ICEE 2006 14th Iranian Conference on Electrical Engineering

Adaptive Data Reusing Normalized Least Mean Square Algorithm Based on Control of Error

Hadi Sadoghi Yazdi
Faculty of Engineering, Tarbiat Moallem University of Sabzevar, Sabzevar, Iran
sadoghi@sttu.ac.ir

Abstract: *Data reusing normalized least mean squares (DRNLMS) algorithms converge often faster than the conventional least mean squares (LMS) algorithm. This paper analyzes an adaptive DRNLMS, ADRNLMS, algorithm which has lower computational complexity relative to DRNLMS algorithm. Convergence behavior of an ADRNLMS algorithm are theoretically derived and analyzed. A large number of reusing times was found to raise the convergence rate but also increase computational complexity. In the proposed ADRNLMS algorithm is shown that number of reusing time is related to boundary of selected error. Decreasing of estimation error from selected threshold is caused decreasing of number of reusing time and vice versa. Simulation results validate the analysis and ensuing method.*

Keywords: Least mean Square, Data Reusing, Convergence Behavior.

1. Introduction

THE least mean square (LMS) algorithm has been widely used due to simplicity. The LMS algorithm gained popularity due to its robustness and low computational complexity. The main drawback of the LMS algorithm is that the convergence speed depends strongly on the eigenvalue spread of the input-signal correlation matrix [1]. To overcome this problem, a more complex recursive least squares (RLS) type of algorithm can be used. However, the faster convergence of the RLS algorithm does not imply a better tracking capability in a time-varying environment [1, 2]. An alternative to speed up the convergence at the expense of low additional complexity is to use the data-reusing LMS (DRLMS) algorithm [3]. The DRLMS algorithm uses current desired and input signals repeatedly in each iteration in order to improve its convergence speed. But disadvantage of the DRLMS algorithm is increasing computational complexity compared to LMS algorithm. In this paper, a new method is proposed with an adaptive iterating number or adaptive data reusing number. This novelty reduces computation significantly. Derivation of convergence behavior and simulation results show superiority of the proposed scheme.

1.1. Related work in the data reusing LMS algorithm

Roy et al.'s proposed DRLMS algorithm [3]. This algorithm is parameterized by the number of reuses (L) of the weight update per data sample, and can be considered to have intermediate properties between the LMS and the normalized LMS algorithms. The basic update in the Data-Reuse algorithm can be written as,

$$W_{i+1}(n) = W_i(n) + 2\mu e_i(n) X(n) \qquad (1)$$

Where

$$e_i(n) = d(n) - W_i^T(n) X(n), \quad i = 1,2,\ldots,L \qquad (2)$$

$e_i(n)$ represents the output error on the ith use of some data, out of total of L passes. The time update recursion is then naturally given by:

$$W_1(n) = W(n); \quad W(n+1) = W_{L+1}(n) \qquad (3)$$

Where, L is the number of times of repeated updating. Some recent work [4] it was demonstrated that the Normalized LMS (NLMS) algorithm was closely related to the DRLMS algorithm, and could be interpreted as a limiting case of the latter. Broadly speaking, the convergence properties of the DR-LMS update is intermediate between those of the LMS and the NLMS algorithms. Repeated updating was first applied to the conventional NLMS by De Campos et al [5]. In this study, the data-reusing NLMS (DRNLMS) algorithm was carried out as shown below,

$$W_{i+1}(n) = W_i(n) + \frac{1}{X^T(n)X(n)} 2\mu e_i(n) X(n) \qquad (4)$$

With the algorithms proposed in [6], which are called normalized and unnormalized new data-reusing LMS (NNDR-LMS and UNDR-LMS), performance can be further improved when past data are also used within each iteration. The binormalized data-reusing LMS (BNDRLMS) algorithm [7] analyzed in this correspondence employs normalization on two

orthogonal directions obtained from current and past data. It can be shown that the BNDRLMS algorithm belongs to the family of normalized algorithms discussed in [8] when sample-by-sample updating is used.

The algorithm known as the set-membership NLMS (SMNLMS) algorithm [9], which is further studied in [10], was shown to achieve both fast convergence and low misadjustment. In this algorithm step size is determined based on error. In SMNLMS, the filter is designed to achieve a specified bound on the magnitude of the output error.

In [11] a modification of the DRLMS algorithm is proposed. They let the number of data reusing runs, L, vary at each iteration. Initially, they make L very large and the value of L is decreased by 1 for each new iteration. This is equivalent to using the normalized LMS with unity step at the beginning of adaptation and moving gradually to the LMS at steady state.

One main problem is seen in [11]. Changing of L independent of error or misadjustment is caused a large number of unnecessary operations; also it is caused decreasing of tracking capability because of changing of L parameter in one direction (decreasing or increasing). We propose data reusing runs (L) which change based on error.

2. Motivation

The DRNLMS algorithm is based upon using the current data vector more than once per time iteration. When a data vector $X(n)$ is received, the weight vector is computed L times. Fixing L parameter gives additional constant computation which in varieties of application is not necessary. For example in acoustic noise cancellation, error microphone is measured using analog to digital converter with definite resolution. So, if boundary of error is specified, L parameter in DRNLMS can be changed adaptively for satisfying boundary of error.

This paper shows relation between convergence rate and reusing time in DRNLMS algorithm. Increasing reusing time is caused independence of convergence speed and step size of NLMS. Relation of reusing time and instance error is derived and conclusion shows that if a boundary checking algorithm is used optimum reusing time is obtained.

3. The proposed method

A few theorems are derived before presenting of proposed method. It is shown that a boundary checking algorithm over error need for controlling of mean square error, computational complexity and convergence speed.

3.1. Convergence behavior of DRNLMS

The basic update in the Data-Reuse algorithm has been written in relation (4). Relation (3) can be written as,

$$W(n+1) = W_{L+1}(n) = W_L(n) + \frac{1}{X^T(n)X(n)} 2\mu e_L(n)X(n) = W_{L-1}(n) + \frac{1}{X^T(n)X(n)} 2\mu e_{L-1}(n)X(n) \quad \frac{1}{X^T(n)X(n)} 2\mu e_L(n)X(n) \tag{5}$$

Abstractly,

$$W(n+1) = W_{L-1}(n) + \frac{1}{X^T(n)X(n)} 2\mu X(n)\left(e_L(n) + e_{L-1}(n)\right) \tag{6}$$

So using (3) and above relation we have,

$$W(n+1) = W(n) + \frac{1}{X^T(n)X(n)} 2\mu X(n)\sum_{i=1}^{L} e_i(n) \tag{7}$$

Above formulae is result of weight update in DRNLMS algorithm.

Now we present a theorem which shows DRNLMS algorithm is same as NLMS algorithm with new step size. So proof of convergence behavior is similar to NLMS algorithm.

Theorem-1: The DRNLMS algorithm is same as NLMS with new step size.

Proof: at first, error is calculated. According to (2),

$$e_1(n) = d(n) - W^T(n)X(n) \tag{8}$$

Also, using (5) we have,

$$W_2(n+1) = W(n) + \frac{1}{X^T(n)X(n)} 2\mu e(n)X(n) \tag{9}$$

Result of using (9) in (8) for second reusing time,

$$e_2(n) = d(n) - W_2^T(n)X(n) = d(n) - \left(W(n) + \frac{1}{X^T(n)X(n)} 2\mu e(n)X(n)\right)^T X(n) \tag{10}$$

After simplification of (10),

$$e_2(n) = d(n) - W^T(n)X(n) - 2\mu e(n) \tag{11}$$

(11) and (2) obtain,

$$e_2(n) = e(n) - 2\mu e(n) \tag{12}$$

Finally,

$$e_k(n) = e(n)(1-2\mu)^{k-1} \tag{13}$$

Term of $\sum_{i=1}^{L} e_i(n)$ is calculated to form of as follow,

$$\Sigma_e = \sum_{i=1}^{L} e_i(n)$$
$$= \sum_{i=1}^{L} e(n)(1-2\mu)^{i-1} = e(n)\sum_{i=1}^{L}(1-2\mu)^{i-1} \quad (14)$$
$$= \frac{e(n)}{2\mu}\left[1-(1-2\mu)^L\right]$$

After substitute of (14) in (7),

$$W(n+1) = W(n) + \frac{1}{X^T(n)X(n)} 2\mu X(n)\Sigma_e \quad (15)$$
$$= W(n) + \frac{\left[1-(1-2\mu)^L\right]}{X^T(n)X(n)} X(n)e(n)$$
$$= W(n) + \frac{\hat{\mu}}{X^T(n)X(n)} X(n)e(n)$$

Above relation is same update formula in the NLMS algorithm which has been appeared in [12] with step size $\hat{\mu}$.

$$\hat{\mu} = \left[1-(1-2\mu)^L\right] \quad (16)$$

The result of derivation shows that,

a) The DRNLMS is NLMS with new step size
b) Step size is controlled with data reusing number (L)
c) Increasing of data reusing number obtain larger step size and so increasing convergence speed

3.2. Mean square error of the DRNLMS algorithm

Error in the DRNLMS algorithm is calculated according (14) or briefly:

$$\Sigma^2{}_e = \left(\frac{\left[1-(1-2\mu)^L\right]}{2\mu}\right)^2 e^2(n) \quad (17)$$

Taking expectations on both sides of (17) yields,

$$MSE_{DRNLMS} = \left(\frac{\left[1-(1-2\mu)^L\right]}{2\mu}\right)^2 MSE_{NLMS} \quad (18)$$

Increasing of data reusing number (L) in (18) shows that MSE in the DRNLMS algorithm increases. So, MSE and convergence speed have opposite behavior against data reusing number (L), whereas large L give further computational complexity. Therefore, a trade-off between the error, computational complexity and number of reusing number can be achieved.

3.3. Optimum reusing number

Calculation of optimum can not obtain easily by optimizing of all constraints as,

- Order (DRNLMS) =L$\times$Order (NLMS)
- MSE equation of the DRNLMS algorithm according (18)
- Convergence speed from (16)

So, a boundary checking algorithm is used for controlling of data reusing number (L) as follows,

$$\textit{If error} < \varepsilon_{\min}$$
$$L = \max(L_{\min}, L-1); \quad (19)$$
$$\textit{else}$$
$$L = \min(L_{\max}, L+1);$$

Where minimum of data reusing is $L_{\min}$ and maximum data reusing number is $L_{\max}$. Relation (19) control data reusing number for decreasing of computational complexity and computer simulation shows effect of using (19) in the DRNLMS algorithm.

4. Simulation experiments

The performance of the DRNLMS algorithm with adaptive reusing number is tested for a familiar channel identification scenario described in [2]. In order to demonstrate the convergence behavior of the proposed approach, DRNLMS and the NLMS algorithms, simulation experiments have been performed with different step size for finding time saving, convergence behavior and computational saving. In all experiments carried out for the system identification, a stationary white noise sequence was used and the system is a 32-tap FIR transversal filter having parameters that are arbitrarily chosen.

The input data were normalized to have unit variance. The norm of the difference between the plant FIR weights and adaptive filter weights plotted as a function of time, as depicted in Fig. 1. The norm is claculated by,

$$\text{norm}(h,W) = \left(\sum_i (h_i - W_i)^2\right)^{1/2} \quad (20)$$

It can be seen that DRNLMS algorithm has much better convergence than NLMS algorithm.

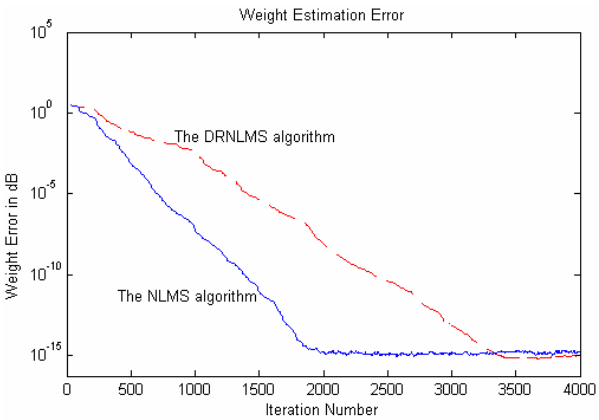

Fig.1: Weight estimation error, norm of the difference weight vector with parameters $\mu = 0.4,\ L = 8$

According (16), increasing L is caused step size of the DRNLMS algorithm ($\hat{\mu}$) increase, so convergence rate increase. If data reusing number (L) increase up to 30 Fig 2 converts to Fig 3. So, we expect for fast convergence upper possible bound of L is selected. But upper bound of L is caused increasing of computational cost. We showed that optimum data reusing number depend on MSE and convergence speed and computational complexity. We can find optimum data reusing number using the proposed boundary checking algorithm relation (19). We expect middle of L have had lower capability relative to the proposed method (ADRNLMS algorithm). Fig 4 shows the proposed algorithm can increase convergence speed regarding to searching of optimum reusing runs. Interesting note in Fig 4 is low effect of step size (μ) on the convergence speed of the proposed algorithm which this subject has been shown in (16).

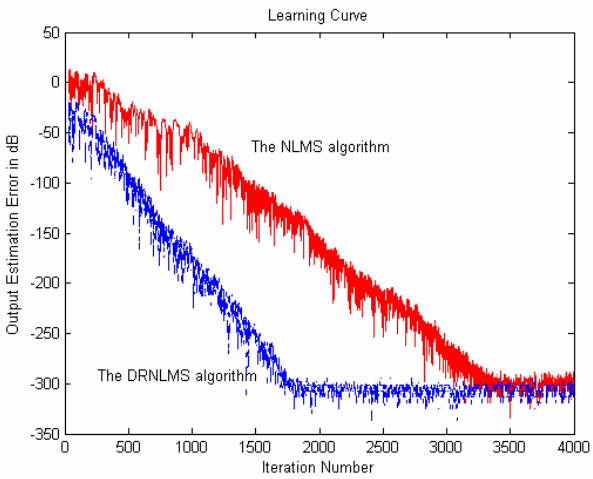

Fig.2: Output estimation error in dB with parameters $\mu = 0.4,\ L = 8$

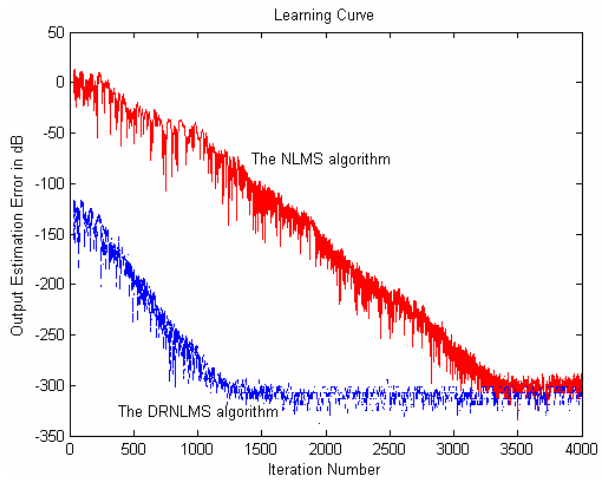

Fig.3: Output estimation error in dB with parameters $\mu = 0.4,\ L = 30$

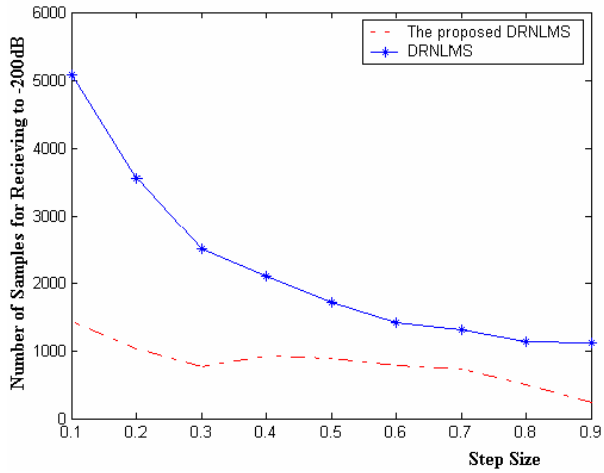

Fig.4: Data reusing number for receiving to -200dB error

If initial data reusing number is valued to eight ($L_{ini} = 8$), the proposed algorithm (ADRNLMS algorithm) decrease it regarding to $\varepsilon_{\min}$ according to (19). Variation of L for $\varepsilon_{\min} = -300dB$ is shown in Fig 5. By defining time saving according to following relation, we can calculate value of saving of computation cost.

$$TimeSaving = \frac{NL_{\max} - \sum_{i=1}^{N} L_i^{ADRNLMS}}{NL_{\max}} \times 100 \qquad (21)$$

Where N is number of samples or total of iteration number and $L_i^{ADRNLMS}$ is data reusing number which is selected based on (19). Simulation result of Fig 6 shows time saving (21) versus step size (μ). Increasing of step size upon to one is caused time saving reach to 38%.

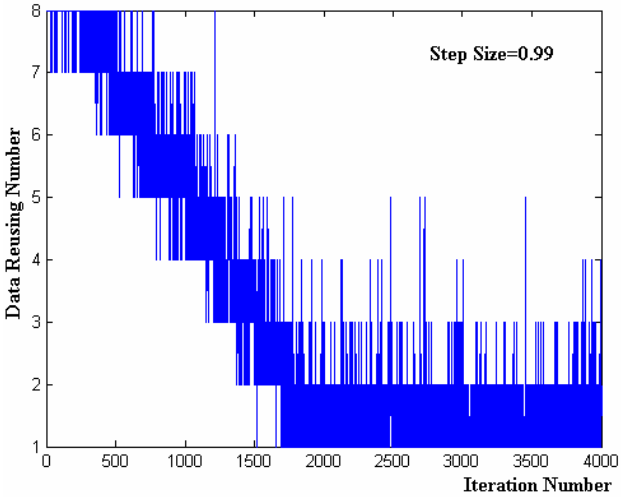

Fig.5: Data reusing Number (L) in convergence phase

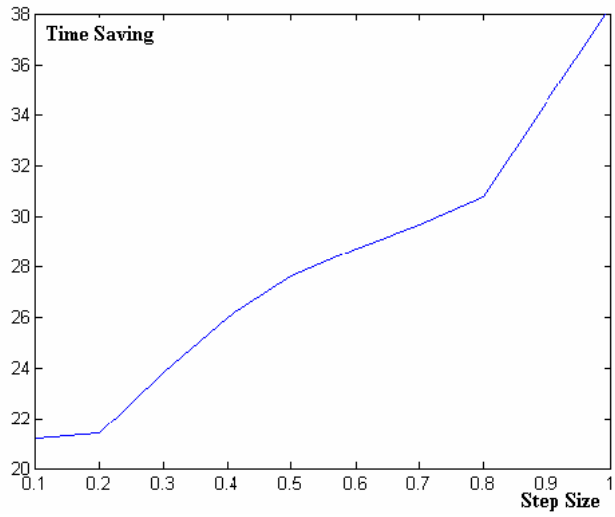

Fig.6: Time saving versus step size by $L_{\max} = 8$

4.1. Evaluating the tracking performance of the proposed algorithm

Tracking is a steady-state phenomenon that is different from the convergence, which is a transient phenomenon. In general, convergence and tracking are two different properties. If an algorithm has good convergence, its tracking ability is not necessarily fast and vice versa. In the tracking phase, a reasonable assumption is that the optimum weights vary according to a first-order Markov process [12], and the filter must track these weights. The following relation shows the variation of the filter's optimum weights,

$$W_{n+1}^{*} = aW_{n}^{*} + \omega_{n} \qquad (22)$$

$$d_{n} = W_{n}^{*T} X_{n} + \nu_{n} \qquad (23)$$

where, a is a constant and ω_n is the process noise vector in the n'th step, which has zero mean with correlation matrix Φ, and ν_n is the measurement noise, which is assumed to be white Gaussian with zero mean and variance σ_{ν}^{2}.

After changing desired signal according (23) the proposed algorithm change L according to (19). Adaptation of $L_i^{ADRNLMS}$ shows in Fig 7 and Fig 8 compares the NLMS and ADRNLMS algorithms. Also Fig 9 shows better performance of the proposed algorithm (ADRNLMS algorithm).

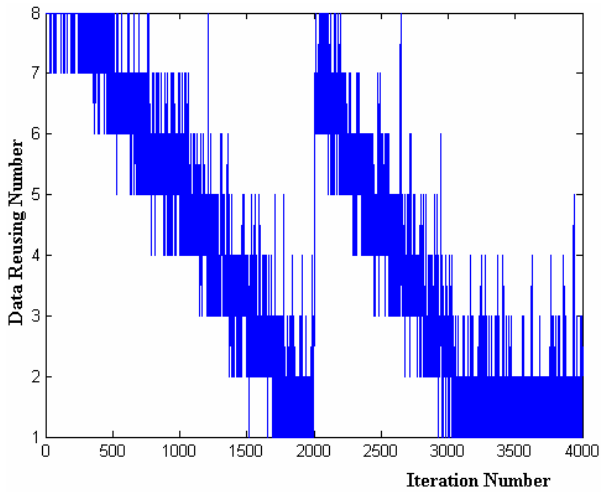

Fig.7: Data reusing Number (L) after changing desired signal

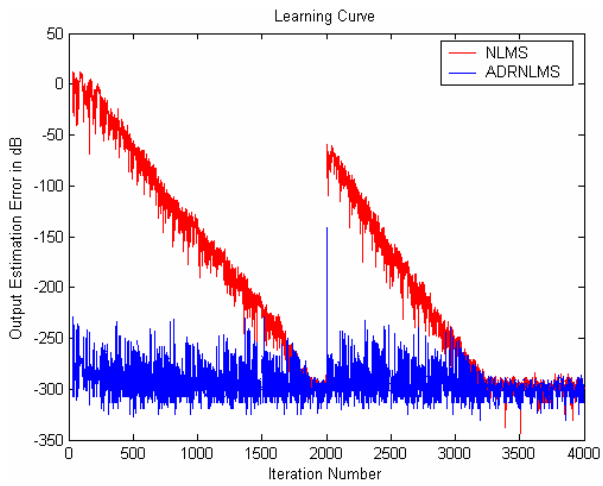

Fig.8: Comparing learning curve of the DRNLMS and ADRNLMS algorithms (ADRNLMS algorithm by L according (19))

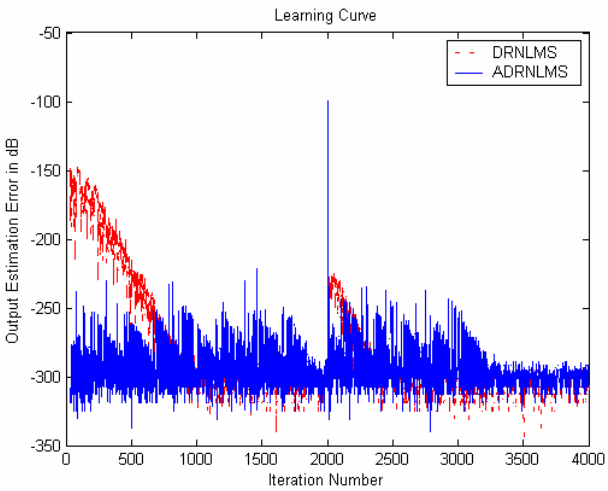

Fig.9: Comparing learning curve of the DRNLMS and ADRNLMS algorithms (DRNLMS algorithm by L=5 and ADRNLMS algorithm by L according (19)) for changing of desired signal only once.

Now, tracking phase is tested. For this purpose after $\frac{N}{2}$ iteration number or receiving samples desired signal change according to (23) continually. This is different by old experiment which desired signal changed only once (Fig 9). Fig 10 shows tracking phase after 2000th sample. Lower error in learning curve shows power of the ADRNLMS algorithm compared to the DRNLMS algorithm in tracking phase.

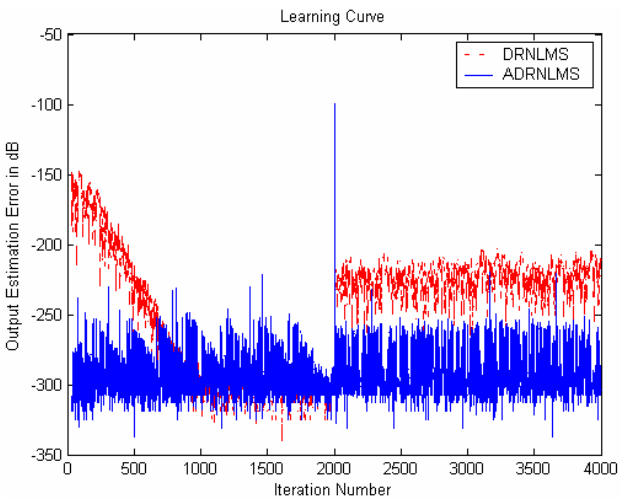

Fig.10: Comparing learning curve of the DRNLMS and ADRNLMS algorithms (DRNLMS algorithm by L=5 and ADRNLMS algorithm by L according (19)) in tracking phase according (22) , (23) for changing of desired signal continually.

5. Conclusions

In this paper, we analyzed an adaptive DRNLMS, ADRNLMS, algorithm with lower computational complexity relative to DRNLMS algorithm. Convergence behavior of the proposed algorithm theoretically derived and analyzed. Also it was shown that tracking capability was increased.

References:

[1] P. S. R. Diniz, Adaptive Filtering: Algorithms and Practical Implementations, Second ed. Boston, MA: Kluwer, 2002.

[2] S. Haykin, Adaptive Filter Theory. Englewood Cliffs, NJ: Prentice-Hall, 1996.

[3] S. Roy, J. J. Shynk, "Analysis of the Data-Reusing LMS Algorithm," Proceedings of the 32nd Midwest Symposium on Circuits and Systems, vol.2, pp.1127-1130, Aug. 1989.

[4] R. Nitzberg, “Application of the Normalized LMS Algorithm to MSLC,” IEEE Trans. on Aerosp. Electron. Syst., vol. AES-21, No. 1, pp. 79-91, Jan 1985.

[5] M. de Campos, P. Diniz and J. Apolinario, “On Normalized Data-Reusing and Affine Projections Algorithms,” Proc. of IEEE Int. Conf. on Electronics, Circuits and Systems, vol. 2, pp.843 –846. 1999.

[6] B. A. Schnaufer, “Practical Techniques for Rapid and Reliable Real-Time Adaptive Filtering,” Ph.D. dissertation, Univ. Illinois Urbana-Champaign, 1995.

[7] J. Jr. Apolinario, M.L.R. Campos, P.S.R. Diniz, "Convergence Analysis of the Binormalized Data-Reusing LMS Algorithm," IEEE Trans. on Signal Processing, vol.48, Issue.11, pp.3235-3242, Nov. 2000.

[8] M. Montazeri and P. Duhamel, “A set of algorithms linking NLMS and block RLS algorithms,” IEEE Trans. Signal Processing, vol. 43, pp.444–453, Feb. 1995.

[9] S. Gollamudi, S. Nagaraj, S. Kapoor, and Y. F. Huang, “Set-membership filtering and a set-membership normalized LMS algorithm with an adaptive step size,” IEEE Signal Processing Lett., vol. 5, pp. 111–114, Apr. 1998.

[10] P. S. R. Diniz, S. Werner, "Set-Membership Binormalized Data-Reusing LMS Algorithms," IEEE Trans. on Signal Processing, vol.51, Issue.1, pp.124-134, Jan. 2003.

[11] M. I. Haddad, M. A. Khasawneh, "A modified variable degree variable step size LMS algorithm," Midwest Symposium on Circuits and Systems, pp.506-509, Aug. 1998.

[12] B. Widrow, and S. D. Stearns, Adaptive Signal Processing, Prentice-Hall, Englewood Cliffs, NJ، 1984.