ICEE 2006 14th Iranian Conference on Electrical Engineering

# Tracking by a New Type of Nonlinear Adaptive Filter

**Hadi Sadoghi Yazdi**[1] **Mojtaba Lotfizad**[2] **Mahmood Fathy**[3]

1- Faculty of Engineering, Tarbiat Moallem University of Sabzevar, Sabzevar, Iran
2- Department of Electrical Engineering, Tarbiat Modarres University, Tehran, Iran
3- Faculty of Computer Engineering, Iran University of Science and Technology, Tehran, Iran
sadoghi@sttu.ac.ir, lotfizad@modares.ac.ir, mahfathy@iust.ac.ir

***Abstract**: In this paper, a new nonlinear adaptive filter is presented. This filter consists of three main parts. In the first part, the input space is mapped into the high dimensional space, HDS, using the RBF kernel. The second part employs the Kalman filter as a smoother in HDS. The RLS adaptation algorithm is used for weight updating in the third part. The innovation of this work lies in using Kalman filter as the smoother in HDS. It is shown that HDS has a smaller mean square error, MSE, and in noisy environment has a smaller signal variance relative to the input space, therefore Kalman smoother was applied to HDS instead of the input space. Experimental results show better performance of the new structure compared to the conventional RLS in tracking a nonlinear noisy chirp sinusoid and for vehicle tracking in the analysis of traffic scene.*

**Keywords**: Nonlinear adaptive filter, RBF kernel, Kalman smoother, RLS algorithm, Vehicle tracking.

## 1. Introduction

The subject of adaptive signal processing has been one of the fastest growing fields of research in recent years. It has attained its popularity due to a broad range of useful applications in such diverse areas as solar systems, weather forecast, chemical reactions, communications, radar, sonar, seismology, navigation, control systems, biomedical electronics and surveillance systems [1, 2].

Usually, the filter is assumed to be linear; the motivation for using linear filters is based on simple equations for describing the signal and noise. While this approach has led to great advances in the past years, the replacement of the linear filter with nonlinear ones should enable us to obtain an accurate description of the signal and noise. This may lead to better performance of practical signal processing applications. Nonlinear processing techniques are potentially useful in many scenarios. For example in vision-based surveillance, nonlinearities arise from the following,

- The system that generates the video signal and/or the noise.
- The signal acquisition system.
- The transmission channel for video signal.
- The behavior of the target.
- The target trajectories.
- The human perception mechanism for interpretation of the behavior of target.

However some problems exist in using nonlinear adaptive filter [3-5] as,

- Inutile of the many traditional linear methods
- Difficulties in the analysis of nonlinear systems
- Increased computational complexity compared to linear filters

Fundamentally, there are two types of nonlinear adaptive filters, which are Volterra-based filters and neural networks [6]. The radial basis function, RBF, neural network has properties of each two aforementioned methods. RBF neural networks can be used for nonlinear system identification [7-8]. Also RBF is used in the prediction and tracking problems [9-11].

For the RBF network development, both the center selection and weight parameter estimation are important. Gradient descent is a suitable method for weight learning [12]. In many applications, the LMS algorithm could still be too slow, especially when the input correlation matrix is ill-conditioned [13]. Superior alternatives have been proposed; for example those relying on orthogonalization procedures [14, 15].

It has been shown that the RBF neural network is based on recursive least squares learning, RLS, algorithm [16, 17]. The Kalman filter is used joined with RBF neural network. In [18] extended Kalman filter is used for learning procedure of RBF neural network at identification problem.

In this work, smoothing of signal at high dimensional space in RBF net by a Kalman filter is presented. Section 2 is devoted to problem formulation. In section 3, the proposed approach is described. Section 4 contains experimental results in nonlinear noisy chirp tracking and also for vehicle tracking and the final section includes the conclusions.

## 2. Problem formulation

Tracking systems rely heavily upon statistical estimation theory. A simple description for nonlinear tracking is,

$$x(k+1) = f(x(k), x(k-1), \ldots, x(k-N)) \qquad (1)$$

Where, $x(k+1)$ is state of the target in future time and $f(.)$ is nonlinear relation between the current and previous states with the future ones. We assume that $x_m(k)$ is the actual state of the signal and $e(k) = x_m(k) - x(k)$ is the prediction or estimation error. The mean square error, MSE, is $MSE = E\{e^2\}$.

We exploit the radial basis function for nonlinear modeling of the input signal. In the high dimensional space generated by the $\Phi$ function, the input space is mapped into $\Phi$-space or $Y = \Phi(X)$. With regard to our theorems which appeared in the Appendix, "the MSE at $\Phi$-space is smaller than the input space", and "variance at $\Phi$-space is smaller than variance at input space" so we apply a Kalman estimator in the $\Phi$-space thereupon $Y = [y_1,\ldots,y_k]$ is filtered to $\hat{Y} = [\hat{y}_1,\ldots,\hat{y}_k]$ or $\Phi$ is smoothed to $\hat{\Phi}$.

The linear combination of $\hat{Y}$ elements yields the desired output.

Briefly, based on what we discussed above,

$$x(k+1) = W_{opt}{}^T \hat{\Phi} \tag{2}$$

Where, the optimum weight vector $W_{opt} = [w_1,\ldots w_k]^T$ is found by the RLS algorithm. $\hat{\Phi}$ is a smoothed version of $\Phi$ by Kalman smoother. The above formulation will be applied to practical tracking problem in the next section.

## 3. The proposed nonlinear tracking system

The proposed nonlinear adaptive filter contains three main parts. At first, nonlinear input space is converted into the linear space using the RBF kernel. The second part uses Kalman filter as smoother in high dimensional space, HDS. The RLS adaptation algorithm is used for weights updating in the third part. Innovation of this work is applying Kalman filter as smoother in HDS. Three parts of the proposed system are shown in Fig 1.

### a) Using the RBF kernel for converting space

Let $X = [x(t),\ldots,x(t-n)]$ be the buffered data in the input space, $\Phi$ is the output of a set of real-valued functions $\varphi(x,c_i)$ (i=1,…,k) at HDS, as shown by,

$$\Phi = [\varphi(x,c_1),\varphi(x,c_2),\ldots,\varphi(x,c_K)]$$

$$\varphi(x,c_i) = \exp\left(-\frac{(x-c_i)^2}{2\sigma^2}\right) \tag{3}$$

Where $c_i$ is i^th center and $\sigma$ is standard deviation of learning sample of signal that can be obtained by K-means or fuzzy K-means algorithms. Thus, input space $X$ is converted to HDS using RBF kernel, which this space according to Cover theorem [19] is linear relative to $X$.

### b) Kalman smoother in $\Phi$ space

Nonlinear formulation of input signal and linear behavior of $\Phi$ space are two main properties of $\Phi$ space. In addition, any dimension of $\Phi$ space has smaller MSE relative to that of the input space according to the Appendix. These ingredients encourage us to utilize Kalman smoother, KS, in $\Phi$ space, HDS, instead of applying KS in the input space [20, 21]. The Kalman filter is well-known estimation procedure to minimize error from available measured data. This filter need to the dynamical system for state or process equation,

$$\hat{x}(k) = F(k)x(k) + G(k)w(k)$$

$$= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} v(k) \tag{4}$$

where T is the sampling period which in vehicle tracking in video is 0.04 second, $x(k)$ is the state vector, $v(k)$ is the random acceleration disturbance and $F(k)$ is a known squared state transition matrix relating the states of the system at time (k) which is $\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$ and measurement equation is

$$y(k) = C(k)x(k) \tag{5}$$

where $y(k)$ is the observation vector and $C(k)$ is measurement matrix. Fig 1 shows the proposed procedure.

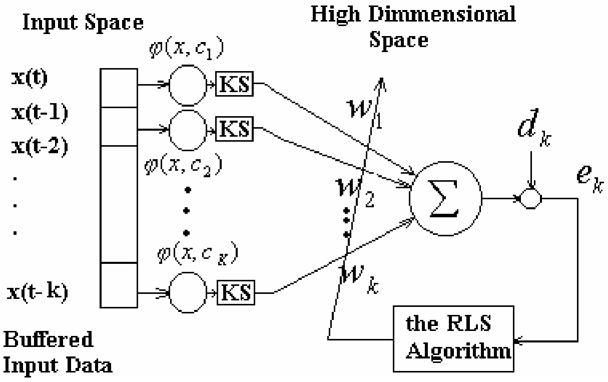



**Fig.1: The proposed nonlinear adaptive filter, KS block is a Kalman smoother**

### c) The RLS adaptation algorithm

Recursive least squares, RLS, filter is an adaptive, time-update, version of the Wiener filter. Fast convergence of the RLS is one of reasons to its usage in weight updating algorithm. The weight update equation can be written as,

$$W_{k+1} = W_k + R_k^{-1}\Phi_k e_k \tag{6}$$

Where, $W_k = [w_1, \ldots w_k]$ is the weight vector of the RLS filter with input signal $\Phi_k = [\varphi(x_k, c_1), \varphi(x_k, c_2), \ldots, \varphi(x_k, c_K)]$ and $e_k$ is the error signal, $e_k = d_k - \Phi_k^T W_k$. $R^{-1}{}_k$ is the inverse autocorrelation function at $\Phi$ space can be written as,

$$R_k^{-1} = \lambda^{-1} R_{k-1}^{-1} - \frac{\lambda^{-2} R_{k-1}^{-1} \Phi_k \Phi_k^T \left(R_{k-1}^{-1}\right)^T}{1 + \lambda^{-1} \Phi_k^T R_{k-1}^{-1} \Phi_k} \quad (7)$$

Where, $\lambda$ is the forgetting factor, $0 \le \lambda \le 1$ and this paper set to 0.42. This approach is better than using Green function for obtaining weights in [13], because of utilizing from recursive inverse autocorrelation function Eq.(7).

## 4. Experimental results

In this section, the proposed nonlinear tracking system is used for noise reduction from a noisy chirp sinusoid and also for tracking vehicles in traffic scenes.

### 4.1. Nonlinear tracking of a noisy chirp sinusoid

Adaptive recovery of a chirp sinusoid buried in noise is of special interest to researchers because the chirp sinusoid represents a well-defined form of nonstationarity. In this experiment, we present nonlinear chirp sinusoid. The nonlinear chirped is given by:

$$x(k) = g(\sqrt{p_s} \exp(j[(2\pi f_c + \psi k/2)k + \varphi])) \quad (8)$$

Where, $\sqrt{p_s}$ denotes the signal amplitude, $f_c$ is the center frequency, $\psi$ is the chirp rate and $\varphi$ is an arbitrary phase shift. $g(.)$ is a nonlinear function (e.g. $\sin(.) + \exp(.)$). The signal $x(k)$ is deterministic but nonstationary because of the chirping.

According to theorem that appeared in the appendix "MSE in $\Phi$-space is smaller than MSE in input space" we measure MSE of input space and $\Phi$-space at different SNRs whose results are shown in Fig 2. We conclude from Fig 2 and the Appendix that $\Phi$-space has a smaller MSE and signal variance relative to the input space, so it is reasonable to applying KS at $\Phi$-space.

According to the proposed nonlinear adaptive filter shown in Fig 1, $x(k)$ that is generated by Eq.(8) is buffered and applied to the proposed system. The obtained results of three algorithms are illustrated in Fig 3. The conventional RLS and nonlinear RLS without KS block and nonlinear RLS with KS are compared at signal to noise ratio, SNR, of 23 dB. The proposed system gives a better result in prediction of next state of signal relative to other methods.

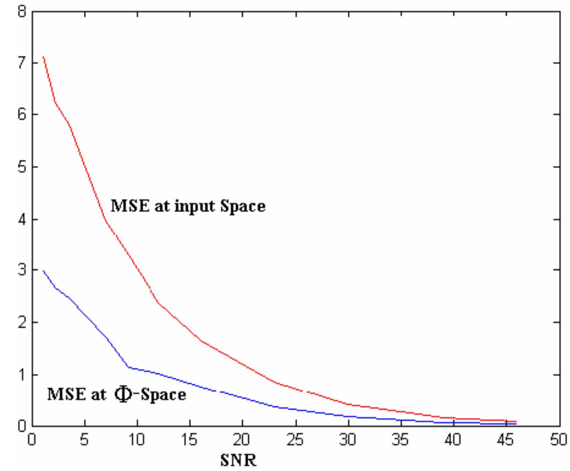



**Fig.2: Comparing MSE at different SNR**

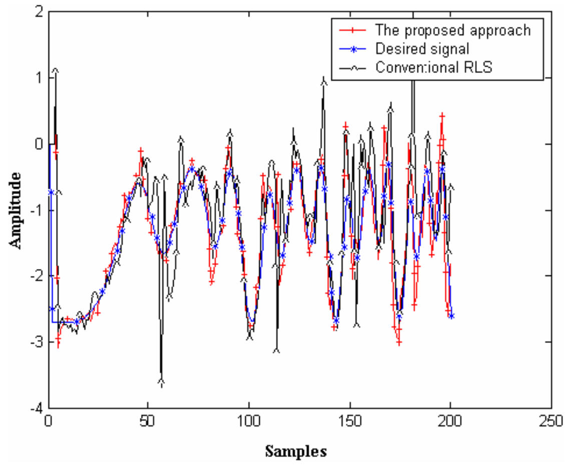



**Fig.3: Comparing predictors at SNR ratio of 23 dB**

The Kalman filter gives smoothed signal in HDS as shown in Fig 4.

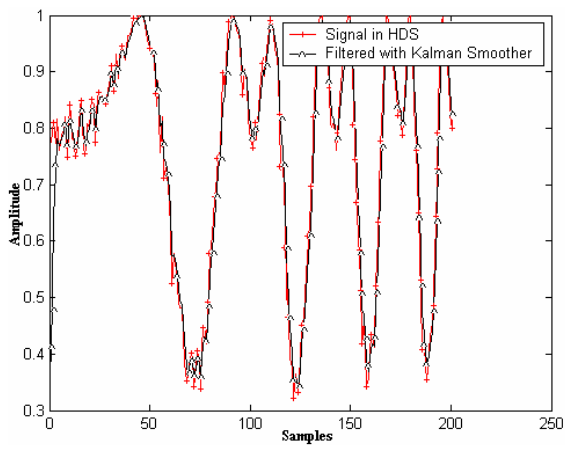



**Fig.4: The Kalman operation in HDS in one dimension**

We compare MSEs of three algorithms, conventional RLS, nonlinear RLS and nonlinear RLS with KS, the proposed approach, for different SNRs on signal with length 201 samples. The result of the simulation is shown in Fig 5.

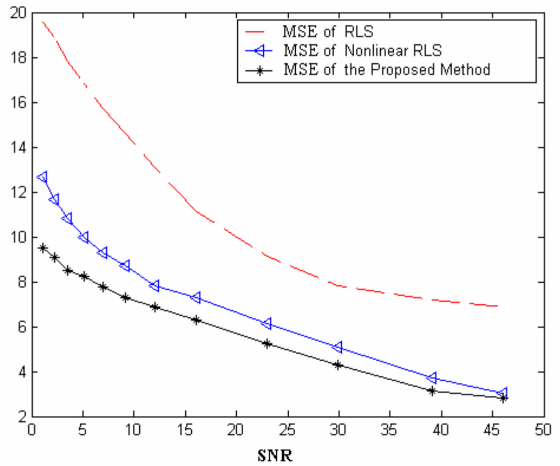



**Fig.5: Comparing of MSE for three algorithm at different SNR**

The above figure shows that the Kalman smoother clearly increases the performance of the nonlinear RLS. The next section is devoted to tracking vehicles by the proposed approach.

## 4.2. Vehicle tracking

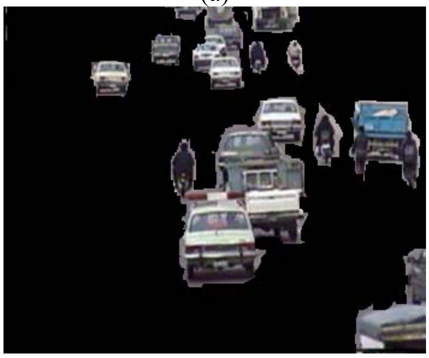
Considerable research has been undertaken in the field of estimation theory in relation to the multi-object tracking problem. This is of interest in both military and civilian applications. In the traffic scene, the estimated trajectories are used to check collision avoidance and regularity of traffic flow and validity check of behavior of drivers.

Vehicle tracking has been studied despite many difficulties such as full or partial occlusion [22, 23, 24]. For vehicle tracking, after detection, similar blobs of the image are found in two consecutive frames and their centers of gravity are obtained. These locations are also applied to an RLS or modified RLS algorithm [6], [25] or nonlinear adaptive filter such as the one that is proposed in this paper.

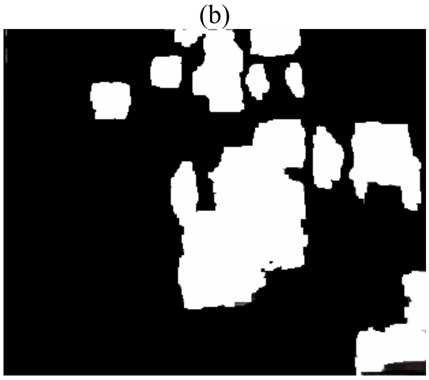
If prediction error is smaller than 7 pixel in x,y direction, we assume that predictor has converged. Then, the predictor can help after convergence of the predictor for each blob. The predictor corrects the unsuitable assigning of the blobs. Briefly, after arrival of each vehicle to the scene, it is labeled and tracked in the area of interest inside the scene by detection and prediction algorithm.

A classic technique in vehicle detection is the background subtraction. Background is obtained by combining of non-moving blobs in the scene. Each pixel in the background image is combined with the corresponding pixels of the received non-moving image blob. This work is done using the coefficient (0.9) of the effect of last background and coefficient (0.1) of the received non-moving blob. Fig 6 shows the result of above detection method including main figure, result of background subtraction and blobs.

(a)



(b)



(c)

**Fig.6: Detection method using adaptive background subtraction a) main image b) removing background c) blob detection**

After performing the background subtraction and finding the existing objects in the scene, vehicle localization is used for determining the object boundaries. Localization helps to solve the partial occlusion problem which is done by finding feature points and grouping them. We use corners as features and extract those according to [26].

After corner detection, grouping technique based on distance criteria is used for finding vehicle boundary. Fig 7 shows grouping technique for car localization, despite partial occlusion.

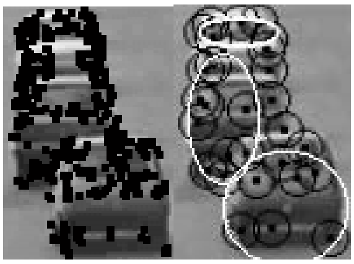


**Fig.7: Car localization using feature grouping**

After finding the vehicle boundary, a color histogram is used for the similarity measurement in consecutive frames. For each detected boundary, an HSV color histogram is achieved and quantized to $18 \times 6 \times 6$ bins, such that one vector of length 648 is obtained. Matching is obtained when the Euclidean distance between each two vectors is smaller than 0.05. Minimum distance is given in search rectangular windows with length of 50 pixels. Then center of gravity of finding vehicle is applied to an estimator or predictor in order that after the convergence of the predictor for each vehicle can help for attribution of similar blobs to an object and generating a smoothed trajectory.

Different types of noise are observed in car tracking that is an incentive for using adaptive filtering for noise reduction. Several sources of noise is presented as follows,

A) Attributing the whole or part of the vehicle to other vehicles because of shadow and partial occlusion

B) Detection of different parts of vehcile because of type of segmentation method.

C) variation of the vehicle position because of partial occlusion.

Thus, we use the predictor algorithm for filtering the vehicle position. In this paper, we discussed only the predictor. We are interested in two types of trajectory, namely, forward and turning motions.

The main arguments that show the evidence of nonlinearities in the vehicles tracking or generally target tracking are as follows,

a) nonlinear behavior of the drivers
b) nonlinear dynamics of vehicles during turn
c) sudden starts and stops of vehicles
d) problems in detection algorithms
   d-1) imperfect detection
   d-2) vehicles occlusion phenomena
   d-3) lighting condition

### 4.2.1 Results

Our experiments are concerned with congestion in junctions of highway to highway, highway to square and locations similar to Fig 10 that contains turning. Experimental results are obtained over about 10000 frames which are selected from set of 140000 frames database. At least 100 vehicles were tracked in sequences with length of about 70 frames.

By adding different simulation noises, we obtain many types of SNR environments. Fig 8 shows that for the forward motion, the RLS algorithm is suitable because of similar MSE relative to the proposed approach and low computational complexity compared to the proposed approach.

However, the nonlinear adaptive filter has better performance for turn motions especially in low SNRs according to Fig 9. Nonlinearities of turn motion have caused superiority of the proposed approach relative to other methods. Many vehicles were tracked; show that tracking of vehicles in highlight locations of Fig 10 by the proposed algorithm give better results comparing with conventional RLS. So detected vehicles in highlight location must be tracked by the proposed algorithm and others are tracked by conventional RLS.

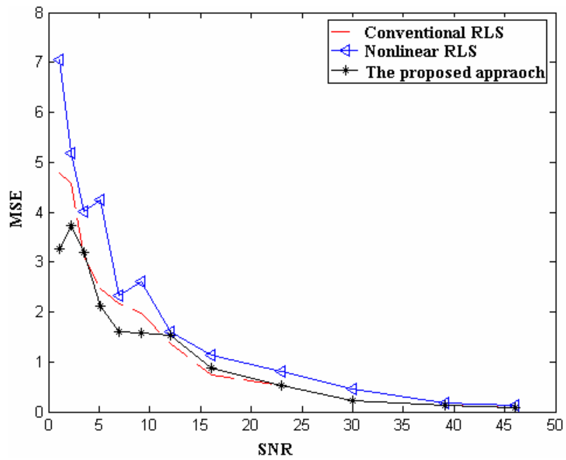



**Fig.8: MSE of tracking for forward motion**

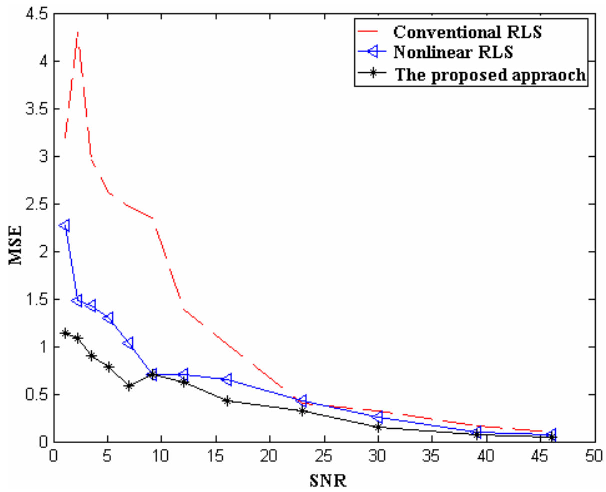



**Fig.9: MSE of tracking in turn motions**

**Fig.10-a: The proposed places for tracking using the proposed algorithm**


**Fig.10-b: The proposed places for tracking using the proposed algorithm**

## 5. Conclusions

We have developed a nonlinear tracking algorithm for handling nonlinear behavior of drivers, nonlinear dynamics of vehicles during turn motion, sudden starts and stops of vehicles and problems in detection algorithms in traffic scenes. Thus, we proposed a nonlinear adaptive filter for tracking. The proposed algorithm contains three main parts. First, non-linear input space is converted into the linear space using the RBF kernel. The second part is using Kalman filter as a smoother in high dimensional space, HDS. The RLS adaptation algorithm is used for weight updating in the third part. The innovation of this work lies in using Kalman filter as the smoother in HDS. It is shown that HDS has a higher signal to noise ratio, SNR, relative to the input space, so a Kalman smoother is applied to HDS instead of the input space. Experimental results showed better performance of the new structure compared to the conventional RLS in tracking noisy chirp sinusoid and in vehicles tracking.

## 6. Appendix

### A. MSE analysis

***Theorem***: If we assume that $MSE_i = E\{e^2\}$ is the mean square error in the input space, then the MSE at $\Phi$-space will be smaller than the input space.

***The proof***: the mapping is according to

$$Y = \Phi(X) \qquad \text{(a-1)}$$

Where, $\Phi = [\varphi(x, c_1), \varphi(x, c_2), \ldots, \varphi(x, c_K)]$ and we can assume that $\varphi(x, c_i) = \exp\left(-\frac{(x-c_i)^2}{2\sigma^2}\right)$. In simple form we can write $\varphi(x, c_i) = \exp(-x^2)$. By substituting of $e(k) = x_m(k) - x(k)$ in $\varphi(x, c_i)$, we have,

$$\varphi(x(k), c_i) = \exp(-x(k)^2) \qquad \text{(a-2)}$$
$$= \exp(-(x_m(k) + e(k))^2)$$
$$= \exp(-x_m(k)^2)\exp(-e_m(k)^2)\exp(-2e_m(k)x_m(k))$$

With assumption of $e_m(k)$ is small enough we can betake $\exp(-e_m(k)^2)$ term. Also we know $\exp(-x_m(k)^2)$ is the desired output in each dimension at $\Phi$-space. For simplification, we substitute $y = \varphi(x(k), c_i)$, thus we have,

$$y = y_m \exp(-2e_m(k)x_m(k)) \qquad \text{(a-3)}$$

Where, $y_m = e(-x_m(k)^2)$.

The Taylor series of term $\exp(-2e_m(k)x_m(k))$ is,

$$\exp(-2e_m(k)x_m(k)) = 1 - 2e_m(k)x_m(k) \qquad \text{(a-4)}$$

Neglecting high order terms of the series,

$$y = y_m - 2e_m x_m y_m \qquad \text{(a-5)}$$
$$= y_m - 2e_m x_m e^{-x^2_m}$$

The term $\alpha = 2x_m e^{-x^2_m}$ always is smaller than one. So we have,

$$y = y_m - \alpha e_m \qquad \text{(a-6)}$$

or $e_\Phi = \alpha e_m$, thus we have,

$$MSE_\Phi = E\{e_\Phi^2\} = \alpha^2 E\{e^2\}$$
$$MSE_\Phi = \alpha^2 MSE_i \qquad \text{(a-7)}$$

The above equation shows that $MSE_\Phi < MSE_i$ or *"MSE in $\Phi$-space is smaller than MSE in the input space"*.

### B. Variance analysis

***Theorem**:* If we assume that $\sigma_x^2$ is the variance in the input space, then the variance at $\Phi$-space, $\sigma_\Phi^2$, will be smaller than $\sigma_x^2$.

***The proof**:*

According to aforementioned mapping $Y = \Phi(X)$ and relation (a-6) in appendix-a, we can write:

$$\begin{aligned}\sigma_\Phi^2 &= E\left\{(y-\bar{y})^2\right\} \\ &= E\left\{\left(y_m - \alpha e_m - \left(\bar{y}_m - \alpha \bar{e}_m\right)\right)^2\right\}\end{aligned} \quad \text{(b-1)}$$

Where $\bar{y} = E\{y\}$ and substituting $e_m = x_m - x$ (b-1) can be written as,

$$\sigma_\Phi^2 = E\left\{\left[(y_m - \bar{y}_m) - \alpha(x_m - \bar{x}_m) - \alpha(x - \bar{x})\right]^2\right\} \quad \text{(b-2)}$$

In noisy environment we assume that $(y_m - \bar{y}_m)$ and $(x_m - \bar{x}_m)$ are small enough so $(y_m - \bar{y}_m) - \alpha(x_m - \bar{x}_m)$ is venial term. In this case we can write (b-2) to following form,

$$\begin{aligned}\sigma_\Phi^2 &= E\left\{\left[-\alpha(x-\bar{x})\right]^2\right\} \\ &= \alpha^2 E\left\{(x-\bar{x})^2\right\}\end{aligned} \quad \text{(b-3)}$$

Or

$$\sigma_\Phi^2 = \alpha^2 \sigma_x^2 \quad \text{(b-4)}$$

Because of $\alpha^2$ is smaller than one we have,

$$\sigma_\Phi^2 \le \sigma_x^2 \quad \text{(b-5)}$$

Or *"variance in $\Phi$-space is smaller than variance at input space"*.

## 7. References


[1] K. P. Fruzzetti, A. Palazoglu, K. A. McDonald, "Nonlinear Model Predictive Control Using Hammerstein Models," Journal of Process Control, vol.7, no.1, pp.31-41, 1997.

[2] S. Haykin, L. Li, "Nonlinear Adaptive Prediction of Nonstationary Signals," Signal Process., vol.43, pp.526-535, Feb. 1995.

[3] P. Maragos, J. F. Kaiser, T. F. Quatieri, "Energy Separation in Signal Modulations with Application to Speech Analysis", IEEE Trans. Signal Processing, vol.41, pp.3024-3051, Oct. 1993.

[4] L. Dorst, R. Boomgaard, "An Analytical Theory of Mathematical Morphology," Proceedings 1st Int. Workshop on Mat. Morphology and its Applications to Signal Processing, Barcelona, May 1993.

[5] O. Stan, E. Kamen, "A Local Linearized Least Squares Algorithm for Training Feed forward Neural Networks," IEEE Trans. on Neural Networks, vol. 11, no. 2, March 2000.

[6] S. Haykin, Adaptive Filter Theory, 3rd-ed, Printice Hall,1996.

[7] K. B. Cho, B. H. Wang, "Radial Based Function Based Adaptive Fuzzy Systems and Their Applications to System Identification and Prediction," Fuzzy Sets and Systems, vol.93, pp.325 – 339, 1996.

[8] C-H. Tsai, H-T. Chuang, "Dead zone Compensation Based on Constrained RBF Neural Network," Journal of the Franklin Institute, vol.341, pp.361-374, 2004.

[9] A. E. Omidvar, "Configuring Radial Basis Function Network Using Fractal Scaling Process with Application to Chaotic Time Series Prediction," Chaos, Solutions and Fractals, vol.22, pp.757-766, 2004.

[10] N. Xie, H. Leung, H. Chan, "A Multiple-Model Prediction Approach for Sea Clutter Modeling," IEEE Trans. on Geosciences and Remote Sensing, vol.41, no.6, June 2003.

[11] S-G. Hong, S-K. Oh, M-S. Kim, J-J. Lee, "Evolving Mixture of Experts for Nonlinear Time Series Modelling and Prediction," Electronics Letters, vol.38, no.1, 3rd Jan. 2002.

[12] Nicolaos B. Karayiannis, "Reformulated Radial Basis Neural Networks Trained by Gradient Descent," IEEE Trans. on Neural Networks, vol.10, no. 3, pp.657-671, May 1999.

[13] S. Haykin, "Neural Networks, A comprehensive Foundation, IEEE Press, New York, 1994.

[14] S. Chen, C. F. N. Cowan, P. M. Grant, "Orthogonal Least Squares Learning Algorithm for Radial Basis Function Networks," IEEE Trans. Neural Networks, vol.2, no.2, pp. 302–309, 1991.

[15] W. Kaminski, P. Strumillo, "Kernel Orthonormalization in Radial Basis Function Neural Networks," IEEE Trans. Neural Networks, vol.8, no.5, pp. 1177–1183, 1997.

[16] T. Knohl, H. Unbehauen, "Adaptive Position Control of Electrohydraulic Servo Systems Using ANN," Mechatronics, vol.10,pp127–173, 2000.

[17] A. Alexandridis, H. Sarimveis, G. Bafas, "A New Algorithm for Online Structure and Parameter Adaptation of RBF Networks," Neural Networks, vol.16, pp.1003-1017, 2003.

[18] D. Simon, "Training Radial Basis Neural Networks with the Extended Kalman Filter," Neurocomputing, vol.48, pp.455-475, 2002.

[19] T. M. Cover, "Geometrical and Statistical Properties of Systems of Linear Inequalities with Applications in Pattern recognition," IEEE Trans. on Electronic Computers, vol.EC-14, pp.326-334, 1965.

[20] M. Savia, H. N. Koivo, "Neural-Network-Based Payload Determination of a Moving Loader," Control Engineering Practice, vol.12, pp.555-561, 2004.

[21] H. Sadoghi, M. Lotfizad, M. Fathy, "Qualitative Interpretation of the Plural Behavior in the Traffic Scene Using a Fuzzy Inference System," 5th Iranian Conf. on Fuzzy Systems, pp.193-199, Sep 2004 (Also accepted in IEEE ICIT2004).

[22] S. Gil, R. Milanese, T. Pun, "Comparing Features for Target Tracking in Traffic Scenes," Pattern Recognition, vol.29, no.8, pp.1285-1296, 1996.

[23] S. Mantri, D. Bullock, "Analysis of Feed forward - Back propagation Neural Networks Used in Vehicle Detection," Transportation Research C. vol.3, no.3, pp.161-174, 1995.

[24] J. Badenas, J. M. Sanchiz, F. Pla, "Motion-Based Segmentation and Region Tracking in Image Sequence," Pattern Recognition, vol.34, pp.661-670, 2001.

[25] H. Sadoghi. Yazdi, M. Lotfizad, E. Kabir, M. Fathy, "Clipped Input Data RLS, CRLS, Applied to Vehicle Tracking," EURASIP Journal on Applied Signal Processing, Issue 8, May 2005.

[26] S. Ando, "Image Field Categorization and Edge/Corner Detection from Gradient Covariance," IEEE Trans. Pattern Analysis and Machine Intelligence, vol.22, no.2, pp.179-180, Feb. 2000.