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Robust Controller Design for Missile Yaw Channel

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Abstract
Quantitative design of robust control systems proposes a transparent and practical controller design methodology for uncertain single-input single-output and multivariable plants. There are several steps involved in the design of such controllers. The main steps involved in the design are template generation, loop shaping and pre-filter design. In this paper, a useful method to design a robust controller for yaw channel of a missile, using quantitative feedback theory (QFT) is proposed. Results are used to show the effectiveness of the proposed method.

Keywords: *"Robust control", "QFT", "Uncertainty", "Dynamic model", "Missile".*

Introduction

In the 1960s, Horowitz continued the pioneering work of Bode and introduced a frequency-domain design methodology (Horowitz 1963) that was refined in the 1970s to its present form, commonly referred to as the quantitative feedback theory (QFT) (Horowitz and Sidi 1972, Horowitz and Wang 1979, Sidi 1973). The QFT is considered as a practical engineering method for the robust controller design of continuous time feedback systems, based on frequency-domain design methodologies. [1], [2]

The quantitative approach provides a design methodology which enables the designer to observe clearly the limitations and trade-off in its design. However, to use this flexibility a compromise between different practical design requirements arise which requires much experience and expertise to select. The selection is mainly based on a time consuming trial-and-error procedure (Horowitz 1991) which can be solved using soft computing techniques.

Missile is a flying vehicle with variable transfer functions. It means that as the speed, height and mass changes, flying parameters change. These changes are called uncertainty. Thus, controlling system for satisfaction of the practical needs, should acquire acceptable performance even in the presence of uncertainty and noise.

Dynamic Model of Missile

For obtaining missile dynamic model two coordinate systems are defined.

1- The origin of the earth-fixed coordinate system is located at the missile launch point.

2- The origin of the body coordinate system is assumed to be at the missile center of gravity. The X_B-axis of the body coordinate system points in the direction of the missile nose, the Y_B-axis points in the starboard direction, and the Z_B-axis completes the right-handed triad Fig 1. [3]

Force and Momentum equations are found as:

$$\begin{aligned}
F_{A_x} &= K_{D_O} + K_{D_A}.\alpha + K_{D_{\delta_a}}\delta_a + K_{D_{\delta_e}}.\delta_e + K_{D_{\delta_r}}.\delta_r \\
F_{A_y} &= K_{Y\beta}.\beta + K_{yr}.r + K_{y_{\delta_r}}.\delta_r \\
F_{A_z} &= K_{l_\alpha}.\alpha + K_{l_q}.q + K_{l_{\delta_e}}.\delta_e \\
M_{x_A} &= L = K_{x_p}.P + K_{l_{\delta_a}}.\delta_a \\
M_{y_A} &= M = K_{m_\alpha}.\alpha + K_{m_q}.q + K_{m_{\delta_e}}.\delta_e \\
M_{z_A} &= N = K_{n_\beta}.\beta + K_{n_r}.r + K_{n_{\delta_r}}.\delta_r
\end{aligned} \tag{1}$$

Finally by ignoring the roll of the missile and rejecting the product of small coefficients, the final equations are as follows:

$$\begin{aligned}
&K_{l_p}p + K_{l_{\delta_a}}\delta_a = I_x\dot{p} \\
&K_{m_\alpha}\alpha + K_{m_q}q + K_{m_{\delta_e}}\delta_e = I_y\dot{q} + (I_x - I_z)pr - x_{bc}(K_{l_\alpha}\alpha + K_{l_q}q + K_{l_{\delta_e}}\delta_e + F_{T_z}) \\
&K_{n_\beta}\beta + K_{n_r}r + K_{n_{\delta_r}}\delta_r = I_z\dot{r} + (I_y - I_x)pq + x_{bc}(K_{y_\beta}\beta + K_{y_r}r + K_{y_{\delta_r}}\delta_r + F_{T_y})
\end{aligned} \tag{2}$$

Modeling of yaw channel

Model of yaw channel is obtained from equation (2)

$$\frac{\psi(s)}{\delta_r(s)} = \frac{k_{n_{\delta_r}} - x_{bc}k_{y_{\delta_r}}}{I_z} \cdot \frac{s + \dfrac{k_{y_{\delta_r}}k_{n_\beta} - k_{y_\beta}k_{n_{\delta_r}}}{mV_m(k_{n_{\delta_r}} + x_{bc}k_{y_{\delta_r}})}}{s^3 + \left(\dfrac{x_{bc}k_{y_r} - k_{n_r}}{I_z} - \dfrac{k_{y_\beta}}{mV_m}\right)s^2 + \left(\dfrac{k_{y_\beta}k_{n_r} - k_{y_r}k_{n_\beta}}{mV_m I_z}\right)s} \tag{3}$$

Quantitative Feedback Theory (QFT)

QFT design includes three main steps which are 1- computing the robust performance bounds. 2-Designing the robust controller. 3- If necessary design a proper pre-filter. At the end, analysis of the design is required.

In figure 2 transfer functions G(s) and F(s) are compensator (strictly proper) and pre-filter (proper) respectively. Also these two transfer functions are checked to be stable. $P(s,\alpha)$ is the transfer function of uncertain process in structured uncertainty system.

Tracking Problem

The specifications overshoot and the settling time are given in the form of upper and lower bounds in frequency domain, usually based on simple second-order models to represent under damped and over damped conditions.

$$a(\omega) < F(j\omega)\frac{P(j\omega)G(j\omega)}{1 + P(j\omega)G(j\omega)} < b(\omega)$$

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Where $a(\omega)$ and $b(\omega)$ are positive real valued functions of ω.

Robust Margins

$$\left|\frac{1}{1+1}(j\omega)\right| \langle \infty \quad \text{for all } \omega > 0 \text{ Where } L(s) = P(s)G(s)$$

Loop Shaping and Pre-filter Shaping

By using the elements of the QFT toolbox we design the controller so that the open loop transfer function exactly lies on its robust performance bounds and does not penetrate the U-contour at all frequency values (ω_i) Fig 3.

$$G = 690.43 + .45\,s + \frac{8.97 \times 10^4}{s} \tag{4}$$

$$F(s) = \frac{1}{\left(\frac{s}{3596}+1\right)\left(\frac{s}{4782}+1\right)} \tag{5}$$

Analysis of Design

The time domain closed loop response with controller and pre-filter is shown in Fig 4 which indicates our design is accurate.

Conclusion

In this article the dynamic model of the missile is found first, and then QFT is introduced as one of the robust controlling method for uncertain SISO and MIMO plants. The main steps involved in the design of these controllers, such as template generation, loop shaping, pre-filter design, manipulation of tolerance bounds within the available freedom, template size considerations and selection of nominal transfer function matrices all are generated based on the experience and expertise of the authors. In this paper, the various steps for yaw control (SISO) are investigated. Finally, design results are presented which shows wonderful results.

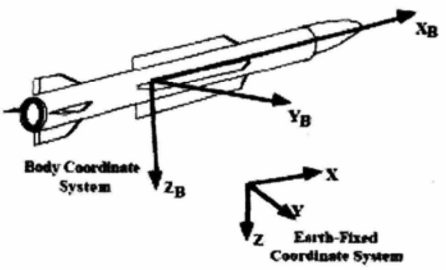

Fig. 1 Missile coordinates system

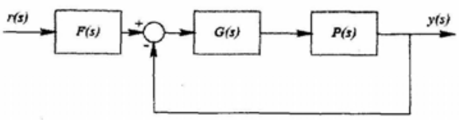

Fig. 2 Feedback Control System Configuration for QFT

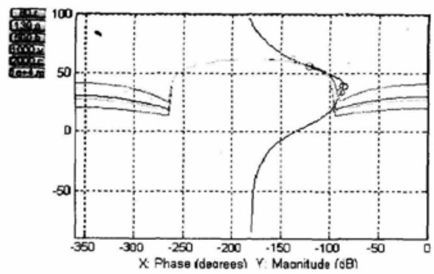

Fig .3 Loop-shaping in Nichols chart

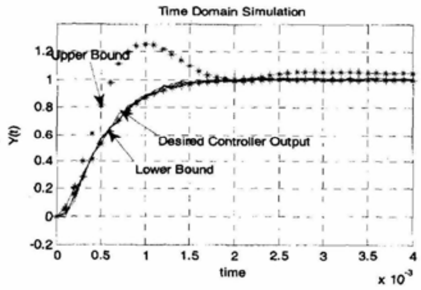

Fig.4 Step response and control effort of system with PID controller and F(s)

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