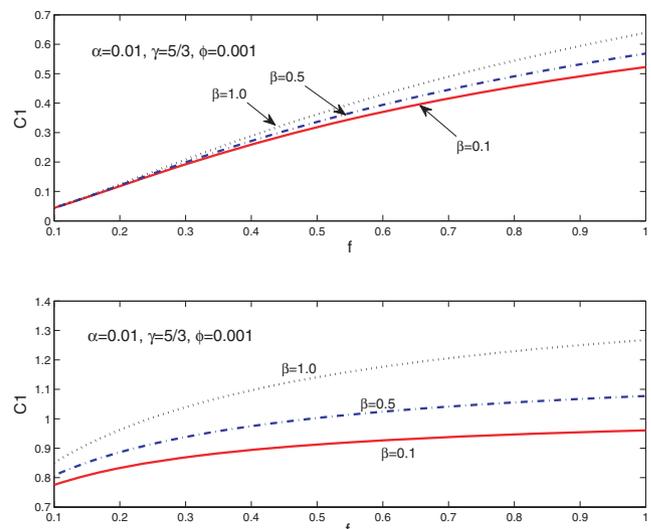
$f$  causes a more radial flow. In the high accretion solution, by adding the role of advection (adding  $f$ ), the effect of thermal conduction was reduced. But, for high accretion solutions, the global effect of thermal conduction increases the radial flow, whereas in a low accretion solution, this role is quite suppressed.

To show the behaviour of the solutions, self-similar radial velocity is demonstrated in Fig. 3 for different values  $f$  with variation of viscosity parameters. The most accepted value for  $\alpha$  is less than 0.1. There are probably, more significantly variations when  $\alpha$  exceeds above 0.1. Recently, King, Pringle & Livio (2007) assert that in a thin and fully ionized disc, the best observational evidence suggests a typical range of  $\alpha \sim 0.1$ – $0.4$  where relevant numerical simulations tend to drive the estimates for  $\alpha$  which are one order of magnitude smaller. However, such large values of  $\alpha$  are unlikely (e.g. Hawley, Gammie & Balbus 1994; Narayan, Loeb & Kumar 1994), so we have not explored this region  $\alpha$ . Here, we investigate these solutions with different values of viscosity less than 0.1. In the low accretion solution (Fig. 3, upper panel), when we increase the viscosity parameter, we see radial inflow increases. But in the case of high accretion solution (Fig. 3, lower panel), this behaviour is inverted. The behaviour of high accretion solution is similar to the solution presented by Ghanbari, Salehi & Abbassi (2007). In this case, increasing the viscous parameter is corresponding to the increase of the heating mechanism, so in a fixed advection regime, there is more energy to advect into the central star.

Fig. 4 shows the coefficient  $C_1$  as a function of  $f$  for different values of  $\beta$ . By adding  $\beta$ , which indicates the role of magnetic field in the dynamics of accretion discs, we will see that the radial flow increases in both solutions. On the other hand, the radial infall velocity increases when the toroidal magnetic field becomes large. This is due to the magnetic tension terms, which dominates the magnetic pressure term in the radial momentum equation that assists the radial infall motion.

#### 4 CONCLUSION

In this paper, we have studied an accretion disc around a black hole in an advection-dominated regime in the presence of toroidal



**Figure 4.** Numerical coefficient  $C_1$  as a function of  $f$  for different values of magnetic field strength,  $\beta$ . Upper panel: for low accretion rate solution and lower panel: for high accretion rate solution.

magnetic field with thermal conduction. We have presented the results of self-similar solutions to show the effect of the viscosity and thermal conduction on magnetically driven accretion flows from a flow threaded by toroidal magnetic fields. The only serious approximation we have here made is the use of an isotropic  $\alpha$  viscosity. Attention has restricted to flow accretions in which self-gravitation is negligible. Considering weekly collisional nature of hot accretion flows, a saturated form of thermal conduction has been used as a possible mechanism for transporting the energy was produced by viscous heating. In a sense, we have accounted for this possibility by allowing the saturated conduction constant,  $\phi$ , to vary in our solutions.

We have found a self-similar solution using similarity technique in analogy to self-similar solutions by NY1994 and Akizuki & Fukue (2006). The self-similar solution is valid in the intermediate region of the advection-dominated discs. The toroidal geometry for magnetic field is a good approximation for magnetic structure in this region. The disc structure, is characterized by the coefficient  $C_1$ , slightly depends on the strength of magnetic field. The geometrical thickness of the discs depends on the magnetic field strength; while the structure of accretion flow will be modified strongly by viscous parameter  $\alpha$  and saturated thermal conduction parameter,  $\phi$ .

We have found two solutions with different infall velocity, where they have different behaviour with thermal conduction. In the high accretion rate solution, when the level of saturated conduction increases, the radial infall velocity also increases, while for the low accretion rate, the solution is quite different.

The presence of magnetic field with toroidal geometry will affect the role of thermal conduction. In both solutions, we have found that a large magnetic field causes a more radial infall velocity.

There are some limitations in our solutions. One of them is that the self-similar hot accretion flow with conduction is one-temperature structure. If we use a two-temperatures structure for the ions and electrons in the discs, it is expected that the ions and electron temperatures will decouple in the inner regions, which will modify the role of conduction. The other limitation of our solution is the anisotropic character of conduction in the presence of magnetic field. Balbus (2001) has argued that the dynamical structure of the

hot flows could be affected by the anisotropic character of thermal conduction in the presence of magnetic field.

However, our results clearly improve the physics of an ADAF around a black hole. It is important to investigate the effect of thermal conduction on the physical structure of hot flow around a black hole. We developed NY1994 standard solutions to a more realistic model of ADAFs by adding the magnetic field on the structure of discs and thermal conduction as a mechanism to transport energy in radial direction. In the future studies, we plan to improve our model with a more realistic viscous model, Case 2, and to follow the effect of thermal conduction to other physical parameters of the discs. Also, we are going to develop our model in a 2D self-similar solution for ADAF around a black hole. Several developments can be investigated to reach a much more realistic description for the physics of hot accretion discs around a magnetized compact object.

## ACKNOWLEDGMENTS

This research has been supported by a grant from FMU. We are grateful to the referee for a very careful reading of the manuscript and for suggestions which have helped us to improve the presentation of our results.

## REFERENCES

- Abramowicz M. A., Chen X., Kato S., Lasota J. P., Regev O., 1995, *ApJ*, 438, L37
- Akizuki C., Fukue J., 2006, *PASJ*, 58, 461
- Balbus S. A., 2001, *ApJ*, 562, 909
- Cowie L. L., McKee C. F., 1977, *ApJ*, 275, 641
- Esin A. A., Narayan R., Ostriker E., Yi H., 1996, *ApJ*, 465, 312
- Fukue J., 2004, *PASJ*, 56, 569
- Ghanbari J., Salehi F., Abbassi S., 2007, *MNRAS*, 381, 159
- Gracia J., Peitz J., Keller C., Camenzind M., 2003, *MNRAS*, 344, 468
- Hameury J. M., Lasota J. P., McClintock J. E., Narayan R., 1997, *ApJ*, 489, 234
- Hawley J. F., Gammie C. F., Balbus S. A., 1994, in Bicknell G. V., Dopita M. A., Quinn P. J., eds, *ASP Conf. Ser. Vol. 54, The First Stromlo Symposium: The Physics of Active Galaxies*. Astron. Soc. Pac., San Francisco, p. 73
- Honma F., 1996, *PASJ*, 48, 77
- Ichimaru S., 1977, *ApJ*, 214, 840
- Kato S., Abramowicz M. A., Chen X., 1996, *PASJ*, 48, 67
- Kato S., Yamasaki T., Abramowicz M. A., Chen X., 1997, *PASJ*, 49, 221
- King A. R., Pringle J. E., Livio M., 2007, *MNRAS*, 376, 1790
- Mahadevan R., Quataert E., 1997, *ApJ*, 490, 605
- Manmoto T., Kato S., 2000, *ApJ*, 538, 295
- Manmoto T., Kato S., Nakamura K., Narayan R., 2000, *ApJ*, 529, 127
- Medvedev M. V., Narayan R., 2001, *ApJ*, 554, 1255
- Meyer F., Meyer-Hofmeister E., 1994, *A&A*, 288, 175
- Meyer F., Liu B. F., Meyer-Hofmeister E., 2000, *A&A*, 361, 175
- Narayan R., Yi I., 1994, *ApJ*, 428, L13 (NY1994)
- Narayan R., Yi I., 1995a, *ApJ*, 444, 231
- Narayan R., Yi I., 1995b, *ApJ*, 452, 710
- Narayan R., Loab A., Kumar P., 1994, *ApJ*, 431, 359
- Narayan R., Yi I., Mahadevan R., 1995, *Nat*, 374, 623
- Narayan R., McClintock J. E., Yi I., 1996, *ApJ*, 457, 821
- Narayan R., Kato S., Honma F., 1997, *ApJ*, 476, 49
- Narayan R., Igumenshchev I. V., Abramowitz M. A., 2000, *ApJ*, 539, 798
- Piran T., 1978, *ApJ*, 221, 652
- Quataert E., Gruzinov A., 2000, *ApJ*, 539, 809
- Shakura N. I., Sunyaev R. A., 1973, *A&A*, 24, 337
- Shakura N. I., Sunyaev R. A., 1976, *MNRAS*, 175, 613
- Shapiro S. L., Lightman A. P., Eardley D. M., 1976, *ApJ*, 204, 187 (SLE)
- Tanaka T., Menou K., 2006, *ApJ*, 649, 345
- Wandel A., Liang E. P., 1991, *ApJ*, 380, 84

This paper has been typeset from a  $\text{\TeX}/\text{\LaTeX}$  file prepared by the author.