

Relations Between Renyi Distance and Fisher Information

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Abstract. In this paper, we first show that Renyi distance between any member of a parametric family and its perturbations, is proportional to its Fisher information. We, then, prove some relations between the Renyi distance of two distributions and the Fisher information of their exponentially twisted family of densities. Finally, we show that the partial ordering of families induced by Renyi distance is the same as that induced by Fisher information.

1 Introduction and Preliminaries

Consider a parametric density $p(x; \theta)$ defined over a probability space \mathcal{X} parameterized by $\theta \in \Omega$. Renyi information (distance) between $p(x; \theta_0)$ and $p(x; \theta_1)$ is defined by

$$D^\alpha(p(x; \theta_1), p(x; \theta_0)) = \frac{1}{\alpha - 1} \log \int_{\mathcal{X}} \left[\frac{p(x; \theta_1)}{p(x; \theta_0)} \right]^{\alpha-1} p(x; \theta_1) dx$$

for all $\alpha > 0$ ($\alpha \neq 1$) (Renyi, 1961).

Kullback-Leibler (K-L) information is a limiting case of the above

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