An Analytical Solution to Investigate the Springback in Unsymmetric Al-Steel Wide Laminates

M. Kadkhodayan
栩 ιÃ□∥ ι□□□v∂ *Associate Professor*
Department of Mechanical Engineering,
Ferdowsi University of Mashhad
Kadkhoda@um.ac.ir

I. Zohourkari
MSc. student
Department of Mechanical Engineering,
Ferdowsi University of Mashhad
Iman_zohourkari@engineer.com

Abstract
The springback of two-ply Aluminum-Steel laminates is investigated. An elastic-plastic analysis is carried out, Stress distribution during loading is obtained and the springback phenomenon after unloading is predicted in various conditions. Analytical results show that the springback of unsymmetric sheet metal laminates is strongly affected by the mechanical properties of each layer, layer thickness ratio, total sheet thickness and bent radius.
Keywords: Sheet Metal Forming, Unsymmetric Laminate, Springback, Pure Bending

Introduction

In recent years many types of sheet metal laminates have been increasingly used in various fields such as automobile, aerospace, home appliances, electrical and defense industries due to their excellent mechanical and functional properties. The laminates studied in this paper consist of dissimilar sheet metals of different thicknesses bonded together by pressure welded hot rolling to achieve properties that cannot be gained by one metal only. Because of the elastic-plastic behavior of the sheet laminate, the deformation imposed on the sheet when it is forced to conform to the die has both elastic and plastic portions which the elastic part of deformation is recovered after unloading. This undesirable phenomenon is known as "springback". It is very important to predict springback and also to find techniques for reducing it. To the best of our knowledge, all the analytical solutions of the deformation pure bending of a plate are based upon the deformation theory. Yuen presented a solution neglecting radial and width direction stresses but his solution is not efficient and need to apply numerical methods [1]. More efficient analytical solution presented by authors using classical lamination theory and von Mises yield criterion for forming symmetric laminates under plane stress condition [2]. Many other analytical studies are focused on single layer plates but in most of them, materials are treated as rigid-plastic[3], Thus, they predict lower values for springback. In this paper more accurate elasto-plastic analysis based on deformation theory and the von Mises yield criterion is developed and used to predict springback in sheet metal laminates subjected to pure bending. This solution is also used to assessing the effect of various parameters including thickness ratio of each layer and yield strength to Young's modulus ratio between layers. Finally, the obtained results are compared with previous works.

Analysis of springback

In order to predict the springback of sheet metal laminates after bending (Figure 1), an elastic-plastic analysis was carried out based on the following assumptions: the component materials are assumed to be isotropic, elasto-plastic, power-law hardening with the stress strain relationship given by

$$\sigma_\varepsilon = \begin{cases} E\varepsilon_\varepsilon, & \varepsilon_\varepsilon < \varepsilon_y \\ \sigma_y\left(\frac{\varepsilon_\varepsilon}{\varepsilon_y}\right)^q, & \varepsilon_\varepsilon \geq \varepsilon_y \end{cases} \qquad (1)$$

strain in width direction is zero (plane-strain condition), plane sections remain planar throughout deformation, springback occurs purely elastic, no delamination occurs in the laminate and no reverse yielding occurs.

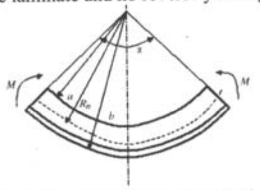

Figure 1. Laminate bending model

As long as reverse yielding does not take place in the yielded fibers, all the fibres satisfy the requirements of the deformation theory of plasticity. Each of the fibers needs to satisfy the equilibrium condition

$$\frac{d\sigma_r^{(k)}}{dr} = \frac{\sigma_\theta^{(k)} - \sigma_r^{(k)}}{r} \qquad (2)$$

It is obvious that for each layer of laminate maximum four zones can be found. Two elastic zones and two plastic zones under tension and compression. Applying Equation 1, a first order ODE for each zone is obtained. Using hook's law for elastic zones and von Mises yield criterion for plastic zones the auxiliary equations can be found

$$\sigma_\theta^{(k)} - \sigma_r^{(k)} = \frac{4E^{(k)}}{3}\left(\ln\left(\frac{r}{R_n}\right)\right) \qquad (3-a)$$

$$\left(\sigma_r^{(k)} - \sigma_\theta^{(k)}\right) = -\left(\frac{2}{\sqrt{3}}\right)^{q^{(k)}+1} \frac{\sigma_y^{(k)}}{\left(\varepsilon_y^{(k)}\right)^{q^{(k)}}}\left(\ln\frac{R_n}{r}\right)^{q^{(k)}} \qquad (3-b)$$

Substituting Equations 3 into 2 and integrating it and imposing appropriate boundary condition radial stress for each zone is obtained. Now using equations 3, $\sigma_\theta^{(k)}$ is determined. Radius (Position) of neutral surface can be determined by use of this fact that the elastic compression and tension zones equations adjacent to N.S have the same radial and also circumferential stress at N.S. Bending moment exerted to laminate during operation is evaluated by integrating as following over the laminate thickness:

$$M_\theta = \int_{r=a}^{r=b} \sigma_\theta . r . dA \,. \qquad (4)$$

As it is known, unloading occurs purely elastic, therefore

$$\Delta M = -\int_A \Delta\sigma_\theta . r . dA \,, \qquad (5)$$

While the moment is decreased to zero

$$M_{Loading} + \Delta M = 0 \qquad (6)$$

By using logarithmic and binomial series expansion and Solving Equ. 6 by Newton-Raphson method change in curvature after unloading is obtained.

Results Discussion

The effect of different parameters such as total laminate thickness, aluminium thickness ratio $(\frac{t_{Al}}{t_{total}})$ and bending curvature on springback were investigated. Figure 2 illustrates springback variations with respect to the various bend curvatures in laminate and monolithic sheets with the same geometry and condition.

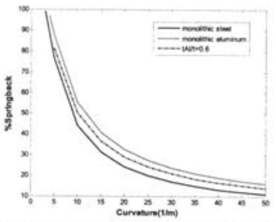

Figure 2. Effect of bent curvature on springback.

Figure 2 illustrates springback variations with respect to the various bend curvatures in laminated and monolithic sheets with the same geometry and condition. It shows that springback in all specimens reduces when bend curvature increases. In addition, in all cases springback of laminate are less than similar monolithic aluminum sheets.

Effect of aluminium thickness ratio on springback is shown in Figure 3. To show the results more clearly, the

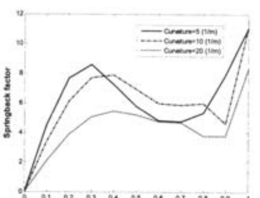

Figure 3. Effect of Al thickness ratio on springback

Springback factor is defined as springback difference between the laminate and similar monolithic steel sheets.

As it may be obtained from the Figure 3, by increasing the aluminum thickness ratio, the springback of laminate remains less than similar monolithic aluminum one. Also the springback of laminate may decrease in special conditions. The influence of laminate total thickness and bend curvature on the springback is Illustrated in Figure 4. It is found that similar to monolithic sheets, when the thickness increases, the springback in the laminate decreases significantly.

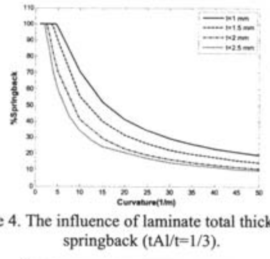

Figure 4. The influence of laminate total thickness on springback (tAl/t=1/3).

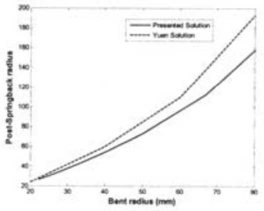

Figure 5. A comparison between the presented solution and Yuen model[1]

Figure 5 shows a comparison between the presented solution for power-law hardening material model and the elastic-perfectly plastic Yuen model [1]. As it is observed clearly, for various bend radii the obtained results from the presented model is followed by Yuen solution and have similar trends to each other. Generally there is a good agreement between the presented model and Yuen results.

Conclusions

For laminates consisting of aluminum and stainless steel the present findings are summarized as follows:

The springback behavior of unsymmetric laminates is strongly affected by the layer thickness ratio, bent curvature and total laminate thicknesses, against the symmetric laminates, neutral surface movement is observed in unsymmetric laminates during forming,

References

[1] Yuen, W. Y. D., 1996, "A generalized solution for prediction of springback in laminated strip", *Materials Processing Technology,* **61,** 254-264.

[2] Kadkhodayan, M. and Zohourkari, I., 2007, "An analytical solution to predict springback in sheet metal laminates", *Proceedings of the Conference on Applications and Design in Mechanical Engineering,* Kangar, Perlis, Malaysia, Paper 68.

[3] Verma, R.K. and Haldar, A., 2007, "Effect of normal anisotropy on springback", *Materials Processing Technology,* **190,** 300-304.