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An Investigation into the application of wavelet transform for crack detection in plates using Finite Element Method

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Abstract
In this paper, a simple method for crack detection in plate structures based on wavelet analysis is presented. The fundamental vibration mode of a cracked plate obtained by Finite Element Method is analyzed using two-dimensional discrete wavelet transform to estimate accurately both the location and length of the cracks. The feasibility of the proposed method is demonstrated through simulation examples, which involves plates with one or more cracks of various depths at different positions and orientations, especially near the plate edges. It is demonstrated that by using this technique one can detect sub-surface cracks, which is a notable result. The proposed detection technique may serve the purpose of structural health monitoring in situations where spatially distributed measurements of structural response in regions of critical concern can be made with, for example, networks of distributed sensors, optical fibers, computer vision and area scanning techniques.
Keywords: Crack detection, Finite Element Method (FEM), Sub-surface defects, Wavelet analysis.

Introduction
Cracks present a serious threat to the performance of structures because most of structural failures are due to material fatigue. For this reason, methods allowing early detection and localization of cracks have been the subject of intensive investigation over the last two decades. A review of the vibration-based damage identification methods for detection of cracked structures is done by (Doubling, 1998) [1]. The vibration of a cracked plate was first investigated by Lynn and Kumbasar (1967) [2]. Liew et al. (1994) studied the vibrational behavior of a plate having an edge or central crack [3]. Cawley and Adams (1979) seem to have been the first to locate defects within a rectangular plate by utilizing natural frequency changes [4]. Araujo dos Santos et al. (1999) [5] and Chen and Bicanic (2000) [6] proposed a method where both natural frequencies and mode shapes were used to detect damage in a laminated rectangular and a cantilever plate, respectively. Cornwell et al. (1999) developed a damage detection method based on changes in modal strain energy [7]. In this paper, the wavelet coefficients of a two-dimensional discrete wavelet transform will be used to identify the location of the crack in a simply supported plate. The wavelet transform is applied to the first mode shape of the cracked plate, which is obtained by ANSYS software. Location of perturbations in wavelet coefficients identify the site of cracks, the maximum value of wavelet transform coefficients and width of perturbation are good criteria to evaluate crack depth and length, respectively.

Discrete wavelet transform (DWT)
In the case of a 1D space, the continuous wavelet transform of a function *f (x)* is defined as a convolution of the function with a function *ψ(x)* called the wavelet function (mother wavelet) in the form

$$Wf(u,s)=\frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(x)\psi^{*}\left(\frac{x-u}{s}\right)dx \qquad (1)$$

where s and u are the scale and translation variables, respectively, and $*$ denotes the conjugate part. For constructing a proper wavelet, which reflects the properties on real applications, Mallat (1989) [8] developed the multiresolution analysis. A scaling function $\phi(x,y)$ is defined and the wavelet function $\psi(x)$ is constructed using the scaling function.

The scaling function $\phi(x,y)$ is associated with a one-dimensional multiresolution approximation $\{V_j\}_{j\in Z}$ where j corresponds to the scale level. Let $\{V_j^2\}_{j\in Z}$ be the separable two-dimensional multiresolution defined by $V_j^2=V_j\otimes V_j$ where $\otimes$ denotes the tensor product. Considering W_j^2 as the detail space equal to the orthogonal complement of the lower resolution approximation space V_j^2 in V_{j-1}^2, we can write

$$V_{j-1}^2=V_j^2\oplus W_j^2 \qquad (2)$$

Where $\oplus$ denotes the direct sum of two vector spaces.
The signal is now a finite energy function $f(x,y)\in L^2(R^2)$. Let $(V_{2j})_{j\in Z}$ be a separable multiresolution approximation of $L^2(R^2)$. Let $\Phi(x,y)=\phi(x)\varphi(y)$ be the associated two-

dimensional scaling function. Let $\Psi(x)$ be the one-dimensional wavelet associated with the scaling function $\phi(x)$. Then, the three “wavelets”

$$\begin{aligned}\Psi^1(x,y)&=\phi(x)\psi(y),\\ \Psi^2(x,y)&=\psi(x)\phi(y),\\ \Psi^3(x,y)&=\psi(x)\psi(y)\end{aligned} \tag{3}$$

are such that

$$\begin{aligned}&[2^{-j}\Psi^1_{2j}\left(x-2^{-j}n,y-2^{-j}m\right),\\ &2^{-j}\Psi^2_{2j}\left(x-2^{-j}n,y-2^{-j}m\right),\\ &2^{-j}\Psi^3_{2j}\left(x-2^{-j}n,y-2^{-j}m\right)]_{(n,m)\in Z^2}\end{aligned} \tag{4}$$

is an orthonormal basis of W_{2j}, and

$$\begin{aligned}&[2^{-j}\Psi^1_{2j}\left(x-2^{-j}n,y-2^{-j}m\right),\\ &2^{-j}\Psi^2_{2j}\left(x-2^{-j}n,y-2^{-j}m\right),\\ &2^{-j}\Psi^3_{2j}\left(x-2^{-j}n,y-2^{-j}m\right)]_{(n,m)\in Z^3}\end{aligned} \tag{5}$$

is as orthonormal basis of $L^2\left(R^2\right)$.The difference of information between $A^d_{2j+1}f$ and $A^d_{2j}f$ is equal to the orthonormal projection of $f(x)$ on W_j^2, and is characterized by the inner products of $f(x)$ with each vector of an orthonormal basis of W_j^2. It is concluded that this difference of information is given by the three detail 2D signals

$$\begin{aligned}D^1_{2j}f&=\left(\left\langle f(x,y),\Psi^1_{2j}\left(x-2^{-j}n,y-2^{-j}m\right)\right\rangle\right)_{(n,m)\in Z^2},\\ D^2_{2j}f&=\left(\left\langle f(x,y),\Psi^2_{2j}\left(x-2^{-j}n,y-2^{-j}m\right)\right\rangle\right)_{(n,m)\in Z^2},\\ D^3_{2j}f&=\left(\left\langle f(x,y),\Psi^3_{2j}\left(x-2^{-j}n,y-2^{-j}m\right)\right\rangle\right)_{(n,m)\in Z^2},\end{aligned} \tag{6}$$

Just as for one-dimensional signals, one can show that in two dimensions the inner products which define $A^d_{2j}f$, $D^1_{2j}f$, $D^2_{2j}f$ and $D^3_{2j}f$, are equal to a uniform sampling of two-dimensional convolution products. Since the three wavelets $\Psi^1(x,y)$, $\Psi^2(x,y)$ and $\Psi^3(x,y)$ are given by separable products of the functions ϕ and ψ, these convolutions can be written

$$\begin{aligned}A^d_{2j}&=\left\{\left(f(x,y)*\phi_{2j}(-x)\phi_{2j}(-y)\right)\left(2^{-j}n,2^{-j}m\right)\right\}_{(n,m)\in Z^2},\\ D^1_{2j}&=\left\{\left(f(x,y)*\phi_{2j}(-x)\psi_{2j}(-y)\right)\left(2^{-j}n,2^{-j}m\right)\right\}_{(n,m)\in Z^2},\\ D^2_{2j}&=\left\{\left(f(x,y)*\psi_{2j}(-x)\phi_{2j}(-y)\right)\left(2^{-j}n,2^{-j}m\right)\right\}_{(n,m)\in Z^2},\\ D^3_{2j}&=\left\{\left(f(x,y)*\psi_{2j}(-x)\psi_{2j}(-y)\right)\left(2^{-j}n,2^{-j}m\right)\right\}_{(n,m)\in Z^2},\end{aligned} \tag{7}$$

The expressions (7) show that in two dimensions $A^d_{2j}f$ and the $D^k_{2j}f$ are computed with separable filtering of the signal along the abscissa and ordinate .
At the rest of this article we use *CH*, *CV* and *CD* as $D^1_{2j}f$, $D^2_{2j}f$ and $D^3_{2j}f$, respectively, for the sake of simplicity.

Detection of crack location and length

To detect the location and length of the crack, two-dimensional discrete wavelet transform is applied to the plate modal data. This modal data is obtained by finite element analysis using ANSYS. The crack has been modeled by plate elements that have infinitesimal elasticity module in crack site. The data is obtained from finite element analysis is available in A mode shape is a particularly regular function and, therefore, a high number of vanishing moments is required for the wavelet function. The wavelet function, in addition, should preserve symmetry, as vision is sensitive to artifacts caused by asymmetrical wavelets. 51 × 51 sample grid, which is dense enough for identification purposes. For the wavelet analysis, level one of decomposition is used. The horizontal detail coefficients matrix, *CH,* is sensitive to defects with orientation parallel to the *x-axis* and for this reason; the detail coefficient magnitude reflects the crack physical dimensions. The vertical detail coefficients matrix*, CV,* is sensitive to defects with orientation parallel to the *y-axis*. The diagonal detail coefficients matrix*, CD*, is not sensitive to defects parallel to *x* or *y-axis*. Hence, the diagonal detail coefficients matrix is only useful for detecting diagonal cracks with respect to the coordinate system. A modeshape is a particularly regular function and, therefore, a high number of vanishing moments is required for the wavelet function. The wavelet function, in addition, should preserve symmetry, as vision is sensitive to artifacts caused by asymmetrical wavelets. The Symlet $N = 6$ two-dimensional wavelet is used in the analysis presented, yet other wavelets satisfying the aforementioned criterion could be equally well used.

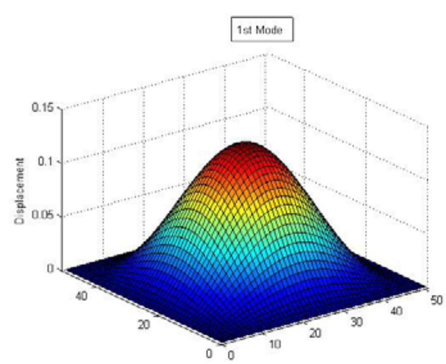
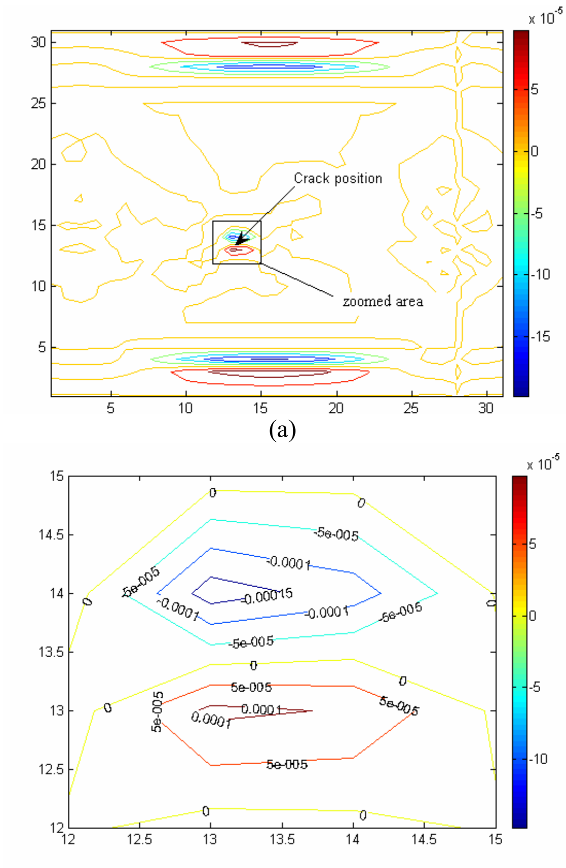


Figure 1: 1st mode shape of a cracked plate with crack position in (*x,y*)=(42.5 cm,41 cm) in an array of 51×51 sample grid

For FEM analysis, a cracked plate, simply supported in four sides with the following dimensions and mechanical properties is considered: length = 1.0 m, width = 1.0 m, depth = 0.08m, E = 200 GPa, ν = 0.3 and ρ = 7860 kg/m3. Using the above-described procedure, the fundamental vibration mode of the cracked plate was calculated by ANSYS. The results are shown in "Figure 1".



(a)

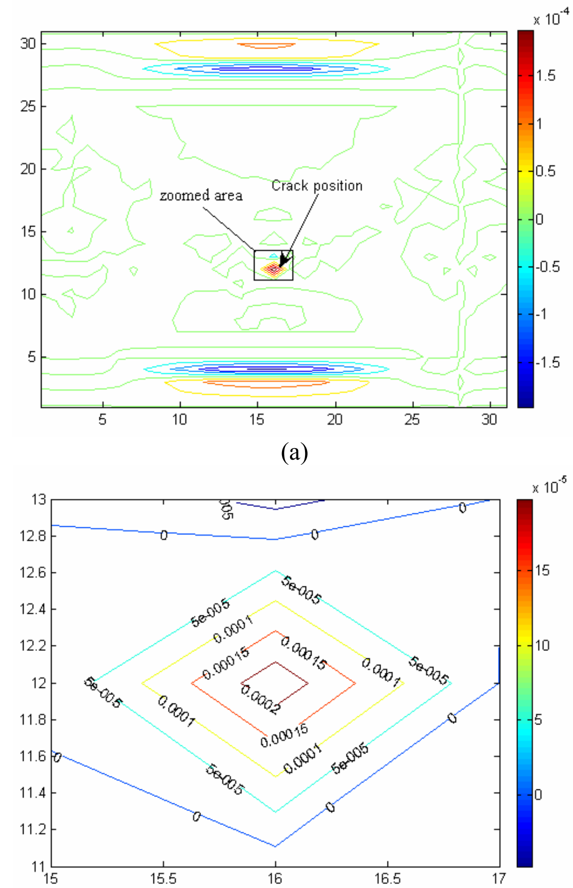
(b)

Figure 2: (a) Contour map of horizontal detial coefficients matrix, CH, for a cracked plate centred at (x,y)=(42.5 cm, 41 cm) in an array of 31×31 sample grid; (b) zoomed area of crack position.

"Figure 2 and 3" illustrate horizontal detail coefficients matrix, *CH*, of the first mode of vibration for plates with a crack centered at (x,y)=(42.5 cm, 41 cm) and (x,y)=(52.5 cm, 31 cm), respectively, having a crack depth of 2 cm.

The maximum value of *CH* is located in the crack center. There are perturbations of *CH* near horizontal edges of plate illustrated in "Figure 2" because of discontinuity at the plate supports. It should be noted that these perturbations are also depicted in other similar figures. The *CH* data are available on an array of 31 × 31 sample grid after using two-dimensional discrete wavelet transform because of down sampling.

"Figure 4 and 5" illustrates horizontal detail coefficients matrix, *CH*, for the first mode of vibration for plates with a crack centered at (x,y)=(54 cm, 31 cm) with length of 8 cm and (x,y)=(55 cm, 31 cm) with length of 10 cm, respectively, having a crack depth of 2 cm. The horizontal extent of *CH* perturbations in the crack position is a measure of the crack length. For example in "Figure 5" the horizontal extent of *CH* perturbations in the crack position is approximately 9.6 cm compared to 10 cm length of the crack.



(a)

(b)

Figure 3: (a) Contour map of horizontal detial coefficients matrix, CH, for a cracked plate centred at (x,y)=(42.5 cm, 41 cm) in an array of 31×31 sample grid; (b) zoomed area of crack position.

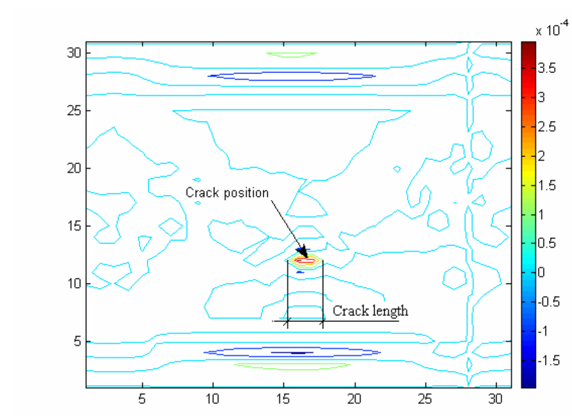


Figure 4: Contour map of horizontal detial coefficients matrix, CH, for a cracked plate centred at (x,y)=(54 cm, 31 cm), with length of 8 cm in a 31×31 sample grid.

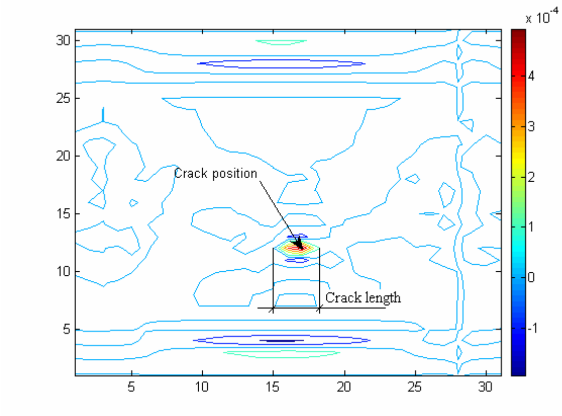


Figure 5: Contour map of horizontal detial coefficients matrix, CH, for a cracked plate centred at (x,y)=(55 cm, 31 cm), with length of 10 cm in an array of 31×31 sample grid.

The maximum value of *CH* at the center of crack can be used as a good criterion for estimating of crack depth. “Figure 6” shows horizontal detail coefficients matrix, *CH*, of a cracked plate with a crack centered at (x,y)=(54 cm, 31 cm) with different depths and length of 6 cm. It is possible to obtain some curves of maximum *CH* at the center of crack versus depth per length of the crack, by analyzing different crack depths with different lengths. “Figure 7” shows these curves.

Determination of embedded cracks

In previous cases, all detected cracks were surface cracks but it is important to know if wavelet analysis method is able to detect sub-surface cracks. “Figure 8” shows the location of a sub-surface crack centered at (x,y)=(53 cm, 31 cm) which is located at depth of 2.5 cm under surface of the plate.

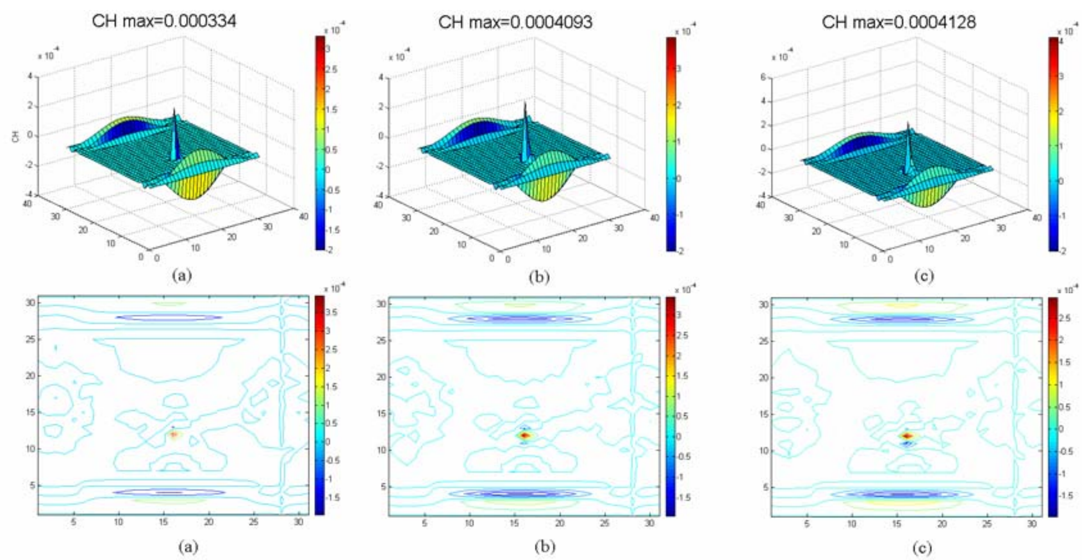


Figure 6: Contour map of horizontal detial coefficients matrix, CH, for a cracked plate with crack depth of (a) 1 cm (b) 2 cm (c) 3 cm in an array of 31×31 sample grid.

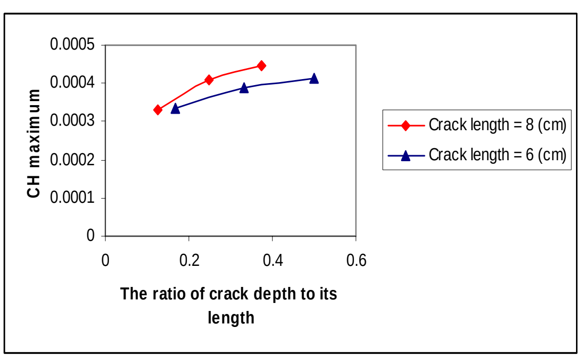


Figure 7: Curves of crack depth.

Analyses for sub-surface cracks in different depths show that until the ratio of crack depth to plate thickness is less than 50 percent, identification of crack is possible (“Figure 9”).

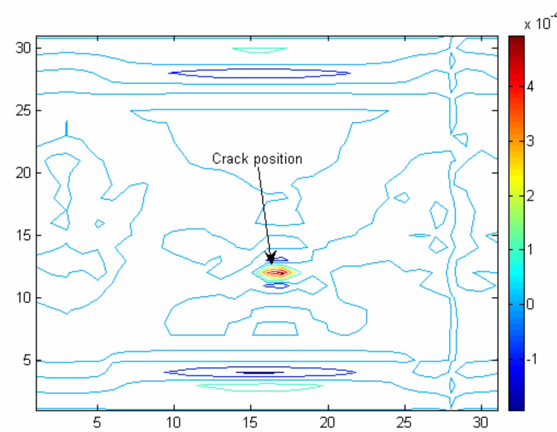


Figure 8: Contour map of horizontal detial coefficients matrix, CH, of a plate with a sub-surface crack in an array of 31×31 sample grid.

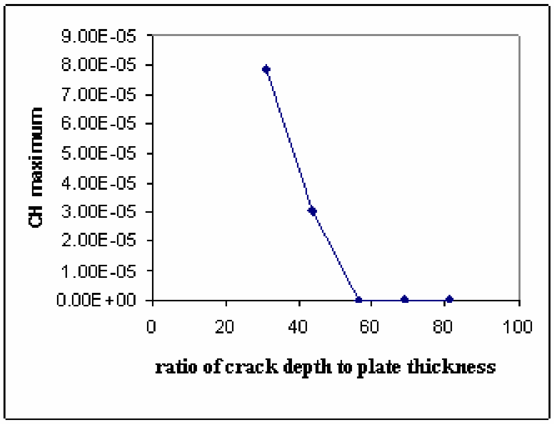


Figure9: maximum CH versus ratio of sub-surface crack depth to plate thickness.

Determination of non-horizontal crack

In section 3, it is mentioned that diagonal detail coefficients matrix, *CD,* is not sensitive to defects parallel to *x* or *y*-axis and hence the diagonal detail coefficients are of low value for a horizontal cracks but useful for detecting diagonal ones. “Figure 10” illustrates that a crack that makes an angle of 45˚ with x-axis can be detected using diagonal detail coefficients matrix, *CD,* but it is not true in the case of using horizontal detail coefficients matrix, *CH.*

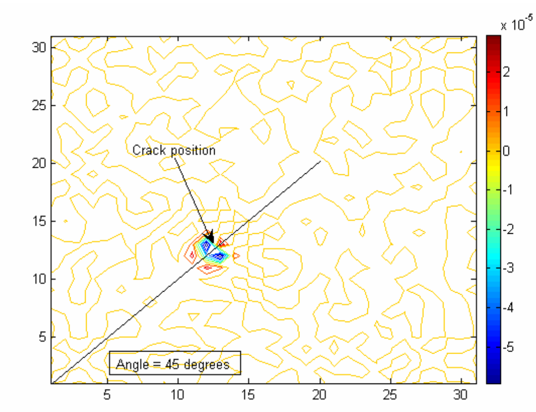


Figure10: Contour map of diagonal detial coefficients matrix, CD, of a plate with diagonal crack in an array of 31×31 sample grid.

Detection of two cracks in a plate

It is obvious that in practical situations there is more than one crack in damaged region. Then it is important to investigate the capability of this method to detect more than one crack. “Figure 11” shows *CH* coefficients matrix of plates that are contain two horizontal cracks with different distances from each other. As illustrated in “Figure 11” this method can detect cracks that are too close to each other. The estimation of crack depth in presence of more than one crack has a certain degree of ambiguity that increase when two cracks are closer to each other.

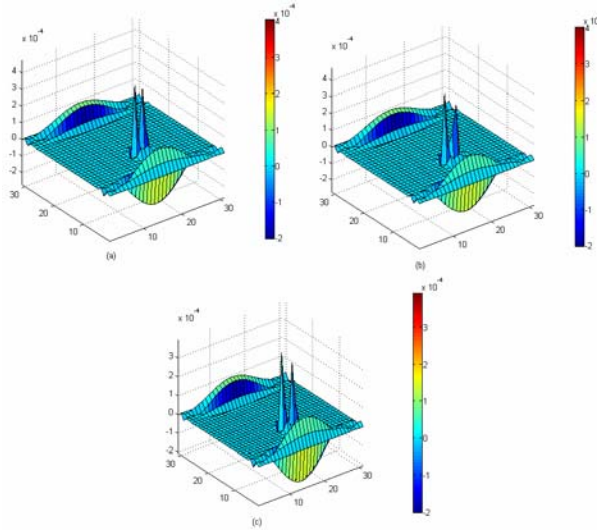


Figure 11: Contour map of horizontal detial coefficients matrix, CH, of plates with two cracks centered at (a) (x1,y1)= (53, 31) & (x2,y2)= (61, 31) (b) (x1,y1)= (53, 31) & (x2,y2)= (63, 31) (c) (x1,y1)= (53, 31) & (x2,y2)= (65, 31) in an array of 31×31 sample grid.

Determination of cracks near edges of plate

There is an ambiguity in detection of cracks near supports of plates. “Figure 12” illustrates the horizontal wavelet coefficients matrix, *CH,* of a plate that contains crack near edge. According to “Figure 12” a good estimation of crack position, length and depth cannot be obtained.

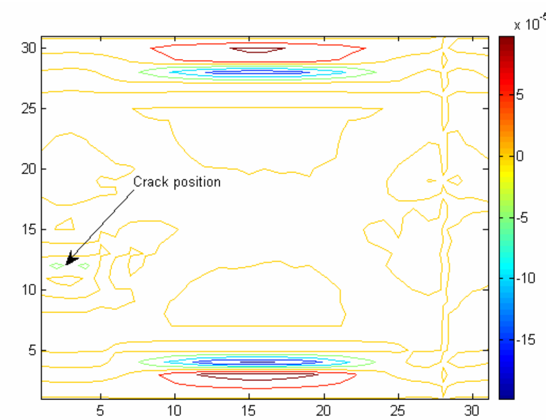


Figure 12: Contour map of horizontal detial coefficients matrix, CH, for a cracked plate centred at (x,y)=(2.5 cm, 31 cm), with length of 5 cm in an array of 31×31 sample grid.

Conclusions and Future works

Viability of wavelet transform method for identification of crack in plate structures was demonstrated by analyzing the vibration mode of cracked plate modeled by ANSYS (FEM software). The location and extent of crack is accurately depicted in the spatial change of transformed response. Analysis of more realistic situations such as multiple cracks, cracks near edges, oblique cracks and sub-surface defects demonstrated the feasibility of the proposed method. Capability of this technique in detection of sub surface defects is an interesting result. It is shown that cracks embedded in the depth of less than 50% of thickness of plate can be detected.

More generally instead of using vibrational analysis (dynamic loading), it is possible to use static analysis (static loading) in order to find displacement field in damaged region that is easier to obtain in some cases.

These results may be use as a basis for detection of cracks in engineering structures. Extension of using this technique for damage detection in different structures such as pipelines, thick walled pressure vessels and so on are possible but more investigation have to be done in this subject.

The work presented provides a foundation for using the two-dimensional wavelet analysis as an efficient damage detection tool for two-dimensional structures. These results make wavelet analysis favorable for use in experimental data analysis. It seems, however, that a key issue for an efficient application of the method is the spatial resolution and the accuracy of the response data.

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