A Polynomial Matrix SVD Approach for Time Domain Broadband Beamforming in MIMO-OFDM Systems

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Abstract — **Singular value decomposition (SVD) is a useful technique to mitigate co-space interference (CSI) in multiple input multiple output (MIMO) systems employing orthogonal frequency division multiplexing (OFDM) scheme. The SVD based frequency-domain broadband beamforming (FBBF) approach uses the beamforming matrices in both transmitter and receiver sides for each subchannel matrix. However, when the MIMO-OFDM system has a large number of subchannels, the FBBF technique is very complex due to computing the SVD of each subchannel separately. In this paper, we propose a new approach to eliminate CSI and intersymbol interference (ISI) in time-domain and frequency-domain, respectively. By the use of polynomial matrix SVD based on SBR2 algorithm, the MIMO-OFDM system is decomposed to the parallel decoupled single input single output (SISO) OFDM systems. In this way, the CSI is eliminated by time-domain broadband beamforming (TBBF) in both transmitter and receiver sides and ISI is mitigated in each SISO-OFDM system by frequency-domain equalization. The performance of the proposed method is evaluated by computer simulations for fixed broadband wireless communication system developed based on the WiMax. Simulation results show that the proposed method achieves good performance in concerned SNR interval.**

Index Terms — **Broadband Beamforming, MIMO, OFDM, Polynomial Matrix, SVD.**

I. INTRODUCTION

SPACIAL diversity employed both in transmitter and receiver sides is a promising technique that increases bandwidth efficiency and link reliability in multiple input multiple output (MIMO) communication systems. However, intersymbol interference (ISI) and co-space interference (CSI) are main challenging issues for exploiting the capacity of the MIMO systems.

By the use of orthogonal frequency division multiplexing (OFDM) scheme in the MIMO system, the ISI can be mitigated based on converting each frequency selective channel to the number of flat and independent subchannels. To overcome CSI, different methods have been proposed. A broadband beamforming for single carrier MIMO system has been proposed in [1] and [2] based on SBR2 algorithm [3] that calculates the eigenvalue decomposition (EVD) of polynomial matrix. A unified framework for transmit and receive beamforming has been proposed in [4] based on convex optimization for multicarrier MIMO system. A joint transmit and receive optimal precoder/decoder (beamforming) has been designed in [5] for a set of parallel flat MIMO subchannels. In [6], a joint precoder and equalization approach has been proposed for broadband MIMO system based on the SBR2 algorithm. Beamforming for each subchannel in frequency domain is a method that mitigates the CSI in MIMO-OFDM system [7]. By utilizing singular value decomposition (SVD) of each flat subchannel matrix, transmit and receive unitary beamforming matrices can be obtained. After multiplying beamforming matrices to the transmitted and received signals of each subchannel, the CSI is eliminated. Although the frequency-domain broadband beamforming (FBBF) method based on the SVD technique is completely able to eliminate the CSI, when a large number of subchannels is employed, this method is very complex due to computing the SVD for all subchannels.

In this paper, we propose a time-domain broadband beamforming (TBBF) method for MIMO-OFDM system. This method uses beamforming matrices in both transmit and receive sides in order to eliminate the CSI based on polynomial matrix SVD approach. The proposed TBBF method can be less complex in comparison with the FBBF method when channel impulse response duration is very less than the OFDM symbol duration. The SVD of channel impulse response polynomial matrix is obtained based on the SBR2 algorithm [3]. Although the SBR2 algorithm attains the polynomial matrix SVD approximately, simulation results show that the proposed TBBF method achieves good performance in concerned SNR interval.

The structure of this paper is as follows. After introduction, the polynomial matrix SVD technique based on the SBR2 algorithm is introduced in section II. Based on the polynomial matrix SVD, the TBBF algorithm is developed In Section III for MIMO-OFDM system. Section IV contains computer simulations and section V concludes the paper.

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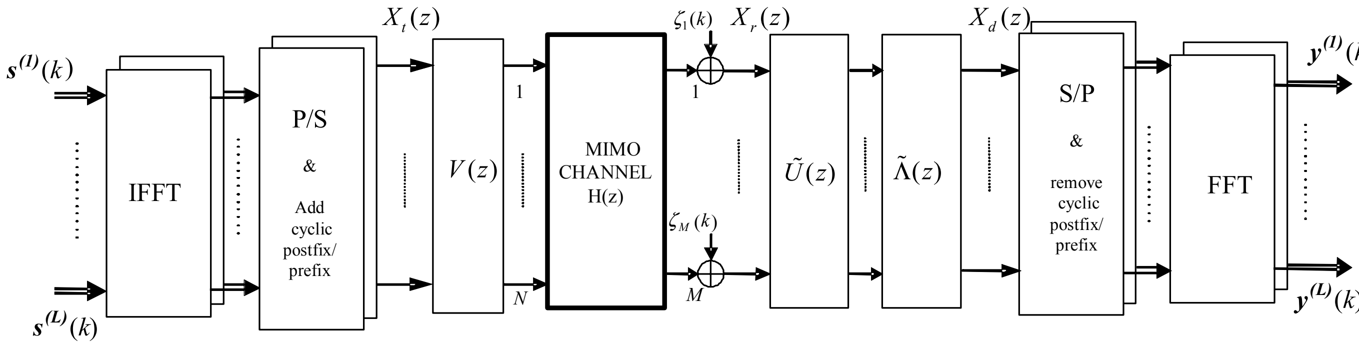

Fig.1. Structure of a MIMO-OFDM system employing time-domain broadband beamforming.

II. Polynomial Matrix SVD

Let define $H(k)$ as impulse response polynomial matrix of a frequency selective MIMO channel with N transmitting antennas, M receiving antennas and L_c taps. Z-transfer function of $H(k)$ is given as

$$\mathbf{H}(z) = \sum_{k=0}^{L-1} H(k)\, z^{-k} \tag{1}$$

$\mathbf{H}(z)$ is a $M \times N$ polynomial matrix with the maximum order of L_c. $\mathbf{H}(z)$ can be decomposed as

$$\mathbf{H}(z) = \mathbf{U}(z)\mathbf{\Lambda}(z)\tilde{\mathbf{V}}(z) \tag{2}$$

where $\tilde{\mathbf{V}}(z)$ is denoted as $\tilde{\mathbf{V}}(z) = \mathbf{V}^H(z^{-1})$. Note that $(.)^H$ denotes transposed complex conjugate operation. $\mathbf{U}(z)$ and $\mathbf{V}(z)$ are paraunitary matrices such that $\tilde{\mathbf{U}}(z)\mathbf{U}(z) = I$, $\tilde{\mathbf{V}}(z)\mathbf{V}(z) = I$ and $\tilde{\mathbf{\Lambda}}(z)\mathbf{\Lambda}(z) = \mathbf{D}(z)$ where $\mathbf{D}(z)$ is a diagonal polynomial matrix. Although $\mathbf{\Lambda}(z)$ is not necessarily a diagonal matrix, similar to flat channel matrix, above decomposition is called polynomial matrix SVD.

To calculate paraunitary matrices $\mathbf{U}(z)$ and $\mathbf{V}(z)$, we use a new decomposition iterative algorithm called SBR2 [3] that has been developed for eigenvalue decomposition (EVD) of a para-Hermitian polynomial matrix. By defining para-Hermitian polynomial matrices $\mathbf{A}_1(z) = \mathbf{H}(z)\tilde{\mathbf{H}}(z)$ and $\mathbf{A}_2(z) = \tilde{\mathbf{H}}(z)\mathbf{H}(z)$ and employing the SBR2 algorithm [3], we have

$$\mathbf{A}_1(z) = \mathbf{G}_1(z)\mathbf{D}_1(z)\tilde{\mathbf{G}}_1(z) \tag{3}$$

$$\mathbf{A}_2(z) = \mathbf{G}_2(z)\mathbf{D}_2(z)\tilde{\mathbf{G}}_2(z) \tag{4}$$

where $\mathbf{U}(z) = \mathbf{G}_1(z)$ and $\mathbf{V}(z) = \mathbf{G}_2(z)$.

III. Time-Domain Broadband Beamforming

We consider a MIMO-OFDM system, which has N transmitting antennas, M receiving antennas and L subcarriers when $S(k) = [\boldsymbol{s}^{(1)}(k),...,\boldsymbol{s}^{(L)}(k)]$ is the k th transmitted OFDM symbol and $\boldsymbol{s}^{(l)}(k) = [s_1^{(l)}(k),...,s_N^{(l)}(k)]^T$ is a symbol vector transmitted from the lth subcarrier. Note that $(.)^T$ represents the transpose operation. The frequency domain symbols are converted to time domain and after adding sufficient cyclic prefix/postfix, the data vector $X_t(n) = [x_{t1}(n),\ldots,x_{tN}(n)]^T$ at the n th time snapshot is formed. The transmit vector $X_t(n)$ is filtered by passing through $\mathbf{V}(z)$ and is transmitted via MIMO channel. The z-transform of the received $X_r(n) = [x_{r1}(n),\ldots,x_{rM}(n)]^T$ vector becomes

$$\mathbf{X}_r(z) = \mathbf{H}(z)\mathbf{V}(z)\mathbf{X}_t(z) + \boldsymbol{\zeta}(z) \tag{5}$$

where $\boldsymbol{\zeta}(z)$ is the z-transform of $\boldsymbol{\zeta}(k)$ that is the additive white Gaussian noise vector and $\mathbf{V}(z)$ is the transmitter paraunitary beamforming matrix. By substituting the polynomial matrix SVD of $\mathbf{H}(z)$ in (5), we have

$$\mathbf{X}_r(z) = \mathbf{U}(z)\mathbf{\Lambda}(z)\mathbf{X}_t(z) + \boldsymbol{\zeta}(z) \tag{6}$$

After passing received $X_r(k)$ vector through $\tilde{\mathbf{U}}(z)$ and $\tilde{\mathbf{\Lambda}}(z)$, respectively, as shown in Fig.1, the z-transform of the output $X_d(k)$ vector becomes

$$\mathbf{X}_d(z) = \mathbf{D}(z)\mathbf{X}_t(z) + \boldsymbol{\rho}(z) \tag{7}$$

where $\mathbf{D}(z) = \tilde{\mathbf{\Lambda}}(z)\mathbf{\Lambda}(z)$ and $\boldsymbol{\rho}(z) = \tilde{\mathbf{\Lambda}}(z)\tilde{\mathbf{U}}(z)\boldsymbol{\zeta}(z)$. Since $\mathbf{D}(z)$ is a diagonal polynomial matrix, the CSI is eliminated and MIMO-OFDM system is decomposed to $P \le \min\{M, N\}$ parallel independent OFDM systems as shown in Fig. 2.

$$X_{dm}(z) = D_m(z)X_{tm}(z) + \rho_m(z) \qquad \text{for } m = 1,\ldots,P \tag{8}$$

where $D_m(z)$ is the mth diagonal element of $\mathbf{D}(z)$. After removing cyclic prefix/postfix and taking fast Fourier transform (FFT), the ISI is mitigated in each subchannel $D^{(l)}$ in frequency domain such that the output vector of the lth subchannel becomes

$$\mathbf{y}^{(l)}(n) = D^{(l)}\mathbf{s}^{(l)}(n) + \upsilon^{(l)}(n) \qquad \text{for } l = 1,\ldots,L \tag{9}$$

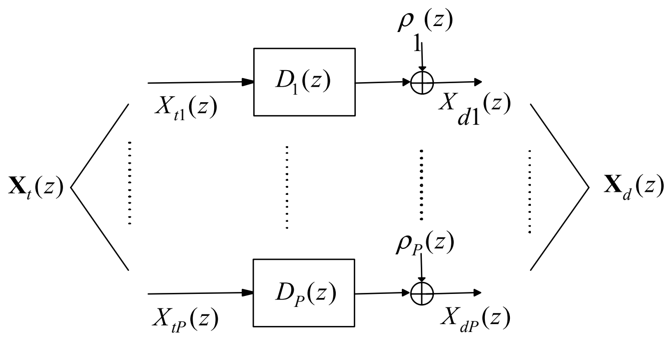

Fig.2. Decomposition of the time-domain MIMO channel to P parallel independent SISO channels.

where $D^{(l)}$ is given as

$$D^{(l)} = \begin{pmatrix} D_1^{(l)} & & 0 \\ & \ddots & \\ 0 & & D_p^{(l)} \end{pmatrix} \tag{10}$$

By multiplying $\mathbf{y}^{(l)}(n)$ to the inverse of $D^{(l)}$ for all $l = 1,\ldots,L$, one can detect the transmitted signal.

IV. Simulations And Results

A MIMO-OFDM system based on IEEE802.16 (WiMAX) standard with $M = N = 2$ and 4 has been considered in simulations. A sequence of independent, identically distributed QPSK or 16-QAM signal vector is sent from transmitter antenna arrays. Each frequency selective channel between Tx-Rx antennas has been realized based on an exponentially decaying power delay profile with L_c=16 resolvable paths.

Since the SBR2 is a diagonalizing algorithm that computes $\mathbf{D}(z) = \tilde{\mathbf{\Lambda}}(z)\mathbf{\Lambda}(z)$ approximately, $\mathbf{D}(z)$ does not become an exact diagonal polynomial matrix. Therefore the CSI is not eliminated completely. To evaluate the performance of the SBR2 diagonalizing algorithm, the powers of diagonal and off-diagonal tap weights of $\mathbf{D}(z)$ polynomial matrix have been depicted in Fig.3 based on different exponentially decay factors, $\beta = 0.3, 0.4$ and 0.5 for one hundred independent channel matrix realizations when $M = N = 2$. As seen in Fig. 3, for tap index interval $-L_c < l < L_c$, the power of diagonal tap weights of $\mathbf{D}(z)$ is significantly higher than that of the off-diagonal ones. In addtion, the total power of the tap weights that are outside this interval is much less than that of the inside interval tap weights. Thus, with good approximation, $\mathbf{D}(z)$ can be assumed to be a diagonal polynomial matrix for $-L_c < l < L_c$.

Moreover, as we mentioned in Section II, although $\mathbf{D}(z) = \tilde{\mathbf{\Lambda}}(z)\mathbf{\Lambda}(z)$ is approximately a diagonal polynomial

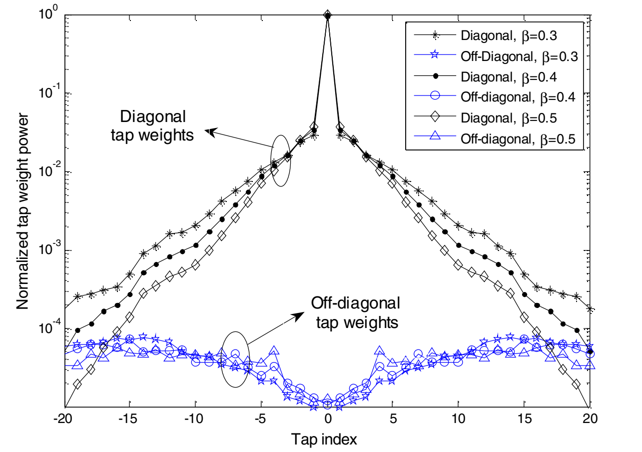

Fig. 3. Powers of the diagonal and off-diagonal tap weights of $\mathbf{D}(z) = \tilde{\mathbf{\Lambda}}(z)\mathbf{\Lambda}(z)$ for different β values and $M=N=2$.

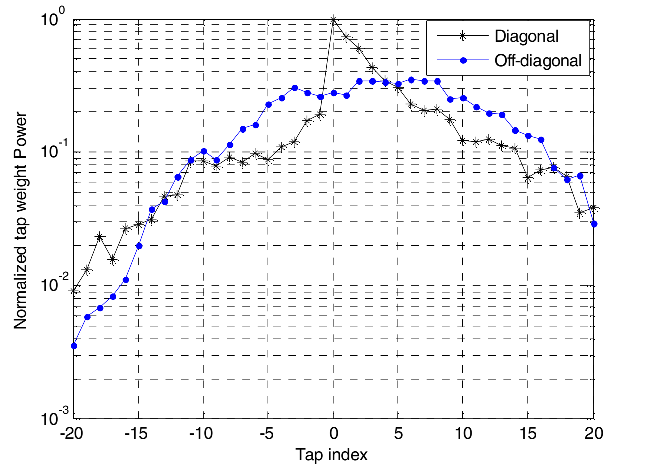

Fig. 4. Powers of the diagonal and off-diagonal tap weights of $\mathbf{\Lambda}(z)$ for $\beta = 0.3$ and $M=N=2$.

matrix, $\mathbf{\Lambda}(z)$, which is computed based on $\mathbf{\Lambda}(z) = \tilde{\mathbf{U}}(z)\mathbf{H}(z)\mathbf{V}(z)$, does not necessarily become a diagonal polynomial matrix. To highlight this phenomenon, the powers of diagonal and off-diagonal tap weights of $\mathbf{\Lambda}(z)$ have been indicated in Fig.4 for one hundred independent channel matrix realizations with $\beta = 0.3$. As seen, the powers of diagonal and off-diagonal tap weights are in the same order that indicates $\mathbf{\Lambda}(z)$ is not a diagonal polynomial matrix.

To evaluate the bit error rate (BER), we have simulated a MIMO-OFDM system based on block diagram shown in Fig.1. Since $\mathbf{D}(z)$ is a non-causal filter, cyclic postfix should be added to the OFDM symbol in addition to cyclic prefix in order to eliminate the ISI. We should note that adding cyclic prefix/postfix because of $\tilde{\mathbf{\Lambda}}(z)$ filter does not affect bandwidth efficiency due to using $\tilde{\mathbf{\Lambda}}(z)$ filter in the receiver side. Both cyclic prefix and cyclic postfix with length equal to channel length $L_c = 16$ are employed.

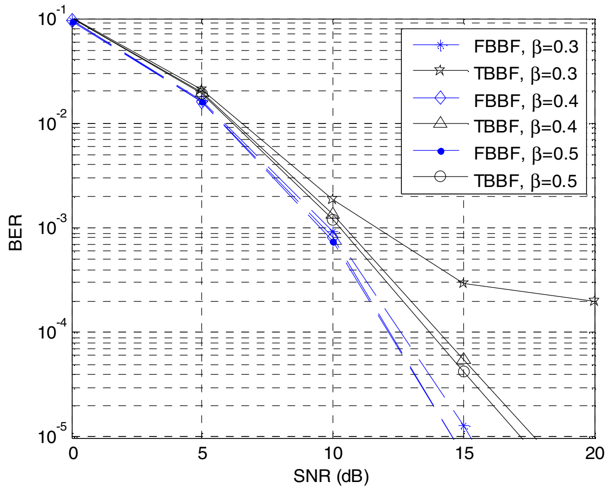

Fig. 5. BER versus SNR for the proposed time domain broadband beamforming (TBBF) algorithm and frequency domain broadband beamforming (FBBF) algorithm for QPSK modulation scheme and different β values when *M=N=2*.

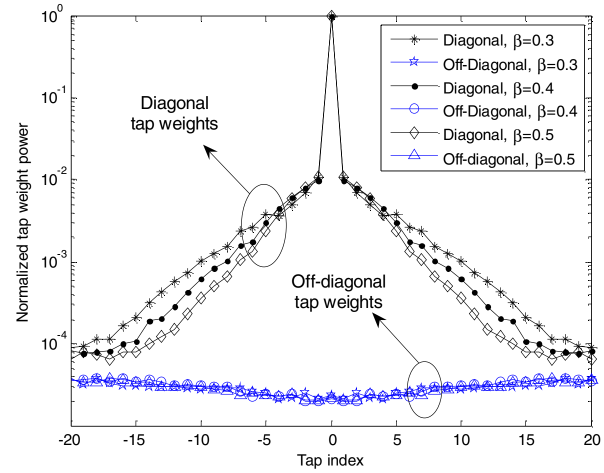

Fig. 7. Powers of the diagonal and off-diagonal tap weights of $\mathbf{D}(z)=\tilde{\mathbf{\Lambda}}(z)\mathbf{\Lambda}(z)$ for different β values and *M=N=4*.

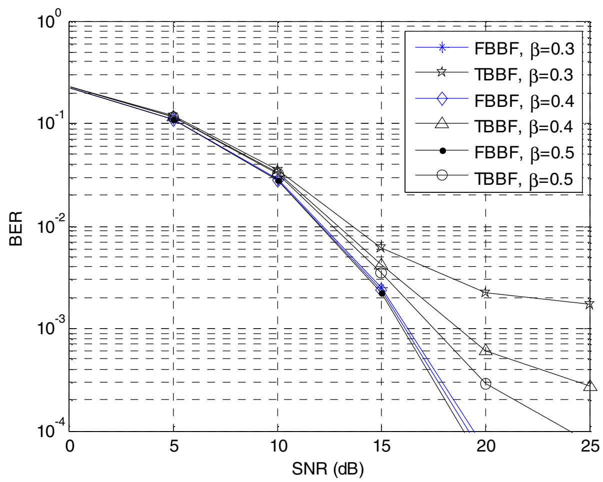

Fig. 6. BER versus SNR for the proposed time domain broadband beamforming (TBBF) algorithm and frequency domain broadband beamforming (FBBF) algorithm for 16-QAM modulation scheme and different β values when *M=N=2*.

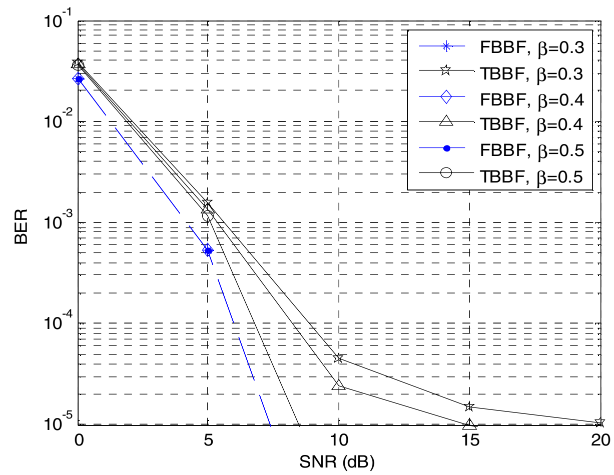

Fig. 8. BER versus SNR for the proposed time domain broadband beamforming (TBBF) algorithm and frequency domain broadband beamforming (FBBF) algorithm for QPSK modulation scheme and different β values when *M=N=4*.

For comparison, frequency-domain broadband beamforming (FBBF) algorithm, which separately utilizes the SVD technique for each flat MIMO subchannel matrix, is considered as well. In this algorithm $\mathbf{V}_l$ and $\mathbf{U}_l^H$ unitary matrices are used in *l*th subchannel as the transmit and the receive beamforming matrices, respectively. The frequency-domain beamforming completely mitigates the CSI. Note that although the performance of the frequency-domain beamforming is optimal, it suffers from high computational complexity due to employing the SVD for each subchannel separately.

In Fig.5 the BER performance of the TBBF algorithm is compared with that of the FBBF algorithm for $M=N=2$ and different β values when QPSK modulation scheme is used. It can be seen that increment of β has no considerable effect on frequency-domain beamforming algorithm but results better performance for time-domain beamforming. The BER performances of both TBBF and FBBF algorithms have been shown in Fig.6 for 16-QAM modulation scheme. Since the 16-QAM modulation scheme is more sensitive to interference, the floor effect in BER performance occurs for $\beta=0.4$ and 0.5 about BER=10^{-4}. However, this effect is not seen in the QPSK modulation scheme even at BER=10^{-5} for $\beta=0.4$ and 0.5.

As SNR increases, the difference in performance between frequency-domain and time-domain beamforming algorithms is increasing as well. The main reason of the time-domain beamforming performance degradation is the residual CSI that is produced by the diagonal tap weights out of the considered interval along with the off-diagonal tap weights.

Note that the SBR2 produces diagonal polynomial channel matrix, $\mathbf{D}(z)$, whose impulse response duration is longer than $2L_c$, however by increasing β, the power of channel impulse response is concentrated in the $-L_c < l < L_c$ interval as seen in Fig.3. Therefore, due to decreasing the residual CSI, the performance of the time-domain beamforming algorithm is improved by increasing β.

When $M = N = 4$, similar to Fig.3, the powers of diagonal and off-diagonal tap weights of $\mathbf{D}(z)$ polynomial matrix have been shown in Fig.7 for different β values based on one hundred independent channel matrix realizations. As seen in Fig.7, there is more CSI for $M = N = 4$ in comparison with $M = N = 2$ due to increasing the number of antennas from two to four. To evaluate the effect of the CSI in performance, the BER of both TBBF and FBBF algorithms have been depicted in Fig.8 for QPSK modulation scheme. In comparison with $M = N = 2$ situation (Fig.5), due to increasing the space diversity order from two to four, the BER performance in $M = N = 4$ situation is totally better for $\text{SNR} \le 20\text{dB}$. Although there is no floor effect in $M = N = 2$ for $\beta = 0.4$, because of increasing CSI the floor effect in BER happens about $\text{BER}=10^{-5}$ for $\beta = 0.4$ in $M = N = 4$.

V. Conclusions

A Time-domain broadband beamforming (TBBF) algorithm has been proposed in this paper for MIMO-OFDM systems based on polynomial matrix SVD approach. Transmit and receive time-domain beamforming matrices are attained based on the SVD of channel impulse response polynomial matrix where the SBR2 algorithm is employed to obtain the SVD of channel polynomial matrix.

The proposed algorithm eliminates the CSI by broadband beamforming in both transmit and receive sides. When the number of subchannels in the MIMO-OFDM system is very larger than the number of channel impulse response taps, the complexity of the proposed TBBF algorithm can be less than the FBBF algorithm.

Simulation results indicate that the performance of the proposed TBBF algorithm is fairly good compared with the performance of the SVD based FBBF algorithm that completely eliminates CSI and ISI in the MIMO-OFDM systems.

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