River bed deformation calculated from boundary shear stress

Déformation d'un lit de rivière calculée à partir de la contrainte de cisaillement

ANDRÉ PAQUIER and SAEED R. KHODASHENAS, *Cemagref, Hydrology-Hydraulics Research Unit, CP 220, 69336 Lyon Cedex 09, France, e-mail: andre.paquier@cemagref.fr*

ABSTRACT
Deformation of various kinds of cross-sections was computed with the hypothesis that scour or deposition were directly related to shear stress computed by the Merged Perpendicular Method. Final stabilised cross section agrees with theoretical stable shape. To estimate the deformation of a river bed, the results of a 1D model which computes the volume of sediment eroded or deposited between two cross sections are used as a basis. Then, these volumes are transversely distributed in every section in relation with shear stress. The method is then applied to a reach of River Rhône.

RÉSUMÉ
La déformation de plusieurs types de sections en travers a été calculée en supposant que l'érosion ou le dépôt sont directement liés à la contrainte de cisaillement calculée par la Méthode des Perpendiculaires Confondues. Finalement, une fois l'équilibre atteint, la section obtenue est proche de la forme de la section stable théorique. Pour estimer la déformation d'un lit de rivière, on utilise les résultats d'un modèle unidimensionnel calculant le volume de sédiment érodé ou déposé entre deux sections comme base. Ensuite, ces volumes sont répartis transversalement selon les valeurs de contrainte dans la section en travers. La méthode est ensuite appliquée à un bief du Rhône.

Key words: shear stress, sediment transport, bed deformation, stable channel

Introduction

The bed of the river is continuously evolving but main changes occur during floods. Engineers tried to simulate such changes in the topography of the bottom by numerical models of sediment transport. Most of the codes which were developed in the past decades, like HEC6 (Thomas and Prashum, 1977), IALLUVIAL (Karim and Kennedy, 1982), SEDICOUP (Holly and Rahuel, 1990), BRALLUVIAL (Holly et al., 1985), CHARIMA (Holly et al. 1990), were built on a one dimensional approach. The one dimensional simulation of the evolution of river bed appears to be not sufficiently complete. To solve the requirements of the engineers and the real problems of the rivers, research was thus carried out to develop the techniques of calculation of mobile bed by using other models: stream tubes, two-dimensional and, even, three-dimensional. Among the models of this type, one will quote: GSTARS (Yang et al., 1988), (Yang and Simoes, 1999), TABS2 (Thomas et al., 1985), MOBED2 (Spasojevic et al., 1990) and USTARS (Lee et al., 1997). The multidimensional codes are often developed for the resolution of local or specific problems, their calculating time is very long; moreover, they require a broad knowledge of the initial data and boundary conditions, which is not obvious in the majority of the cases, and, because of the lack of the data, they consider simplified assumptions, which will move away the final results from reality. Thus, to build a tool at the scale of a whole reach, in rivers in which problems are linked with bed-load transport, it was considered that the 1-D approach needs to be investigated more. In such rivers, modification in the form of an alluvial channel due to scour or deposition of sediments may be often modelled through the computation of boundary shear stress. For engineering purposes, sediment parameters are not accurately known. Thus, shear stress should be computed by a simple and rapid method rather than an accurate one. A simple method used in FLUVIAL 14 (Chang, 1997) consists in supposing that the shear stress is proportional to water depth. As this method appeared not precise enough, a geometrical method was developed to compute the shear stress in an irregular cross section. This method called Merged Perpendicular Method was derived from the normal area method but gave more precise results (Khodashenas and Paquier, 1999). The main hypothesis of the model described here below is the relation between the computed shear stress at one point of the bottom and the displacement of this point. Then, two objectives are emphasised: on the one hand, developing a method that might be applied to simulate the deformation of any cross section and, on the other hand, integrating this method in a more general model of evolution of an alluvial channel.

Description of the model

Three steps are proposed:

1. computation of hydrodynamics through de Saint Venant equations;
2. computation of erosion or deposit by conservation of the mass of sediments;
3. distribution of eroded mass or deposited mass inside every cross section.

The details are as follows.

Computation of hydrodynamics

The results of a one dimensional hydrodynamic model is used as a basis, which decreases the computing time and also the number

Revision received January 25, 2002. Open for discussion till February 28, 2003.

of initial data and of boundary conditions. For hydraulic computation in 1D, RUBAR 3 was used. This code solves de Saint Venant equation by an explicit second order Godunov type numerical scheme (Paquier, 1996). RUBAR 3 can simulate all kinds of floods even flash floods.

Conservation of the mass of sediments

1D hydrodynamic equations are completed by the equation of conservation of the sediment mass (Paquier,1995). One size of sediment and constant Manning coefficient are supposed. The basic equation (1) is:

$$\frac{\partial S_{1D}}{\partial t} + \frac{\partial Q_s}{\partial X}\frac{1}{1-\lambda} = 0 \qquad (1)$$

in which S_{ID} sediment section (thus ΔS_{ID} is the mean bed deformation), X distance along channel axis, t time, λ porosity, $Q_s = L.q_s$ sediment discharge, L active width, q_s sediment discharge rate that is computed by Meyer-Peter and Muller's relation (2) (Graf and Altinakar, 1996):

$$\begin{cases} q_s = 8\sqrt{gd_m^3(s_s-1)}.\left(\zeta\tau^* - \tau_c^*\right)^{3/2} \text{ if } \zeta\tau^* > \tau_c^* \\ q_s = 0 \text{ in the other cases} \end{cases} \qquad (2)$$

in which d mean sediment size, $s_s=\rho_s/\rho$ relative density of sediment, g acceleration of gravity, $\zeta = (K_s/K_s')^{3/2}$ is a roughness parameter in which K_s is total Manning-Strickler coefficient, K'_s grain Manning-Strickler coefficient is often computed by $K_s' = 21/d^{1/6}$, τ^* dimensionless shear stress and τ^*_c dimensionless critical shear stress.

The active width is generally computed as the width at the surface of the water but can be reduced. For instance, the active width will be limited to the width of the main bed when there is overflow.

For the presented examples, the sediment discharge was computed from the relation (2); however, if the considered problem involves suspended load in addition to bed load, the relation providing sediment discharge may be changed; this implies that same change is performed in the third step of the method. Further, the sediment continuity equation viz. Eq. (1) should be modified to include the effects of presence of suspended load; for instance, in the model described, a loading length can be introduced to represent the lag between sediment discharge and sediment transport capacity (Paquier et al., 1997).

Distribution in every cross section

The sediment scour or deposit of one section ΔS_{ID} obtained by the 1D model is distributed in the section transversely by dividing the wetted perimeter in small segments which move separately. A 3D evolution of the bed is then possible; the 3 dimensions are the river axis, the transversal direction from one bank of the river to the other one and the vertical direction.

Various relations between the deformation and the shear stress were tested. Although several ones provide suitable results, it finally appears that the best method consists in using a relation similar to the one used for computing the sediment discharge at the second step (equation (2)). Moreover, both erosion and deposition should be allowed in the same cross section. Then, inside one section, change in bottom elevation in point j or segment j, ΔS_{jin}, is first supposed to be proportional to the sediment discharge rate, q_s, computed in point j which means that in Meyer-Peter and Muller's relation τ^* and τ^*_c are computed in point j.

$$\begin{cases} \Delta S_{jin} = \dfrac{8\Delta t\, l_j}{(1-\lambda)\Delta X}\sqrt{gd^3(s_s-1)}.\left(\zeta\tau_j^* - \tau_{jc}^*\right)^{3/2} \text{ if } \zeta\tau_j^* > \tau_{jc}^* \\ \Delta S_{jin} = -\dfrac{8\Delta t\, l_j}{(1-\lambda)\Delta X}\sqrt{gd^3(s_s-1)}.\left(\tau_{jc}^* - \zeta\tau_j^*\right)^{3/2} \text{ if } \tau_{jc}^* \geq \zeta\tau_j^* \end{cases} \qquad (3)$$

in which: Δt and ΔX are respectively the time step and the space step used in the discretisation of the second step, τ^*_j dimensionless boundary shear stress in point j calculated by the Merged Perpendicular Method, l_j length of the segment j centred on point j, τ^*_{jc} dimensionless critical shear stress in point j that is computed by a relation from (Ikeda, 1982):

$$\tau^*c\theta = K\ \tau^*c0 \qquad (4)$$

$$K = \frac{-\alpha Tan^2\phi Cos\theta + (Tan^2\phi Cos^2\theta + \alpha^2 Tan^2\phi Sin^2\theta - Sin^2\theta)^{.5}}{(1-\alpha Tan\phi)Tan\phi} \qquad (5)$$

in which: τ^*_{c0} is dimensionless critical shear stress for horizontal bottom (which may vary from one point to an other one considering, for instance, the bed sediment size or the vegetation), $\tau^*_{c\theta}$ dimensionless critical shear stress for bank with slope θ, $\alpha = F_L / F_D$, F_L and F_D are respectively dimensionless lift and drag forces, ϕ angle of internal friction of sediment, θ side slope of cross section. For the tests here below, the following values are

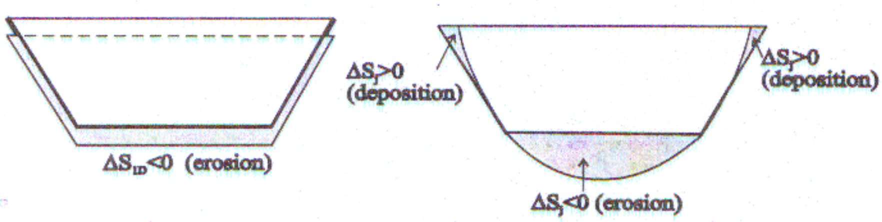

Fig. 1. Differences between deformation distributions.

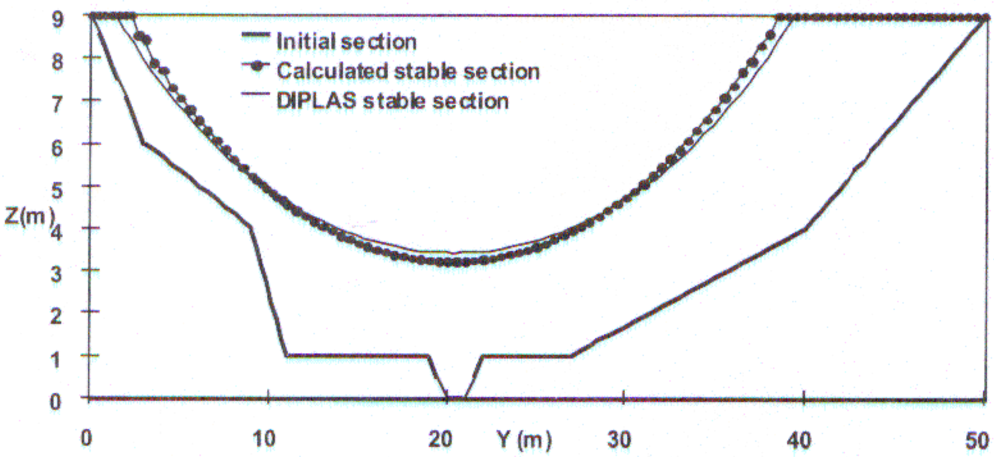

Fig. 2. Comparison of calculated stable section with (Diplas and Vigilar, 1992) stable section.

selected $\phi=35°$, $\alpha=0.85$

The total mass of the sediments that have moved, should not change during the distribution of the sediments inside the section, thus equality should be kept (see Fig. 1) between the deformation area obtained by the two first steps (uniform distribution) and the deformation area finally obtained: $\Sigma\Delta S_j=\Delta S_{ID}$. At first, ΔS_{ID} and $\Sigma\Delta S_{jin}$ are different; then, the areas from erosion and deposition are separated and different coefficients are applied to each area as follows (equations (6a), (6b), (7a), (7b)).

Initial 1-D Model (2 steps) 1-D Model with distribution (3 steps)

First case : when $\Sigma\Delta S_{jin}>\Delta S_{ID}$

$$C_c\sum_{\Delta S_{jin}\geq 0}\Delta S_{jin}+\sum_{\Delta S_{jin}<0}\Delta S_{jin}=\Delta S_{1D}\Rightarrow C_c=\frac{\Delta S_{1D}-\sum_{\Delta S_{jin}<0}\Delta S_{jin}}{\sum_{\Delta S_{jin}\geq 0}\Delta S_{jin}} \tag{6a}$$

$$\begin{cases}\text{If } \Delta S_{jin}\geq 0\Rightarrow \Delta S_j = C_c\times\Delta S_{j\,in}\\ \text{If } \Delta S_{jin}<0\Rightarrow \Delta S_j=\Delta S_{j\,in}\end{cases} \tag{6b}$$

Second case: when $\Sigma\Delta S_{jin}<\Delta S_{ID}$

$$\sum_{\Delta S_{j}\geq 0}\Delta S_j+C_c\sum_{\Delta S_{j}<0}\Delta S_j=\Delta S_{1D}\Rightarrow C_c=\frac{\Delta S_{1D}-\sum_{\Delta S_j\geq 0}\Delta S_j}{\sum_{\Delta S_j<0}\Delta S_j} \tag{7a}$$

$$\begin{cases}\text{If } \Delta S_{jin}\geq 0\Rightarrow \Delta S_j = \Delta S_{jin}\\ \text{If } \Delta S_{jin}<0\Rightarrow \Delta S_j=C_c\times\Delta S_{jin}\end{cases} \tag{7b}$$

Validation of model by comparison with stable channel

For this validation, for reason of computational time, the computation by the developed method was simplified by taking into account only one cross section. Discharge and water level were kept constant and slope continuously adapted to remain in uniform regime (which means that the computation is equivalent to compute one full reach supposed to be always at uniform flow). Erosion or deposition volumes were supposed proportional to sediment discharge. Later, it was verified that the conclusions concerning the final shape applied to the full model although the scour or the deposition does not make possible to remain with uniform flow.

In the steady and uniform conditions, in the absence of outside influences, a channel reaches a stable shape. Diplas and Vigilar (1992) have studied the geometry of a stable channel. They proposed a fifth-polynomial bank profile to represent the shape of threshold bank.

Fig. 2 shows that the developed method leads to a stabilised shape similar to the stable shape from Diplas and Vigilar (1992) even when starting from an irregular cross section. The main parameters are: initial water depth h=9m, water discharge rate Q= 30 m^3/s, initial longitudinal slope S=0.0001.

In a small flume, Stebbings's (1963) sent a discharge into a flat bed of sediments in order to form a stable channel. The comparisons for three parameters (top width, area and centreline channel

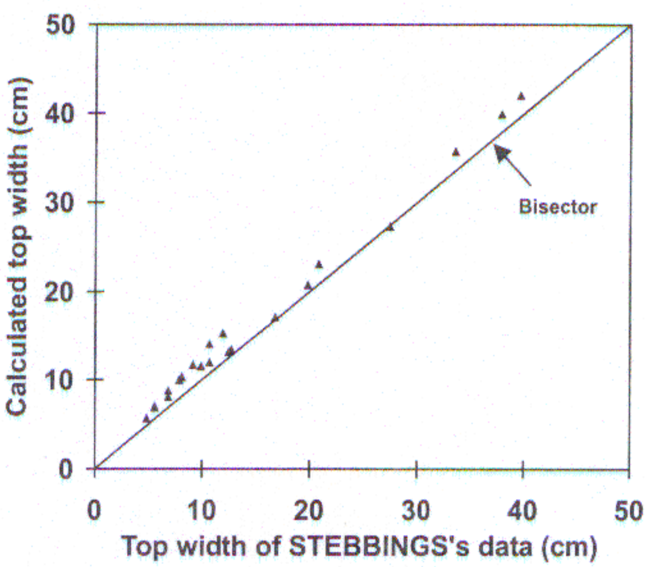

Fig. 3. Comparison of top width of stable section.

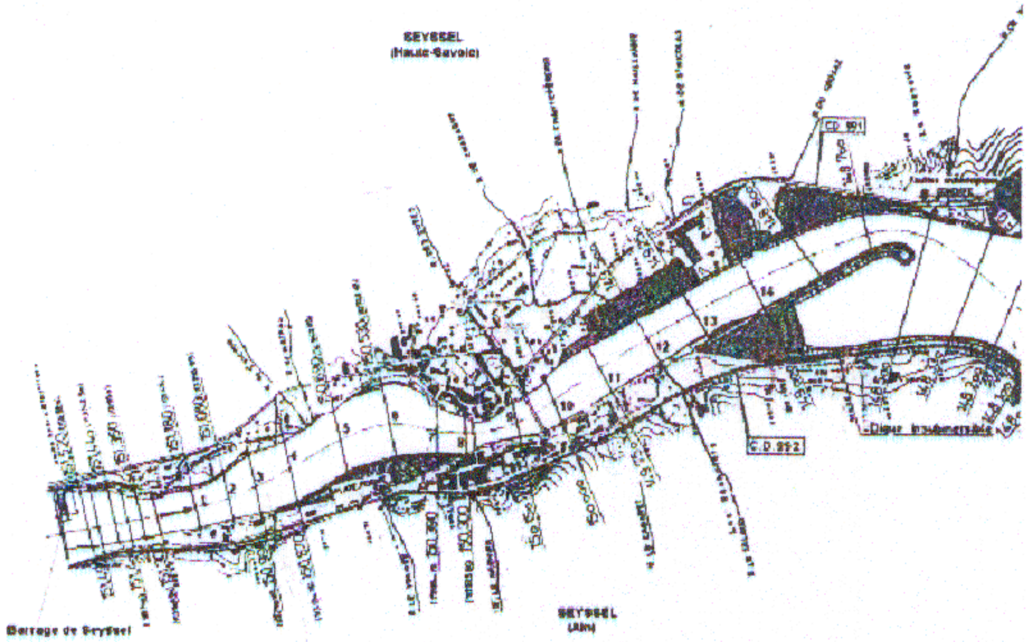

Fig. 4. Map of the studied reach of Rhône river.

depth of the stable channel) showed that the calculated values are in satisfactory agreement with the experimental data. Fig. 3 shows the comparison for top width of stable section.

Application to a reach of Rhône River

The figures 4 and 5 show the map and the longitudinal profile of the reach studied . This reach is situated between two dams, the Dam Seyssel and the Dam Motz on the Rhône river in the French Alps. The length of the reach is 2180 m and it includes 14 cross sections.

The observations show that the diameter of the sediments in this part of Rhône is very large. A median diameter (d=3 cm), a porosity (λ = 30%) and a density of the sediments (s_s=2.6) were proposed. Geometry of the bottom is defined by 14 cross sections for two dates June 1990 and April 1993.

The slope of the bottom is very varied and one sees holes and opposite slopes in some points (see Fig. 5). Fig. 6 shows the daily discharge input of the studied reach. Although it should be noted that critical discharge varies from one section to another one, the minimum discharge for noticeable sediment movement can be estimated to 500 m3/s. This discharge is exceeded 148 days in the period considered (14 %); thus a reduction of the computation time is possible by eliminating the periods corresponding to low discharges.

There is no information on the sediment discharge upstream of the reach. Maximum capacity of sediment transport in the first section upstream of the reach is then introduced in the reach. Note that this information is the only one necessary for sediments as the distributions of shear stress and rate of deposition inside one cross section are computed from sediment discharge and mean hydraulic variables in the cross section (in addition to the general characteristics of the sediments). Of course, this hypothesis of maximum capacity upstream leads to a sediment equilibrium in the upstream cross section for all the discharges.

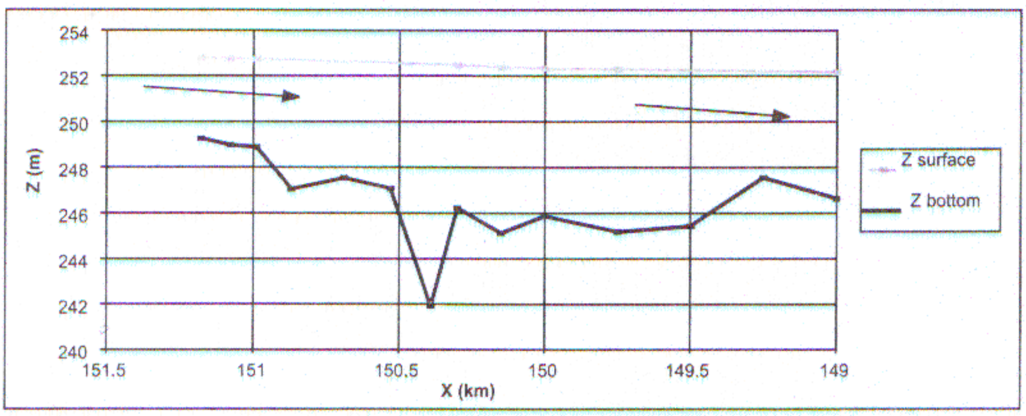

Fig. 5. Longitudinal profile of the bottom and the water surface at initial time (6/06/1990).

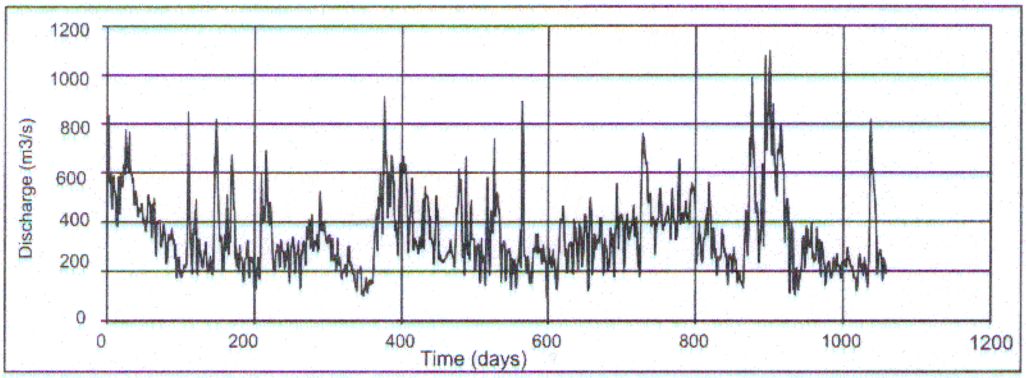

Fig. 6. Rhône river - Discharge hydrograph.

Results

The model was used to simulate the period from June 1990 to April 1993. In the majority of the sections, there is no significant deformation and the model confirms this point. In sections 7 and 8, which have changed a lot, the model (Fig. 7) is not far from measurements.

In section 7, the calculated deformation has a shift to the left compared to measurements. This phenomenon can be due to the meander of section 7. A correction can be applied by modifying the distribution of shear stress in order to take into account the extra stresses due to the meander. The relation could be the following one:

$$\Delta\tau = \rho J \frac{V^2}{2} \frac{x}{R_c} \tag{8}$$

in which $\Delta\tau$ is the correction of shear stress, ρ the water volumetric mass, J the energy slope, V the mean velocity in the section, x the transversal distance from the axis of the river, R_c the meander radius. Directly applied to the case of Rhône River, this relation does not seem to improve the results.

In section 8, eroded sediment in the middle of the bed was a pre-

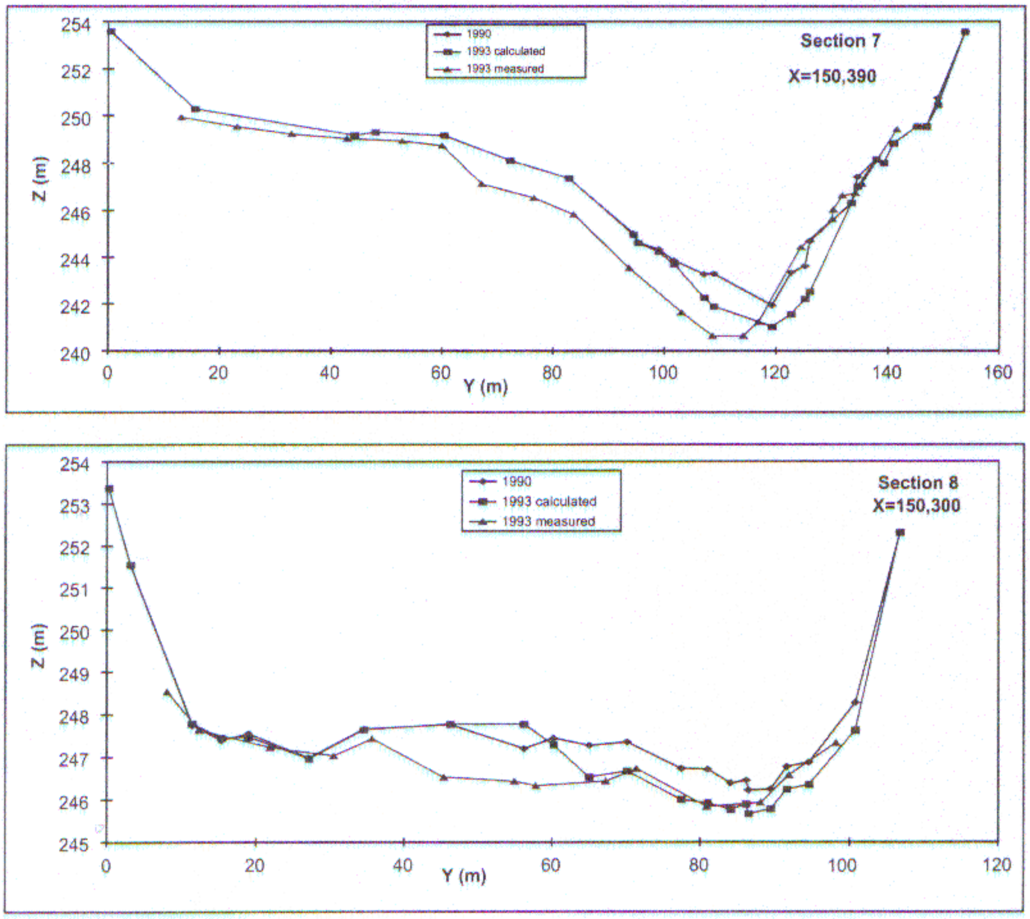

Fig. 7. Deformation of the reach studied of Rhone during the period 1990-1993 in the 2 sections 7 and 8.

vious deposit of finer sediment. It is possible to approach more closely the measured section by changing the critical shear stress in the part of the bottom which has been observed as "eroded" and computed as "not modified". A careful calibration of this critical shear stress, computation point after point, provides results close to the measurements but it is nearly impossible to do such calibration for real case studies.

More generally, the differences may be due to the uncertainty of the sediment and hydraulic data, to the errors of the topographical measurements used as references, to the existence of phenomena and obstacles like the meanders and the bridges which are not taken into account in a suitable way by the assumptions of the model and to the numerical errors of the computer code.

Moreover, due to the hypothesis of sediment homogeneity, the geometry is somewhat smoothened during the changes, may be in a quicker way than the real one. It can be guessed that, after a long period, with enough sediment transport, the cross sections would become completely smoothed as in Fig. 2.

Conclusion

The introduction of a detailed geometrical method (Merged Perpendicular Method) for computing boundary shear stress in a classical 1D bed-load sediment model provides a model that changes the 3D topography of the river bed in a realistic way. Compared to theoretical examples or simple laboratory experiments, the accuracy of the model is sufficient.

The study of a real reach shows that the model provides rather satisfactory results. The difficulties in such real cases stand in the data requirements (precise topography, size of the sediments,...) and the complexity of the real processes. Then, it is not obvious that the extra complexity brought by the computation of the distribution of shear stress is essential for the management of the river bed. Finally, more validations are necessary on real cases, for instance, in order to examine rivers with finer sediments in which the influence of suspended sediment cannot be neglected.

List of Symbols

C_c coefficient to adjust change in section
d mean sediment size
g acceleration of gravity
h water depth
j index of the point of discretisation of the wetted perimeter
J energy slope
K coefficient for computation of shear stress
K_s Manning-Strickler coefficient
K'_s grain Manning-Strickler coefficient
l_j length of the segment of wetted perimeter at point j
L active width
Q discharge
Q_s total sediment discharge
q_s sediment discharge per unit width
R_c meander radius
S longitudinal slope
s_s relative density of sediment
t time
X co-ordinate in flow direction
x transversal distance from the axis
S_{ID} bed section
ΔS_j final change of section related to point j
V mean water velocity
α ratio of dimensionless lift and drag forces
ϕ angle of repose of sand
λ porosity
ρ water volumetric mass
θ angle of bottom to horizontal
$\zeta = (K_s/K_s')^{3/2}$ roughness parameter
τ^* dimensionless boundary shear stress
τ^*_c dimensionless critical shear stress
$\tau^*_{c\theta}$ dimensionless critical shear stress for wall with angle θ to horizontal
τ^*_j dimensionless shear stress in point j
τ^*_{jc} dimensionless critical shear stress in point j

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