Numerical Solution of the Cavitation over Axisymmetric Bodies using the Boundary Element Method Based on Potential

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ABSTRACT: *In this paper, supercavitation and partial cavitation over axisymmetric bodies have been solved, using the Boundary Element Method (BEM), based on potential. In this method, the cavity and the wetted surface of the body will be estimated by some panels. Then, the cavitation will be modeled, by means of Green's third identity integral. For this purpose, the rings of the sources are distributed on the cavity surface, and the ring of the dipoles is distributed on the body and the cavity surface. The high velocity and also proper accuracy in calculating the geometry of the cavity and the drag coefficient are considerable advantages of this method.*

1. INTRODUCTION

Cavitation is recognized as an inadvisable problem in most phenomena, but in some circumstances, cavitation is remarked as a beneficial problem. The most important example is the submerged projectiles, in which cavitation is desired because of intense decrease in drag force. The dimensionless parameter which is represented for introducing cavitation is the cavitation number, which is defined as below:

$$\sigma = \frac{p_\infty - p_v}{1/2\rho V_\infty^{\ 2}} \tag{1}$$

Where p_∞ is atmosphere pressure, p_v is vapor pressure, ρ is the fluid density, and V_∞ is the fluid velocity. If bodies move with relatively high velocities inside fluids, cavitation starts at a point in which its local pressure reaches fluid vapor pressure. In low velocities or in high cavitation numbers, cavity is closed over the body and is called partial cavitation. With increase in velocity and decrease in cavitation number, cavity grows and covers all the body, which is called supercavitation.

Early studies of cavitation were performed by Efors and Tulin, using theoretical methods. Cavitation stream can also be solved with boundary element method (BEM). In this method, a distribution of potential flow elements (vortex, source, sink, doublet and dipole) is located over the boundary of the flow. In 1993, Fine and Kinnas devised a nonlinear boundary element method based on potential for solving partial cavitation flow over a hydrofoil [2]. Partial cavitation flow over torpedoes was conducted by Uhlman et al [2], using BEM method, and source and dipole distribution over body surface and cavity in 2003. All of the performed studies are limited to the specific geometrics, but in this paper, partial cavitation and supercavitation have been studied, using the boundary element method on the different bodies.

2. GOVERNING EQUATIONS

All of the parameters have become non – dimensional, by the fluid density, the diameter of the cavitator and the velocity of the free stream. Governing equation on the field of the flow is the Laplace equation:

$$\nabla^2 \Phi = 0 \tag{2}$$

The total potential (Φ) is the sum of the disturbance potential (φ) and the free stream potential.

$$\Phi = x + \varphi \tag{3}$$

The disturbance potential satisfies the Laplace equation, and also Green's third identity integral. Therefore, the potential in any points on the body surface and the cavity is obtained from the following equation:

$$2\pi\varphi(\mathrm{r},\mathrm{x}) = \oiint_{s} \left\{ \frac{\partial \varphi}{\partial \mathrm{n}} \mathrm{G}(\mathrm{x},\mathrm{r};\xi,\mathrm{R}) - \varphi(\mathrm{r},\mathrm{x}) \frac{\partial \mathrm{G}(\mathrm{x},\mathrm{r};\xi,\mathrm{R})}{\partial \mathrm{n}} \right\} \mathrm{R} d\varphi ds \tag{4}$$

$\mathrm{G}(\mathrm{x},\mathrm{r};\xi,\mathrm{R})$ Is the potential function related to the fluid sources distributed along a ring of radius *R* located on the axis at $x = \zeta$ .This equation states that the potential flow on any surface can be shown by means of the ring distribution of sources and dipoles. For this purpose, the rings of the sources is distributed on the cavity surface, and also the rings of the dipoles is distributed on the body and the cavity surface (Fig.1)

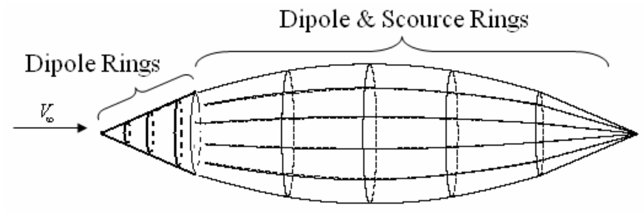

Figure 1. Applying superposition of the free stream, with distributions of the dipoles and sources rings on the interface of the body and the cavity to solve cavitation.

3. BOUNDARY CONDITIONS

Applying Bernoulli equation, the relation between the total velocity on the cavity surface, and the cavitation number can be obtained:

$$\mathrm{U}_c = \sqrt{1+\sigma} \tag{5}$$

This is called the dynamic boundary condition. The kinematic boundary condition states that the flow does not have any vertical component on the body and the cavity surfaces.

$$\frac{\partial \Phi}{\partial \mathrm{n}} = 0 \qquad \text{or} \qquad \frac{\partial \varphi}{\partial \mathrm{n}} = -\mathrm{n}_x \tag{6}$$

4. B.E.M BASED on POTENTIAL

In this method, the body and the cavity surfaces are respectively estimated by N_c and N_b number of the elements, which totally form N elements on the aforementioned surfaces.

By discretization the equation (4), and applying it on the surfaces of the body and the cavity, N number of the algebraic equation is obtained. The unknowns include: N_b number of dipole strengths on the body surface, N_c number of source strengths on the cavity surface, and a cavitation number of $\sqrt{1+\sigma}$.

Therefore, the numbers of the unknowns are N+1, which is one more than the number of the equations. In order to resolve this problem and also solving the system of equations, an auxiliary equation is needed. To obtain this equation, the definition which states that the algebraic sum of the sources powers on the cavity surface must be equal to zero, is used. The integrated form of this equation is as the following:

$$\iint_{S_c} \frac{\partial \varphi}{\partial \mathrm{n}} ds = \iint_{S_b} \mathrm{X}_n ds \tag{7}$$

5. RESULTS

In figure (2), the supercavitation behind of a disk cavitator with the cavity length of 6 (nondimensional length) is shown. To investigate the accuracy of the boundary element method in modeling the cavitation, the Dimensionless cavity length, drag coefficient and Dimensionless cavity diameter versus the cavitation number is compared with the experimental results of the reference [3], and Uhlman boundary element results [2] in figure(3) up to (5). It can be seen that the boundary element method predicts the length of the cavity region with good accuracy. As it was mentioned, one of the advantages of the cavitation is decreasing the drag coefficient. As it is shown in figure (5), the drag coefficient reduces by decreasing the cavitations number or increasing the cavity length, and also the results of the boundary element method have a good coincidence with the experimental results. In figure (6), the supercavitation behind of a conical cavitator with the cavity length of 6 (nondimensional length) is shown. In figure (7) the drag coefficient versus cavitation number for conical cavitator is compare with the experimental data [4]. To represent the capability of boundary element method in modeling the cavitation over more complex geometries, the partial cavitation over a type of torpedo in the non – dimensional length of 40 is shown in figure (8) and (9).

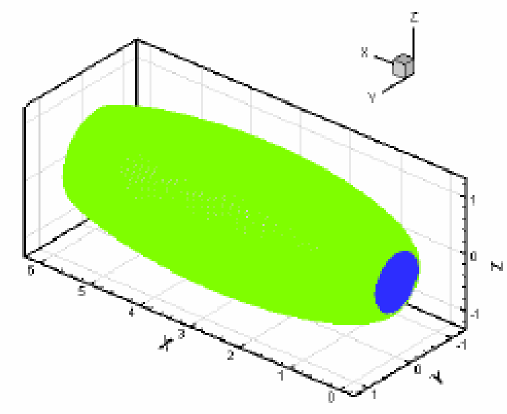

Figure 2. Supercavitation behind the disk cavitator, $l_{cav}/D_{disk} = 6\ ,\sigma = 0.235$

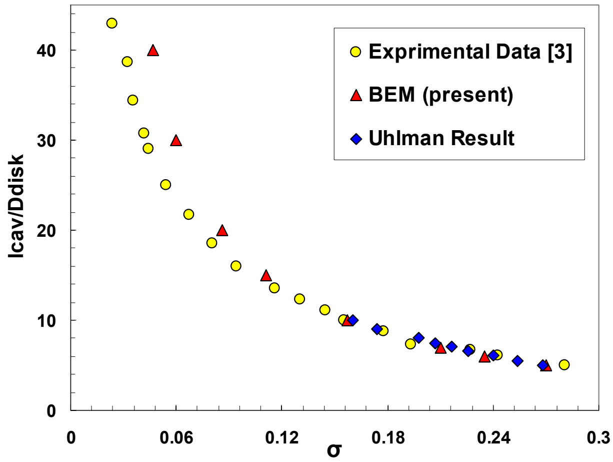

Figure 3. Dimensionless cavity length versus the cavitations number for a disk cavitator.

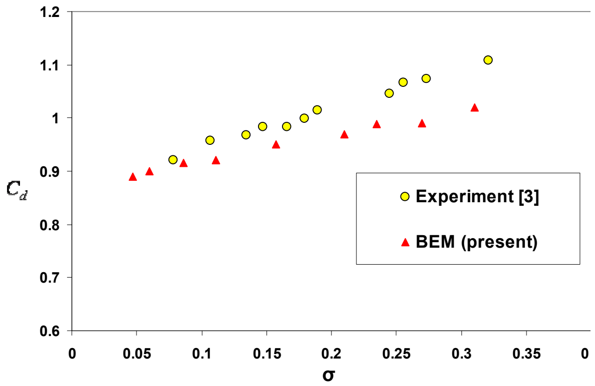

Figure 4. The drag coefficient versus the cavitations number for a disk cavitator

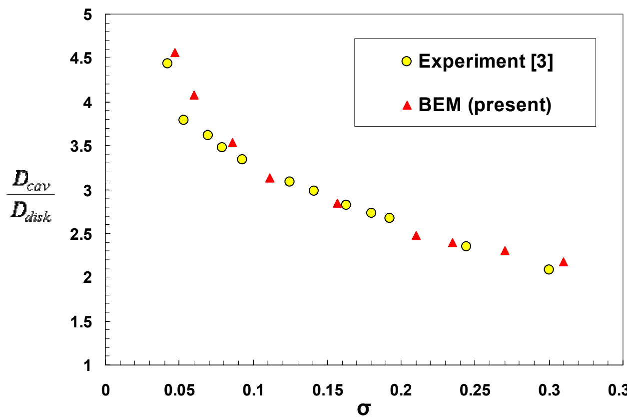

Figure 5. Dimensionless cavity diameter versus the cavitations number for a disk cavitator

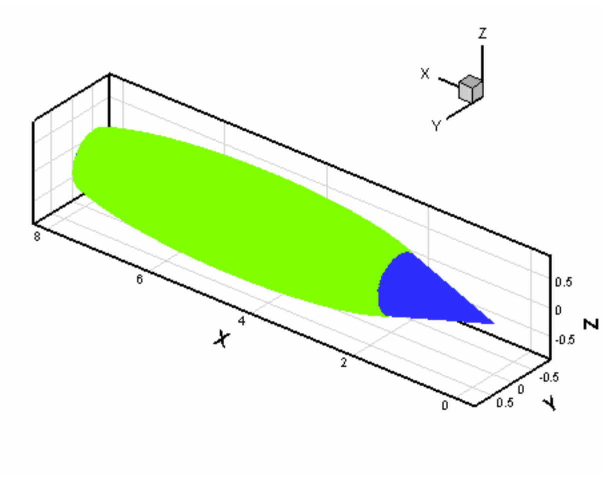

Figure 6. Supercavitation behind the conical cavitator, cone angle=30°,° $l_{cav} / D_{disk} = 6$, $\sigma = 0.1$

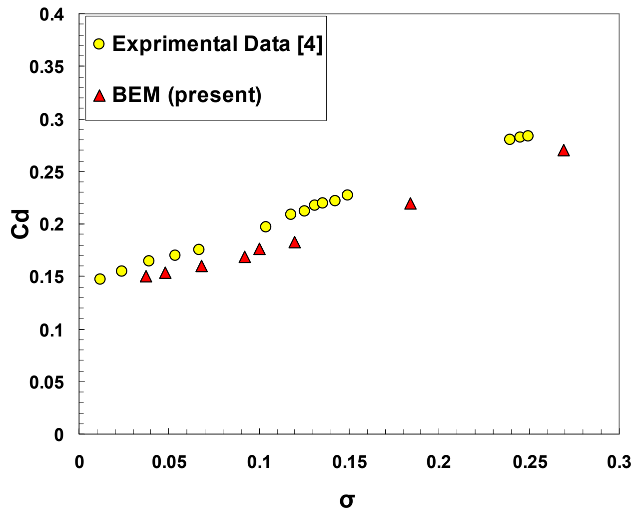

Figure 7. The drag coefficient versus the cavitations number for a conical cavitator with cone angle=30°.

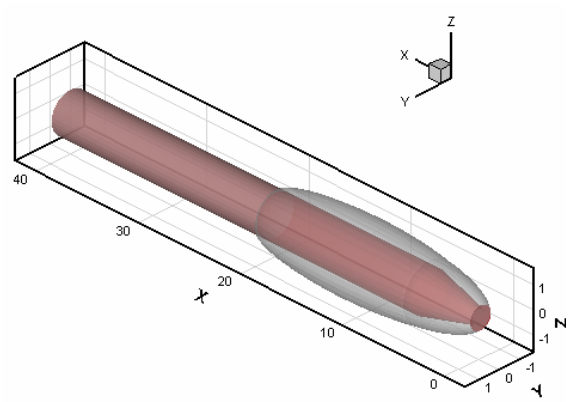

Figure 8. The partial cavitation over the torpedo in non – dimensional length of 20 and $\sigma = 0.075$

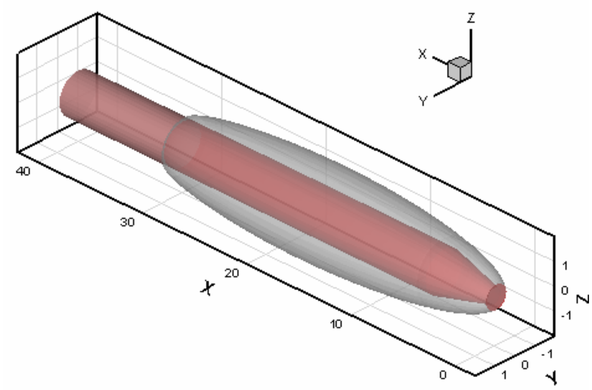

Figure 9. The partial cavitation over the torpedo in non – dimensional length of 30 and $\sigma = 0.055$

6. CONCLUSIONS

The results state the preciseness of the boundary element method in modeling supercavitation, and also show relatively good results for the partial cavitations around the axisymmetric bodies. The high velocity and proper accuracy in calculating the cavity geometry and the drag coefficient are the considerable advantages of this method.

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