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A novel approach in integral solution of laminar natural convection induced by a line heat source

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Abstract
In this study, the natural convection induced by a line heat source is investigated. Boundary layer and energy equations are solved analytically using integral method Since the number of equations is less than number of unknown variables; a new additional equation is applied. This equation is obtained by applying momentum equation on the centerline of plume. Some different velocity and temperature profiles are used. Results show that The Gaussian profile in comparison with other studied profiles has a better agreement with the results of similarity approach.
Keywords: integral method, natural convection, heat transfer, plume, laminar

Introduction

Natural convection induced by a heat sources in an infinite fluid space is relevant in many engineering applications. In particular, laminar natural convection generated around the horizontal line heat source, and from heated circular cylinders and it has been extensively investigated numerically and experimentally. Bejan [1] presented an integral solution for turbulence natural plume. The applied solution is based on some simplifying assumptions and is dependent to experimental data. Fujii [2] solved the two-dimensional boundary layer equations for *Pr* of 0.01, 0.7, 2 and 10 by using similarity approach, which reduces the set of four partial differential equations into two ordinary differential equations. Ayani et al. [3] investigate the effect of radiation on laminar natural convection induced by a line heat source and concluded a considerable departure from the Boussinesq-based solution and from the boundary layer results. All the additional equations applied in the analytical solutions are strongly dependent to experimental data and some simplifying assumptions. In this paper, the additional equation is based on the physics of the flow and obtained using the governing equations of the problem. In addition, no more simplifying assumptions are applied.

Governing Equations

Laminar natural convection flow from a horizontal line heat source, assuming the end effects of the source, is negligible, is governed by continuity equation, the two-dimensional Navier-Stocks equations using Boussinesq approximation and the energy equation [3]. In order to obtain the integral form of governing equations, they are Integrated from y=0 up to y=Y (Figure 1). It is known that $v|_{y=0}$ and $u|_{y=\infty}$ is equal to zero. Also due to symmetry, all the derivatives on the centerline are zero, that is $\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$, $\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0$. Applying the mentioned conditions lead to the following equations.

$$\frac{d}{dx}\left(\int_0^Y u dy\right) = -v_\infty \tag{1}$$

$$\frac{d}{dx}\left(\int_0^Y u^2 dy\right) = \int_0^Y g\beta(T - T_\infty)dy \tag{2}$$

$$\int_0^Y u(T - T_\infty)dy = \frac{q'}{\rho C_p} \tag{3}$$

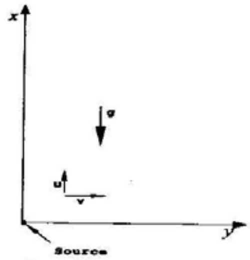

Figure 1- problem under consideration

Temperature and velocity profiles

In pursuit of solving the integral form of the governing equations, proper profiles for both velocity and temperature fields should be guess. Gaussian, exponential, polynomial and sinusoidal profiles are applied. For example The Gaussian profiles are presented below:

$$u = u_c exp\left(-\left(\frac{y}{b}\right)^2\right) \tag{4}$$

$$(T - T_\infty) = (T_c - T_\infty) exp\left(-\left(\frac{y}{b_T}\right)^2\right) \tag{5}$$

In which b and b_T are hydrodynamic and thermal boundary layer thicknesses, respectively. It is assumed

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In this study the natural convection induced by a line heat source is investigated. Boundary layer and energy equations are solved analytically using integral method. Since the number of equations is less than the number of unknown variables; a new additional equation is applied. This equation is obtained by applying momentum equation on the center line of the plume. Some different velocity and temperature profiles are used. Results show that The Gaussian profile in comparison with other studied profiles has a better agreement with the results of similarity approach.
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Introduction
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Linan and Kurdymov [6] studied numerically the laminar free convection induced by a line heat source at small Grashof numbers, using the Boussinesq equations, in stream function-vorticity $(\psi-\omega)$ variables. Wang et al. [7] studied the transient laminar natural convection from horizontal cylinders, and they used the $(\psi-\omega)$ equations. Ayani et al. [8] investigate the effect of radiation on laminar natural convection induced by a line heat source and concluded a considerable departure from the Boussinesq-based solution and from the boundary layer results.
All the additional equations applied in the analytical solutions are strongly dependent to experimental data and some simplifying assumptions. In this paper, the additional equation is based on the physics of the flow obtained using the governing equations of the phenomena. In addition, no more simplifying assumptions are applied.

Governing Equations
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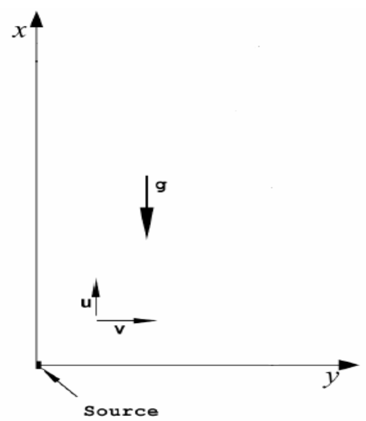

Figure 1. Problem under consideration

Based on the physical configuration shown in Figure 1, the governing equations in the Cartesian coordinate system take the following form :

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

In order to obtain the integral form of the governing equations, they are integrated from y=0 up to y=Y (Figure 1).

$$\int_0^Y \frac{\partial u}{\partial x}dy+\int_0^Y \frac{\partial v}{\partial y}dy=0 \tag{4}$$

$$\int_0^Y \frac{\partial u^2}{\partial x}dy+\int_0^Y \frac{\partial (uv)}{\partial y}dy=\int_0^Y \nu\frac{\partial^2 u}{\partial y^2}dy+\int_0^Y g\beta(T-T_\infty)dy \tag{5}$$

$$\int_0^Y \frac{\partial (uT)}{\partial x}dy+\int_0^Y \frac{\partial (vT)}{\partial y}dy=\int_0^Y \alpha\frac{\partial^2 T}{\partial y^2}dy \tag{6}$$

It is known that $v\big|_{y=0}$ and $u\big|_{y=\infty}$ are equal to zero. Also due to symmetry, all the derivatives on the centerline are zero, that is $\left(\partial u/\partial y\right)_{y=0}=0$ and $\left(\partial T/\partial y\right)_{y=0}=0$. Applying the mentioned conditions lead to the following equations.

$$\frac{d\left(\int_0^Y u\,dy\right)}{dx}=-v_\infty \tag{7}$$

$$\frac{d\left(\int_0^Y u^2 dy\right)}{dx}=\int_0^Y g\beta(T-T_\infty)dy \tag{8}$$

$$\int_0^Y u(T-T_\infty)dy=\frac{q'}{\rho C_P} \tag{9}$$

Boundary conditions

The flow is symmetric about a vertical plane passing through the axis of the heat source (Figure 1). The boundary conditions for the symmetry plane (y = 0) are as follows:

$$y=0 \;\;\Rightarrow T=T_c,\, u=u_c,\, \frac{\partial T}{\partial y}=0,\, \frac{\partial u}{\partial y}=0 \tag{10-a}$$

The other boundaries are located relatively far away from the heat source, and the pressure is assumed to have a constant value.

$$y\to\infty \;\;\Rightarrow T=T_\infty,\, u=0,\, \frac{\partial T}{\partial y}=0,\, \frac{\partial u}{\partial y}=0 \tag{10-b}$$

In which u_c and T_c are centerline velocity and centerline temperature respectively.

Temperature and velocity profiles

In pursuit of solving the integral form of the governing equations, proper profiles for both velocity and temperature fields should be guessed. Gaussian, exponential, polynomial and sinusoidal profiles are applied. It should be noted that for polynomial and sinusoidal profiles the latter boundary condition for relatively far points from the heat source turns to following boundary condition:

$$y=b \;\;\Rightarrow T=T_\infty,\, u=0,\, \frac{\partial T}{\partial y}=0,\, \frac{\partial u}{\partial y}=0 \tag{10-c}$$

In the solution procedure, Pr is assumed to be equal to 1 which is an acceptable assumption in the case of air as the understudy fluid. Therefore, thickness of hydrodynamic boundary layer (b) can be taken to be equal to the thickness of thermal boundary layer (b_T).

A- Gaussian profile:

Applying the boundary conditions, the Gaussian profile takes the following form:

$$u=u_c e^{\left(-\left(\frac{y}{b}\right)^2\right)} \tag{11}$$

$$(T-T_\infty)=(T_c-T_\infty)\,e^{\left(-\left(\frac{y}{b_T}\right)^2\right)} \tag{12}$$

Substituting the Gaussian profiles in the equations 7 to 9 leads to the following relations:

$$\frac{\sqrt{\pi}}{2}\frac{d(u_c b)}{dx}=-v_\infty \tag{13}$$

$$\frac{\sqrt{2\pi}}{4}\frac{d(u_c^2 b)}{dx}=\frac{\sqrt{\pi}}{2}g\beta(T_c-T_\infty)b \tag{14}$$

$$\frac{\sqrt{2\pi}}{4}u_c b(T_c-T_\infty)=\frac{q'}{\rho C_p} \tag{15}$$

B- Third order polynomial:

Applying the boundary conditions, the polynomial velocity and temperature profiles take the form below:

$$u=u_c\left(2\left(\frac{y}{b}\right)^3-3\left(\frac{y}{b}\right)^2+1\right) \tag{16}$$

$$T=(T_c-T_\infty)\left(2\left(\frac{y}{b}\right)^3-3\left(\frac{y}{b}\right)^2+1\right) \tag{17}$$

Following the same procedure done for the Gaussian profile the below relation will be obtained for the third order polynomial:

$$\frac{1}{2}\frac{d(u_c b)}{dx}=-v_\infty \tag{18}$$

$$\frac{13}{35}\frac{d(u_c^2 b)}{dx}=\frac{1}{2}g\beta(T_c-T_\infty)b \tag{19}$$

$$\frac{13}{35}u_c b(T_c-T_\infty)=\frac{q'}{\rho C_p} \tag{20}$$

C- Sinusoidal profile:

$$u=u_c\left[\left(1-\frac{y}{b}\right)\cos(\frac{\pi}{b}y)+\frac{1}{\pi}\sin(\frac{\pi}{b}y)\right] \tag{21}$$

$$T - T_\infty = (T_c - T_\infty)\left[\left(1 - \frac{y}{b}\right)\cos\left(\frac{\pi}{b}y\right) + \frac{1}{\pi}\sin\left(\frac{\pi}{b}y\right)\right] \tag{22}$$

Substituting in the equations 7 to 9, following relations will obtain:

$$0.4053\frac{\partial}{\partial x}\left[u_c b\right] = -v_\infty \tag{23}$$

$$0.894\frac{\partial}{\partial x}\left[u_c^2 b\right] = g\beta(T_c - T_\infty)b \tag{24}$$

$$0.362u_c(T_c - T_\infty)b = \frac{q'}{\rho C_p} \tag{25}$$

D- Exponential profile:

$$u = u_c e^{\left(-y/b\right)}\left(2 - e^{\left(-y/b\right)}\right) \tag{26}$$

$$T - T_\infty = (T_c - T_\infty)e^{\left(-y/b\right)}\left(2 - e^{\left(-y/b\right)}\right) \tag{27}$$

after substituting in integral form of governing equations, we have:

$$1.5\frac{\partial}{\partial x}\left[u_c b\right] = -v_\infty \tag{28}$$

$$\frac{11}{18}\frac{\partial}{\partial x}\left[u_c^2 b\right] = g\beta(T_c - T_\infty)b \tag{29}$$

$$\frac{11}{12}u_c(T_c - T_\infty)b = \frac{q'}{\rho C_p} \tag{30}$$

Solution approach

As done in the pervious step, initially the guessed profiles for temperature and velocity fields are replaced in integral form of the governing equations. This leads to three equations in which u_c, $(T_c - T_\infty)$, b and v_∞, that is, centerline velocity, centerline temperature difference, boundary layer thickness and entrainment velocity, respectively are unknowns. These variables are assumed to have the following form:

$$u_c = Ax^m \tag{31}$$

$$(T_c - T_\infty) = Bx^n \tag{32}$$

$$b = Cx^p \tag{33}$$

$$v_\infty = Ex^l \tag{34}$$

As could be seen there is 4 unknowns and only 3 equations. Therefore, another additional equation is required.

Substituting these relations in latter equations obtained from replacing the guessed profiles in the equations 7 to 9.

the subsequent Non-linear equations will be gained :

A- Gaussian profile:

$$\frac{\sqrt{\pi}}{2}(m+p)ACx^{m+p-1} = -Ex^l \tag{35}$$

$$\frac{\sqrt{2}}{2}(2m+p)A^2x^{2m+p-1} = g\beta Bx^{n+p} \tag{36}$$

$$\frac{\sqrt{2\pi}}{4}ABCx^{m+n+p} = \frac{q'}{\rho C_P} \tag{37}$$

B- Third order polynomial:

$$\frac{1}{2}(m+p)ACx^{m+p-1} = -Ex^l \tag{38}$$

$$\frac{13}{35}(2m+p)A^2x^{2m+p-1} = \frac{1}{2}g\beta Bx^{n+p} \tag{39}$$

$$\frac{13}{35}ABCx^{m+n+p} = \frac{q'}{\rho C_P} \tag{40}$$

C- Sinusoidal profile:

$$0.4053(m+p)ACx^{m+p-1} = -Ex^l \tag{41}$$

$$0.894(2m+p)A^2x^{2m+p-1} = g\beta Bx^{n+p} \tag{42}$$

$$0.362ABCx^{m+n+p} = \frac{q'}{\rho C_P} \tag{43}$$

D- Exponential profile:

$$1.5(m+p)ACx^{m+p-1} = -Ex^l \tag{44}$$

$$\frac{11}{18}(2m+p)A^2x^{2m+p-1} = g\beta Bx^{n+p} \tag{45}$$

$$\frac{11}{12}ABCx^{m+n+p} = \frac{q'}{\rho C_P} \tag{46}$$

Since the number of equations are less than the number of unknown variables, therefore an additional equation is needed which could be obtained by applying the momentum equation on the centerline.

$$u_c\left.\frac{\partial u}{\partial x}\right|_{y=0} + v_c\left.\frac{\partial u}{\partial y}\right|_{y=0} = \nu\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} + g\beta(T_c - T_\infty) \tag{47-a}$$

Knowing $\left.v\right|_{y=0} = 0$ at the centerline the additional equation reduce to the following equation:

$$u_c\left.\frac{\partial u}{\partial x}\right|_{y=0} = \nu\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} + g\beta(T_c - T_\infty) \tag{47-b}$$

This equation shows the balance of inertia term along the perpendicular plane to the plume by diffusion in y direction and buoyancy term. These terms serve as the main basis in forming a plume.

Applying the assumption for u_c, $T_c - T_\infty$ and b in the latter equation lead to an additional equation which serves as an equation to complete the set of four equations and 4 unknowns.

A- Gaussian profile:

$$\overbrace{A^2mx^{2m-1}}^{Inertia} = \overbrace{\frac{-2\nu Ax^{m-2p}}{C^2}}^{Friction} + \overbrace{g\beta Bx^n}^{Buoyancy} \tag{48}$$

B- third order polynomial:

$$A^2mx^{2m-1} = \frac{-6\nu Ax^{m-2p}}{C^2} + g\beta Bx^n \tag{49}$$

C- Sinusoidal profile:

$$A^2 m x^{2m-1} = \frac{-\pi^2 \nu A x^{m-2p}}{C^2} + g\beta B x^n \qquad (50)$$

D- Exponential profile:

$$A^2 m x^{2m-1} = \frac{-2\nu A x^{m-2p}}{C^2} + g\beta B x^n \qquad (51)$$

In order to solve the set of 4 equations, three conservation equations and one additional equation, powers and constants for every equation should be equal to each other. Applying the equalities, the following powers will obtain for all of the profiles:

$$\begin{cases} m + p - l = 1 \\ 2m - n = 1 \\ m + n + p = 0 \\ m - n - 2p = 0 \end{cases} \Rightarrow \begin{cases} m = 0.2 \\ n = -0.6 \\ p = 0.4 \\ l = -0.4 \end{cases} \qquad (52)$$

Till this step it is found that centerline velocity, centerline temperature difference, boundary layer thickness and entrainment velocity are polynomial functions of powers,0.2 , -0.6, 0.4 and -0.4, respectively.
Applying the equality of constants will lead to subsequent equations for each of the under study profiles and unknowns of the problem will be completely determined.
A- Gaussian profile:

$$\begin{cases} \frac{\sqrt{\pi}}{2}(m+p)\ A\ \ C = -E \\ \frac{\sqrt{2}}{2}(2m+p)A^2\ B^{-1} = g\beta \\ \frac{\sqrt{2\pi}}{4}\ A\ B\ C = \frac{q'}{\rho C_P} \\ 2\nu\ A\ \ B^{-1} C^{-2} + A^2 m B^{-1} = g\beta \end{cases} \qquad (53)$$

Solving equation 53 and applying the powers obtained from equation 52 determine the four unknowns and subsequently velocity field and temperature field will be concluded.

$$\begin{cases} u_c = 0.817(g\beta)^{0.4}\left(\frac{q'}{\rho C_P}\right)^{0.4} \nu^{-0.2} x^{0.2} \\ (T_c - T_\infty) = 0.377(g\beta)^{-0.2}\left(\frac{q'}{\rho C_P}\right)^{0.8} \nu^{-0.4} x^{-0.6} \\ b = b_T = 2.59(g\beta)^{-0.2}\left(\frac{q'}{\rho C_P}\right)^{-0.2} \nu^{0.6} x^{0.4} \\ v_\infty = -1.125(g\beta)^{0.2}\left(\frac{q'}{\rho C_P}\right)^{0.2} \nu^{0.4} x^{-0.4} \end{cases} \qquad (54)$$

Subsequently for third order polynomial, sinusoidal and exponential profiles we have:
B- third order polynomial profile:

$$\begin{cases} u_c = 1.06(g\beta)^{0.4}\left(\frac{q'}{\rho C_P}\right)^{0.4} \nu^{-0.2} x^{0.2} \\ (T_c - T_\infty) = 0.67(g\beta)^{-0.2}\left(\frac{q'}{\rho C_P}\right)^{0.8} \nu^{-0.4} x^{-0.6} \\ b = b_T = 3.8(g\beta)^{-0.2}\left(\frac{q'}{\rho C_P}\right)^{-0.2} \nu^{0.6} x^{0.4} \\ v_\infty = -1.2084(g\beta)^{0.2}\left(\frac{q'}{\rho C_P}\right)^{0.2} \nu^{0.4} x^{-0.4} \end{cases} \qquad (55)$$

C- Sinusoidal profile:

$$\begin{cases} u_c = 0.951(g\beta)^{0.4}\left(\frac{q'}{\rho C_P}\right)^{0.4} \nu^{-0.2} x^{0.2} \\ (T_c - T_\infty) = 0.647(g\beta)^{-0.2}\left(\frac{q'}{\rho C_P}\right)^{0.8} \nu^{-0.4} x^{-0.6} \\ b = b_T = 4.49(g\beta)^{-0.2}\left(\frac{q'}{\rho C_P}\right)^{-0.2} \nu^{0.6} x^{0.4} \\ v_\infty = -1.04(g\beta)^{0.2}\left(\frac{q'}{\rho C_P}\right)^{0.2} \nu^{0.4} x^{-0.4} \end{cases} \qquad (56)$$

D- Exponential profile

$$\begin{cases} u_c = 0.936(g\beta)^{0.4}\left(\frac{q'}{\rho C_P}\right)^{0.4} \nu^{-0.2} x^{0.2} \\ (T_c - T_\infty) = 0.428(g\beta)^{-0.2}\left(\frac{q'}{\rho C_P}\right)^{0.8} \nu^{-0.4} x^{-0.6} \\ b = b_T = 2.724(g\beta)^{-0.2}\left(\frac{q'}{\rho C_P}\right)^{-0.2} \nu^{0.6} x^{0.4} \\ v_\infty = -2.29(g\beta)^{0.2}\left(\frac{q'}{\rho C_P}\right)^{0.2} \nu^{0.4} x^{-0.4} \end{cases} \qquad (57)$$

Results Discussion

In order to verify the results and also in order to determine the best profile which could explain the plume phenomena best, results are compared with results obtained from semi-analytical solution of Fujii [2].
For the fluid flowing over plume which is considered to be air, the properties are determined for the ambient temperature of $T_\infty = 20°C$. Other properties are presented in table 1.

Table1-properties considered for air and heat source

$q'(W/m)$	$\nu(m^2/s)$	$g\beta(m/(K.s^2))$	$C_p(J/(kg.K)$	$\rho(kg/m^3)$
70	0.15×10^{-4}	3.338×10^{-2}	1006	1.205

In order to compare the solutions obtained from fully analytical procedure with fujii results, results are presented in terms of similarity parameters, which are defined as below:

$$\xi = Gr^{1/5}\, y/x \qquad Gr = \frac{g\beta x^3 \theta_0}{\nu^2}$$
$$\theta_0 = \frac{q'}{C_p\,\mu} \qquad T - T_\infty = Gr^{-1/5}\theta_0 h(\xi) \qquad (58)$$
$$\frac{ux}{\nu} = Gr^{2/5} f' \qquad \frac{vy}{\nu} = -\xi\left(\frac{3}{5} f - \frac{2}{5}\xi f'\right)$$

Figure 2 presents the variation of centerline velocity which was shown to be a polynomial function of power 0.2.
It should also note that centerline velocity is a function of expansion coefficient(β), source power(q') and also diffusivity coefficient(α). The Gaussian profile agrees well with the Fujii results and exponential, polynomial and sinusoidal, respectively, deviate more and more from Fujii results. For the case of sinusoidal and polynomial profiles, the second boundary condition for far away borders is somehow a non-physical condition, therefore these profiles are not proper for precise capture of the parameters derived from this phenomena. In the case of temperature difference it treats as a polynomial function of power, -0.6. That is the centerline temperature tends to ambient temperature as ξ increases. Also it should be mentioned that the temperature difference just like the centerline velocity is a function of expansion coefficient(β), source power(q') and also Diffusivity coefficient(α). Figure 3 presents the variation of centerline temperature difference versus ξ.
Variation of similarity functions, f, f' and h with respect to similarity parameter, ξ is compared with the results of integral method and are presented in figure 4, 5 and 6, respectively.
As could be seen in these figures Gaussian profile is the best profile among others. Also it should be noted that other profiles over predict the similarity functions f, f' and h.

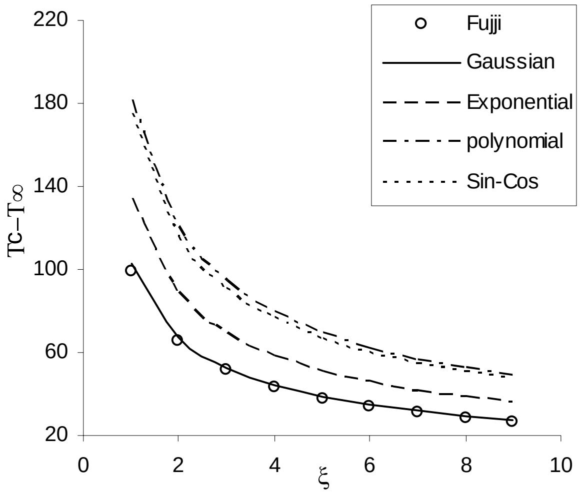

Figure3-Centerline temperature difference versus ξ

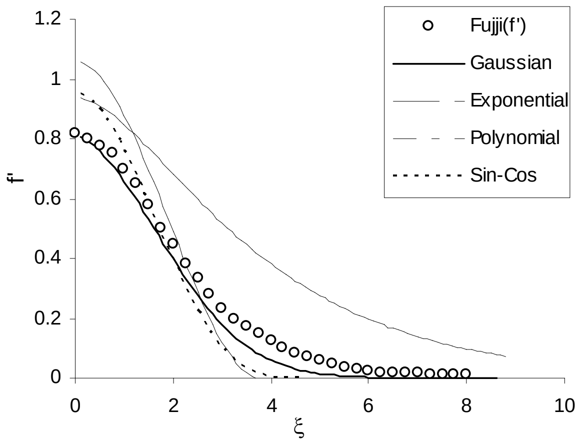

Figure4- variation of f' versus ξ

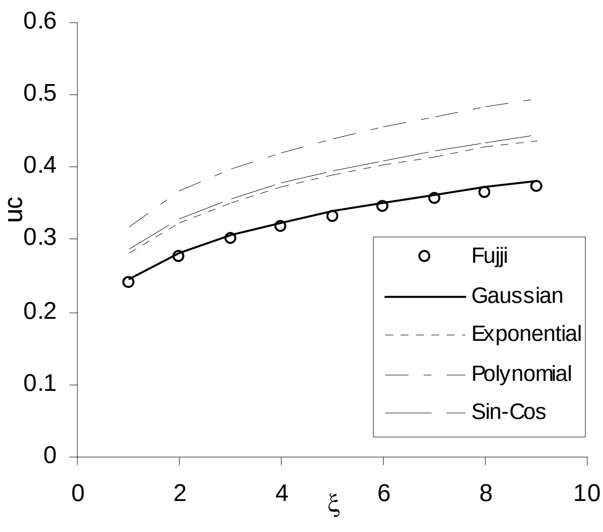

Figure2- centerline velocity variation versus ξ

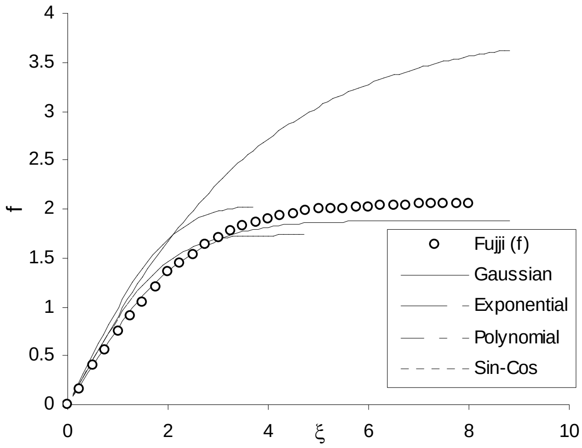

Figure5- variation of f versus ξ

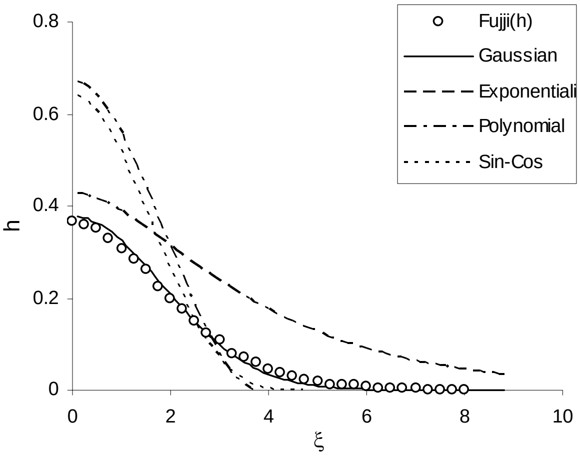

Figure6- variation of h versus ξ

Figure 7 presents the similarity parameters for Gaussian profile with respect to Fujii results. In this figure it is obviously shown that the Gaussian profile obtained from full analytical procedure discussed in this article, agrees well with Fujii results and doubtlessly could predict laminar plume induced by a line heat source.

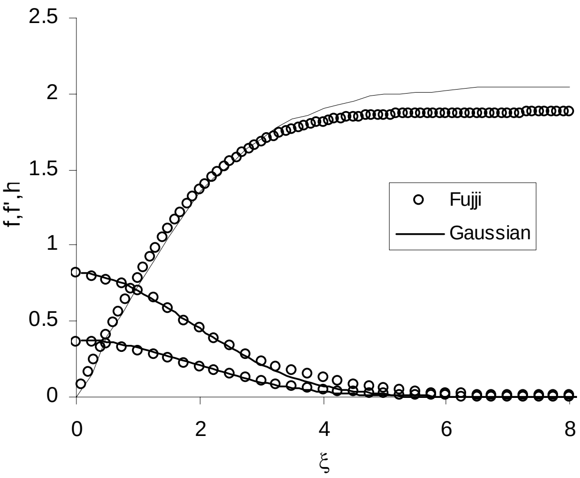

Figure7- variation of f, f' and h versus ξ

Conclusion

In this paper, natural convection induced by a line heat source is investigated using integral method. Prior works mostly treat this phenomenon numerically or semi-analytically. There are some analytical solutions which are strongly dependent to experiment results. In this paper the solution to this problem is obtained in fully analytical procedure applying a new additional equation. The additional equation is obtained applying the momentum equation at centerline of plum.

Results of this fully analytical solution are compared with the similarity results obtained by Fujii [2]. Results show the Gaussian profile could predict this phenomenon appropriately and it has a higher accuracy with respect to other profiles. It should be noted that other profiles, nearly for all cases, over predict the phenomenon and desperately deviate from Fujii results.

References:

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