DYNAMICS RESPONSE OF FUNCTIONALLY GRADED THICK HOLLOW CYLINDER UNDER DYNAMIC LOADING

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SUMMARY: This article presents the analysis of functionally graded hollow cylinder under dynamic load. The functionally graded cylinder is assumed to be made from many subcylinders. Each subcylinder is considered as an isotropic layer. Material's properties in each layer are constant and functionally graded properties are resulted by suitable arranging of layers in multilayer cylinder. The properties are controlled by volume fraction that is exponential function of radius. The shell is assumed to be in plane strain condition, and is subjected to axisymmetric dynamic loading. The Navier equation is solved by Galerkin finite element and Newmark methods. In each interface between two layers, stress and displacement continuity are satisfied. By using the fast Fourier transform (FFT), the time response is transferred to the frequency domain and natural frequencies found, then the dynamic behavior of functionally graded thick hollow cylinder is discussed. Finally the functionally graded cylinder is assumed isotropic and the results are compared with analytical results.

KEYWORDS: Functionally graded materials, Thick hollow cylinder, Dynamic response, Multilayer.

INTRODUCTION

Functionally graded materials (FGMs) are new kind of materials. This material is spatial composite within which mechanical properties vary continuously in the macroscopic sense from one surface to the other. FGMs for use at high temperature are special composites that are usually made of ceramics and metals. In the recent years researchers have studied vibration and dynamic behavior of functionally graded materials and multiplayer cylindrical shells.

The static and dynamic thermoelastic responses of functionally graded material plates were studied by Praveen and Reddy [1]. This work was investigated by varying the volume fraction of the ceramic and metallic constituents using a simple power law distribution and numerical results for the deflection and stresses were presented. A study on the vibration of cylindrical shells made of a functionally graded material composed of stainless steel and nickel was presented by Loy et al [2]. Buckling and steady state vibrations of a simply supported functionally gradient isotropic polygonal plate resting on a Winkler-Pasternak elastic foundation and subjected to uniform in–plane hydrostatic loads were studied by Cheng and Batra [3]. They used Reddy's third order plate theory. A study on the vibration of functionally graded cylindrical shell was presented by Pradhan et al [4]. In this work, Love's theory and Rayleigh method were employed for the analysis and natural frequencies were computed for various constitutive volume fractions. Analyzing transient waves was developed in a cylinder

made of FGM by using a hybrid numerical method [5]. A computational method was presented to investigate SH wave in functionally graded material plates by Han and Liu [6]. The material properties were assumed as a quadratic function in the thickness direction. The transient heat conduction in a strip of a functionally graded material with continuous and piecewise differentiable properties was carried out by Jin [7]. In this work a multi–layered material model was first used to obtain the Laplace transforms of temperature at the interfaces between the layers.

The free vibration of simply supported, fluid–filled, cylindrically orthotropic functionally graded cylindrical shells with arbitrary thickness was investigated by Chen et al [8]. In this paper the effects of related parameters on natural frequencies were discussed. The thermoelasticity problems of hollow cylinder whose boundaries are subjected to time–dependent temperatures and pressures were discussed by Lee [9]. The two–dimensional quasi–static axisymmetric coupled thermoelastic problem of a finitely long hollow cylinder was discussed in this paper.

This article presents the analysis of functionally graded hollow cylinder under dynamic load.

EQUATION OF MOTION AND FINITE ELEMENT MODELING

Consider a thick hollow cylinder of inside radius a and outside radius b made of FGM. The cylinder's material is graded through the r-direction. The shell is made of a combined ceramic-metal material, the mixing ratio of which is varied continuously and smoothly in the r-direction. The inner surface of shell is pure ceramic and outer surface is pure metal. The material distribution is shown by:

$$P = \left(P_m - P_c\right) \left(\frac{r-a}{b-a}\right)^n + P_c \tag{1}$$

Where P is material property, n is a non-negative volume fraction exponent and subscripts c and m stand for ceramic and metal. The material properties P are Young's modulus E and mass density ρ . The FGM cylinder is divided to m sub-cylinder (m layers) and it is assume that the non-homogeneous sub-cylinders are homogeneous. (Fig. 1)



Fig. 1: Schematic of layers

Figure 2. Distribution of modulus of elasticity in layers

The material properties of Jth layer are found by:

$$P = \left(P_m - P_c\right) \left[\frac{(J-1)t}{b-a}\right]^n + P_c$$
⁽²⁾

Where t is:

$$t = \frac{b-a}{m} \tag{3}$$

The change of material property with n, using Eqn.2, is shown in Fig. 2. For plane strain conditions (infinite length) and axisymmetry loading, the governing equation is:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_{\theta}}{r} = \rho \frac{\partial^2 u}{\partial t^2}$$
(4)

Where u is the displacement component in radial direction. Then the strain-displacement relations are:

$$\varepsilon_r = \frac{\partial u}{\partial r}$$

$$\varepsilon_{\theta} = \frac{u}{r}$$
(5)

The constitutive equations of each layer are stated as:

$$\sigma_r = c_1 \varepsilon_r + c_2 \varepsilon_{\theta}$$

$$\sigma_{\theta} = c_2 \varepsilon_r + c_1 \varepsilon_{\theta}$$
(6)

Where c_1 and c_2 are found by:

$$c_{1} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

$$c_{2} = \frac{E\nu}{(1+\nu)(1-2\nu)}$$
(7)

The governing equation in terms of displacement for each layer of cylindrical shell under axisymmetric load becomes:

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} = \frac{\rho}{c_1} \frac{\partial^2 u}{\partial t^2}$$
(8)

The boundary conditions on the inner and outer surfaces of the shell are taken as:

$$\sigma_r = F(t) \qquad \tau_{r\theta} = 0 \quad at the inner surface \qquad (9)$$

$$\sigma_r = 0 \qquad \tau_{r\theta} = 0 \quad at the outer surface$$

The continuity conditions to be enforced at any interface between two layers, can be written as:

$$(\sigma_r)_k = (\sigma_r)_{k+1}$$
(10)
$$(u)_k = (u)_{k+1}$$

The Galerkin method is used to obtain the finite element model of shell. Considering linear shape functions for the variable u as:

$$u = \prec N_1 \succ \{u\} \tag{11}$$

And applying the Galerkin method to the governing Eqn. 7, results into the following dynamic finite element equilibrium equation for each element:

$$[M]_{e} \{ \ddot{X} \} + [K]_{e} \{ X \} = \{ f \}_{e}$$
(12)

For nodes which are located at any interface between kth and (k+1)th layers as Fig.1, the continuity conditions are written as:

$$u_{kl} = u_{kl+1}$$
(13)
$$\sigma_{rkl} = \sigma_{r(kl+1)}$$
(14)

Deriving Eqn. 14 in terms of displacements and expressing the derivatives in backward and forward finite difference for kth and (k+1)th layers result:

$$c_1^k \frac{u_{kI} - u_{kI-1}}{h} + c_2^k \frac{u_{kI}}{R_k} = c_1^{k+1} \frac{u_{kI+2} - u_{kI+1}}{h} + c_2^{k+1} \frac{u_{kI+1}}{R_{k+1}}$$
(15)

Where h is thickness of element and R_k , R_{k+1} are mean radius of kth and (k+1)th layers. the subscript k stand for kth layer. The Eqn. 15 and Eqn. 13 are used in assembling of global stiffness and mass matrices. The global dynamic equilibrium equations for shell become:

$$[M] \{ \ddot{X} \} + [K] \{ X \} = \{ f \}$$
(16)

Once the finite element equilibrium equation is established, different numerical method can be employed to solve them in space and time domains. The Newmark direct integration method with suitable time step is used and the equilibrium equation is solved. To find the first natural frequency, the displacement responses are transferred to frequency domain by using the fast Fourier transform.

NUMERICAL RESULTS AND DISCUSSION

As an example, consider a thick hollow cylinder of inner radius a=0.25m and outer radius b=0.5m. The modulus of elasticity and the mass density at the inner radius (Alumina) are $E_c = 380Gpa$, $\rho_c = 3800 \frac{kg}{m^3}$ and the outer radius (Aluminum) are $E_m = 70Gpa$, $\rho_m = 2707 \frac{kg}{m^3}$. The loading equation is assumed as:

$$F(t) = P_0 t \qquad for \qquad t \le 0.005 \text{ sec}$$
(18)

$$F(t) = 0 \qquad for \qquad t \succ 0.005 \text{ sec}$$



Where P_0 is $P_0 = 4 \frac{GPa}{Sec}$. The shell is excited by unloading at t = 0.005 sec.

Fig. 3: Time history of the radial displacement at middle point of thickness for two values of n

Fig. 4: Time history of the radial displacement at middle point of thickness for two values of n

Fig. 3 shows the time history of the radial displacement at the middle point of thickness for two values of the power law exponent n.The power law exponent n influences conversely the amplitude of vibration. For large values of n, the amplitude will be constant. Fig. 4 shows the constant amplitude of vibration for large values of n. A comparison of the displacement responses between various values of the power law exponent n is presented in Fig. 3 and Fig. 4.





Fig. 5: Time history of radial displacement at r = 0.3125 m for n = 0.5

Fig. 6: Time history of radial displacement at r = 0.375 m for n = 0.5

The comparison of the displacement responses between various point across thickness in certain power law exponent (n = 0.5) are presented in Fig. 5 to Fig. 7.The amplitude decrease from inner surface (ceramic) to outer surface (metal).





Fig. 7: Time history of radial displacement at r = 0.4375 m for n = 0.5

Fig. 8: The displacement responses in frequency domain for various values of n

Fig. 8 shows the displacement responses in frequency domain. In this figure a comparison of first natural frequencies between various values of the power law exponent n is illustrated. The first natural frequencies are located at the peak of diagram. The natural frequency increase when the power law exponent n increase, table 1.

Table 1: First halural frequencies for various values of h				
n	0.01	0.5	5	20
First natural frequency (Hz)	2441.5	3906.2	4394.5	4394.5

Table 1: First natural frequencies for various values of n

Conclusion:

In this paper, functionally graded thick hollow cylinder is studied by using the multiplayer theory. Material's properties in each layer are constant and functionally graded properties are resulted by suitable arranging of layers in multiplayer cylinder. To find the first natural frequency, the displacement responses are transferred to frequency domain by using the fast Fourier transform. A comparison of the radial displacement responses between various values of the power law exponent is presented. The first natural frequencies for various values of n are studied.

REFERENCES

- 1. Praveen G.N., Reddy J.N., "Nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates" *Int. J. solids structures*, Vol. 35(1998), pp. 4457 4476.
- 2. Loy C.T., Lam K.Y., Reddy J.N, "Vibration of functionally graded cylindrical shells" ,*Int. J. of Mech. Sci.*, vol.41 (1999), pp. 309-324.
- Cheng Z.Q., Batra R.C., "Exact correspondence between eigenvalues of membranes and functionally graded simply supported polygonal plates" *J. of sound and vibration*, Vol.229 (4) (2000), pp. 879-895.

- 4. Pradhan S.C., Loy C.T., Lam K.Y., Reddy J.N, "Vibration characteristics functionally graded cylindrical shells under various boundary conditions" *Applied Acoustics*, vol. 61(2000), pp. 111-129.
- 5. Han X., Liu G.R., Xi Z.C., Lam K.Y., "Transient waves in a functionally graded cylinder", *Int. J. of solids and structures*, vol.38 (2001), pp. 3021-3037.
- 6. Han x., Liu G.R., "Effects of SH waves in a functionally graded plate" *Mechanics Research Communications*, Vol. 29 (2002), pp. 327-338.
- 7. Jin Z.H., "An asymptotic solution of temperature field in a strip of a functionally graded material", *Int. comm. Heat Mass Transfer*, Vol. 29 (2002), No. 7, pp. 887-895.
- 8. Chen W.Q., Bian Z.G., Ding H.J.," Three-dimensional vibration analysis of fluid-filled orthotropic FGM cylindrical shells" *,Int. J. of Mech. Sci.*, accepted 18 December 2003.
- 9. Lee Z.Y., "Hybrid numerical method applied to 3-D multilayer hollow cylinder with time-dependent boundary conditions", *Applied mathematics and computation*, vol. 150 (2004), pp. 25-43.