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AN ANALYTICAL SOLUTION FOR BENDING AND VIBRTION ANALYSIS OF TAPERED COMPOSITE BEAMS

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Summary. *In this paper an analytical method is implemented for solving the bending and free vibration of tapered composite beams. In this method, polynomial power series solution is used. A parametric study on tapered composite beams is conducted, considering the effects of taper angle, boundary conditions and laminate configuration.*

1 INTRODUCTION

Tapered composite beams are increasingly being used in various engineering applications such as robot arms, helicopter yoke and turbine blades. In tapered composite beams some plies were terminated at discrete locations along the length of the beam to change the stiffness of the beam [1]. Zabihollah and Ganesan [2] investigated free vibration of tapered composite beams by using higher-order finite element method.

In this paper, an analytical method is presented for the bending and free vibration analysis of midplane tapered composite beams. In this method polynomial power series solution is used for solving the governing equations of bending and free vibration of tapered beams. Also parametric study is carried out to investigate the influence of some parameters as taper angle, laminate configuration and different boundary conditions.

2 FORMULATION

A midplane tapered composite beam is shown in Figure 1. The governing equation of motion of free vibration for symmetric tapered composite beam is [2]:

$$\frac{\partial^2}{\partial x^2}\left[D_{11}(x)\cos^4\phi\frac{\partial^2 w}{\partial x^2}\right]+\rho_s(x)\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

In order to solve Equation (1) $w(x,t)$ is considered as $w(x,t)=Y(x)e^{i\omega t}$.Upon substitution of $w(x,t)$ into Equation (1) the following result will be obtained:

$$\frac{\partial^2}{\partial x^2}\left[D_{11}(x)\cos^4\phi\frac{\partial^2 Y}{\partial x^2}\right]-\rho_s\omega^2 Y=0 \tag{2}$$

where ω is the fundamental natural frequency of the beam. Since x=0 is an ordinary point for

Equation (2), $Y(x)$ can be replaced by $Y(x)=\sum_{n=0}^{N} c_n x^n$ **.** By substituting $Y(x)$ into Equation (2) and set the coefficients of each power of x equal to zero, the recurrence formula is obtained for c_n**.** Furthermore all of the coefficients of each power of x is obtained as a function of four coefficients c_0, c_1, c_2, c_3 and ω, as shown below:

$$Y(x)=c_0+c_1x+c_2(F)+c_3(G) \tag{3}$$

In Equation (3) F and G are functions of different powers of x and ω. The fundamental natural frequency ω is obtained by applying the boundary conditions and setting the determinant of the coefficients c_0, c_1, c_2 and c_3 equal to zero.

3 NUMERICAL RESULTS

In this section a tapered composite beam is considered. The laminate configuration in the thick section is $[0,90]_{2s}$ and the laminate configuration in the thin section is $[0,90]_s$ (For mechanical properties of composite materials see [1]). The variations of the natural frequency versus taper angle for different boundary conditions are shown in Figure 2.

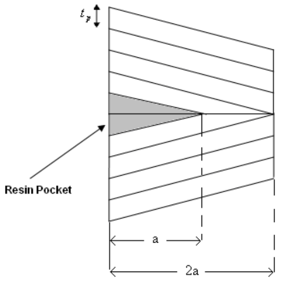

Figure 1: Tapered composite beam

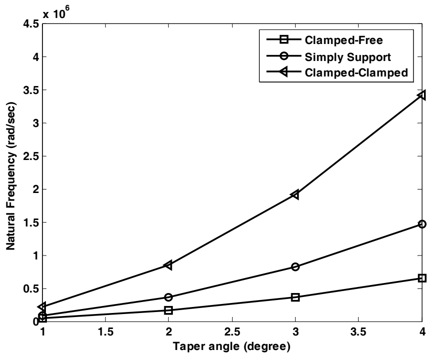

Figure 2: Variation of natural frequent versus taper angle

4 CONCLUSION

In this paper an analytical method was presented for solving the bending and free undamped vibration of tapered composite beams. A parametric study was performed considering the effects of taper angle, boundary conditions and laminate configuration. The obtained results showed that with increasing the taper angle, the natural frequency is also increased.

REFERENCES

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