

Optimal Supply Bidding with Risk Management in an Electricity Pay-as-Bid Auction

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Abstract - It is very important for the participants in a competitive electricity market, which is concerned to various uncertainties, to develop an optimal bidding to achieve maximum profit, especially in a discriminatory pricing (Pay-as-Bid) auction. In this research, the market clearing price (MCP) of each hour assumed to be known as a probability density function (pdf). In this paper, the bidding problem is modeled from a supplier viewpoint and its optimal solution is obtained analytically based on the classical optimization theory. Also, the analytical solution for a multi step bid protocol is generalized and properties of the generalized solution will be discussed. The model is developed to consider concept of risk in the bidding decision making problem. Two different methods for handling the risk are introduced. The proposed two methods are compared using some numerical examples and the results are interpreted. In addition, the effect of variation of MCP's pdf parameters on supplier profit is studied and the results are presented.

Keywords: power market, discriminatory pricing auctions, optimal bidding, supplier profit, risk.

1. INTRODUCTION

The electricity industry worldwide has experienced unprecedented restructuring for breaking traditional monopoly, introducing competition and establishing power market. Master concept of restructuring is that electricity can be traded as a commodity with unbundling the transmission and distribution as ancillary services [1]. Increasing the customer choice flexibilities, more social welfare and asset absorption of private sector are the general destinations of this process.

Auction is one of the useful mechanisms for establishing competition. Under auction based

electricity market environment, the participants submit their buy and sell bids on an hourly basis, which are then aggregated to form the total supply and demand curves. Based on the bid curves, an independent systems operator (ISO) determines the hourly market clearing price (MCP) and the power awarded to each bidder by solving a price-based unit commitment problem [2]. In this process, how well a participant garners profits depends, to a large extent, on how good its bidding strategy is. As a result, how to develop the optimal bidding strategy to obtain a maximum profit has become a major concern of generation companies (GenCos).

Because of strategic behavior of rival, demand fluctuation, forced outage of network components and uncertainty is underlying inherent in the market. Therefore, price volatility is unavoidable that result in more price forecasting complexities and importance. Thus, risk management is most important item that should be taken into account in bidding decision making problem.

There are two pricing rules, uniform price auction (UPA) and Pay-as-Bid (PAB) auction. Payments by the auctioneer to the producers differ across the two auction forms; under UPA all producer which bid below or at the MCP obtain this price, whilst under PAB producer are paid their bids, as long as this is below or equal to the MCP. According to this, it is seen that decision making in bidding strategy is more important in PAB auction than UPA.

So far, many research works have been done in building bidding strategies in the power market. In [3] under the assumption that the probabilistic distributions of competitors' offering prices are known, an optimal offering strategy for a single step bidding at a particular hour is derived by ignoring intertemporal unit constraints. Since bid information is revealed with a significant delay (e.g., five months in New England),

assuming that the probabilistic distributions of offering prices for hundreds of generators are known may not be practical. An iterative auction structure is recommended in [4] and [5], where GenCos are allowed to revise their offers iteratively, and the MCPs are updated and made public during the process until the market closes. Under this structure, a method based on genetic programming and finite state automata is presented in [5] for iteratively revising the offers. In [4] a Lagrangian relaxation-based method is presented for iteratively revising offers considering revenue adequacy. Unfortunately, there currently exist no power markets with the iterative auction structure. A literature survey on strategic bidding in competitive electricity markets is presented in [6].

The recently developed ordinal optimization approach is applied to select “good enough with high probability” offers in [7]. Game theory has also been applied to model market competitors in simplified systems [8-9]. In view of the problem complexity, it may have difficulties to derive payoff matrices.

In [10] based on cobweb equilibrium theory in market using residual demand, optimal strategy for a price taker GenCo is derived. Linear formulation for the bidding problem with self-scheduling according to residual demand is suggested in [11]. In [12] the congestion influence on bidding strategy is studied. Congestion represented as an opportunity to mark up bid price even in perfect competition. However taking into account this opportunity need a lot of data. Risk management and self-scheduling are considered simultaneously in [13]. Reserve and energy markets are modeled as a Markov joint process and the variance of MCP for each of markets considered as risk index. In [14], GenCos are divided into three groups (risk taker, risk neutral, risk averse) using their utility function and optimal strategy is derived for each participant.

The electricity market in IRAN started to operate on October 2003. The pricing rule in this market is based on PAB auction. In this paper, regarding IRAN electricity market structure, at first the bidding problem is modeled from the viewpoint of a supplier in a PAB based market. The analytical solution of this bidding problem is obtained based on the classical optimization theory. Also, the analytical solution for a multi step bid protocol is generalized and properties of the generalized solution are discussed.

To introduce uncertainty in bidding problem, a probability density function (pdf) for MCP is considered. The parameters of this pdf are supposed to be known, which can be forecasted through historical data and statistical techniques.

In another part of this paper the above model is developed to consider concept of risk in the bidding decision making problem. Two different methods for handling the risk are introduced. In the first method, the risk concept is defined based on probability of winning and is taken into account as a nonlinear inequality constraint in the optimization problem. Using Kuhn-Tucker theory, the optimal solution is determined

according to whether or not the risk constraint is activated.

In the second method, the risk is considered by variance of the profit random variable and is formulated by adding a penalty term in the objective function of the optimization problem.

The proposed two methods are explained using some numerical examples and the results are interpreted. Effect of market parameter integrated in variables of pdf of MCP is analyzed and discussed.

2. PROBLEM DESCRIPTION AND FORMULATION

Generally, GenCos have some generation units. Profit of GenCo is depending on position of units in merit list of auction. Therefore, optimal bidding decision making problem is to find the optimal bid (price and quantity) for each unit to offer to the market. Since market rules and structure affect the bidding problem, assuming a particular structure is necessary for formulating the problem.

2.1. Market structure

The bidding problem in this research is studied under perfect competition market assumption. In the other word, it is assumed there is no apparent effect of the genco bid decision on market prices.

According to the market rules, bids in step wise protocol should be offered hourly for each generating unit separately (fig.1).

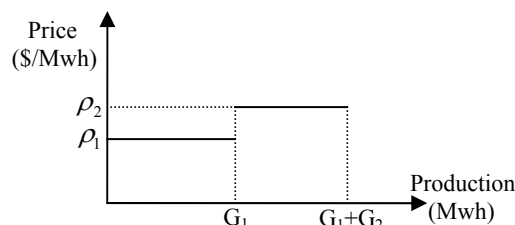


Figure 1: Two step bidding function

where, ρ_i and G_i are price and product quantity correspond to i th step respectively.

Offering price has upper and lower bounds. Winning units and power quantities to be produced by each unit are determined by running the PAB auction.

2.2. Problem formulation

If the most appropriate estimation of MCP is possible, it is sufficient that the offering price be fewer than estimated MCP. However, exact MCP forecasting is very difficult due to existence of different uncertainties. In this area many research works have been done to forecast and probabilistic modeling [15-17].

In this paper, MCP of each hour assumed to be known as a pdf ($f_{\rho_m}(\rho_m)$) and ρ_m is the MCP). In one hour of bidding period, profit of GenCo can be formulated as (1).

$$profit = f(\rho_m, \bar{\rho}, \bar{G}) = \sum_{i=1}^N (\rho_i - c_i) G_i \quad (1)$$

ρ_i , c_i and G_i are the bid price, average cost and the production quantity for i th step respectively. $\bar{\rho}$ and \bar{G} are the price and product quantity vectors that contain ρ_i and G_i .

Due to uncertainty, deterministic profit is not computable. Since the profit is depend on whether the bid will be accepted or not. Therefore, profit is naturally probabilistic variable. Since the pdf of MCP considered to be known, the profit can be defined like below as a mixed probabilistic variable.

$$profit = f(\rho_m, \bar{\rho}, \bar{G}) = \sum_{i=1}^n (\rho_i - c_i) G_i I_i(\rho_m, \rho_i) \quad (2)$$

$$I_i(\rho_m, \rho_i) = \begin{cases} 1 & c_{min} \leq \rho_i \leq \rho_m \\ 0 & \rho_m < \rho_i \leq \rho_{max} \end{cases}$$

c_{min} is the minimum average cost that can be offered to market without profit. $I_i(\rho_m, \rho_i)$ is a discrete random variable that shows the probabilistic nature of price acceptance according to ρ_m and ρ_i . when ρ_m is lower than ρ_i , bid price will not be accepted.

Since each bidding step is independent from the others, expected value of profit probabilistic variable can be computed separately as follows.

$$E(f_i(\rho_m, \rho_i, G_i)) = \int_{-\infty}^{\infty} (\rho_i - c_i) G_i I_i(\rho_m, \rho_i) f_{\rho_m}(\rho_m) d\rho_m \quad (3)$$

based on definition of I_i in eq.2, we have:

$$E(f_i(\rho_m, \rho_i, G_i)) = \int_{\rho_i}^{\rho_{max}} (\rho_i - c_i) G_i f_{\rho_m}(\rho_m) d\rho_m = (\rho_i - c_i) G_i (1 - F_{\rho_m}(\rho_i)) \quad (4)$$

$E(f_i(\rho_m, \rho_i, G_i))$ the expected value of profit for i th step and $F_{\rho_m}(\bullet)$ is the cumulative density function (cdf) of MCP.

Probability of acceptance of i th step price in the auction is equal to $P(\rho_i \leq \rho_m)$ and can be expressed on cdf of MCP.

$$Probability\ of\ acceptance = 1 - F_{\rho_m}(\rho_i) \quad (5)$$

As a result of eq.3, calculated expected profit is the function of offering bid. If GenCo's objective is to obtain the maximum profit without considering the risk, it will face with the following problem.

$$\max f(\rho_m, \bar{\rho}, \bar{G}) = \sum_{i=1}^n (\rho_i - c_i) G_i (1 - F_{\rho_m}(\rho_i)) \quad (6)$$

To maximization the above objective function, each step profit should be maximized independently. If production quantity of each step were fixed, driving the optimal price Using first order derivative rule is achievable as follows.

$$\frac{df(\rho_m, \rho_i, G_i)}{d\rho_i} = 0 \Rightarrow \frac{d}{d\rho_i} (\rho_i - c_i) G_i (1 - F_{\rho_m}(\rho_i)) = 0 \quad (7)$$

Then we can obtain ρ_i^* , the optimal offering price as follows:

$$(\rho_i^* - c_i) = \frac{(1 - F_{\rho_m}(\rho_i^*))}{f_{\rho_m}(\rho_i^*)} \quad (8)$$

We know the sequence of $\{c_i\}$, regarding to the nature of generating unit cost function, is a monotone

increasing sequence. Furthermore, the right side of eq.8 is positive. Therefore, it is obvious that the sequence of optimal bid price $\{\rho_i^*\}$ is also monotone increasing. This fact is illustrated in fig.2 where the left and right side of eq.8 are drawn to solve graphically this equation.

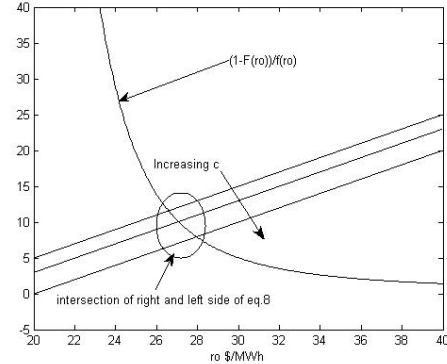


Figure 2: Graphical solution of eq.8 for different c and typical normal pdf ($N \sim (30, 16)$)

3. RISK CONSIDERATION

To this point the discussion focused on presenting methods to find the optimal bid price without considering the risk. But this concept is an important subject from GenCo's viewpoint. In this section two different methods for handling the risk are introduced. In the first method, the risk concept is defined based on probability of winning and in the second method, the risk is considered by variance of the profit random variable.

3.1. Method I

A simple and proper definition of risk is the probability that offered price ρ_i is not accepted in the auction. Therefore we define:

$$Risk_i = P(\rho_i > \rho_m) = F_{\rho_m}(\rho_i) \quad (9)$$

Formulation of eq.6 can be generalized in order to considering the risk as a constraint.

$$\max f(\rho_m, \bar{\rho}, \bar{G}) = \sum_{i=1}^n (\rho_i - c_i) G_i (1 - F_{\rho_m}(\rho_i)) \quad (10)$$

subject to: $1 - F_{\rho_m}(\rho_i) \geq \alpha_i$ $i = 1, \dots, N$, $0 \leq \alpha_i < 1$

In eq.10 $(1 - F_{\rho_m}(\rho_i))$ is acceptance probability of ρ_i in auction according to pdf of MCP.

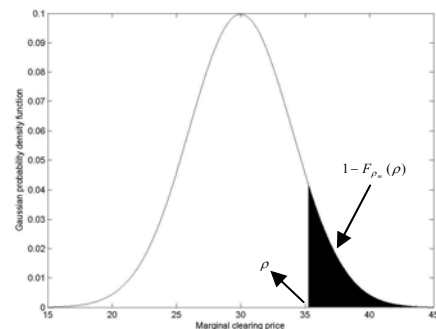


Figure 3: Graphical interpretation for acceptance probability

Graphic interpretation of $1-F_{\rho_m}(\rho_i)$ is shown in fig.3. Considering $1-F_{\rho_m}(\rho_i) \geq \alpha_i$ as a constraint in optimization problem, lead to introduce the acceptance probability as risk management in solving the bidding problem.

α_i is the measure of degree of risk for a participant in the bidding of step i (e.g. for risk averse participant, α_i is large).

The solution of eq.10 depends on whether constraint is active or not (let one step bidding ($\alpha_i = \alpha$)).

Case 1: Constraint is not active. Therefore eq.10 and eq.6 are the same. Thus, the optimal bid price can be obtained from eq.8.

Case 2: Constraint is active. Optimal bid price is obtained as follows:

$$1-F_{\rho_m}(\rho^{**}) = \alpha \Rightarrow \rho^{**} = F_{v_m}^{-1}(1-\alpha) \quad (11)$$

Where ρ^{**} is the optimal bid price obtained from eq.10.

Therefore the solution of eq.10 can be summarized as follows:

$$\rho^{**} = \begin{cases} \rho^* : \rho^* - c = \frac{1-F_{\rho_m}(\rho^*)}{f_{\rho_m}(\rho^*)} & \alpha \leq \alpha^* \\ F_{v_m}^{-1}(1-\alpha) & \alpha \geq \alpha^* \end{cases} \quad (12)$$

In this equation, α^* is the smallest value of α which activate the constraint, and it can be calculated from the following equation:

$$F_{\rho_m}^{-1}(1-\alpha^*) - c = \frac{\alpha^*}{f_{\rho_m}(F_{\rho_m}^{-1}(1-\alpha^*))} \quad (13)$$

Since the steps are independent, the procedure for ρ^{**} calculation can be easily developed for multi step bidding.

3.2. Method II

Considering the variance of the profit as a measure of risk is another way to introducing the risk. Variance represents the diversity of probabilistic variable, so that greater variance is index for greater risk. Since profit has been defined as a probabilistic variable, its variance can be calculated as follows:

$$Var(f_i(\rho_m, \rho_i, G_i)) = E(f_i^2(\rho_m, \rho_i, G_i)) - E^2(f_i(\rho_m, \rho_i, G_i)) \quad (14)$$

after manipulating this equation we have:

$$Var(f_i(\rho_m, \rho_i, G_i)) = (\rho_i - c_i)^2 G_i^2 (1-F_{\rho_m}(\rho_i)) F_{\rho_m}(\rho_i) \quad (15)$$

$Var(f_i(\rho_m, \rho_i, G_i))$ is the profit variance of step i.

If a finance problem is formulated as a decision making problem, decision based only on variance quantity is not appropriate [18]. Because variance quantity in comparison with expected value represent the risk (fig.4)

For example in fig.4 two finance options have different profit pdfs. Expected value of option 'b' is larger than option 'a'. However, variance of option 'b' is greater too. Consequently, the risk index for bidding problem can be defined as the ratio of variance to expected value.

$$R_i(\rho_m, \rho_i, G_i) = \frac{Var(f_i(\rho_m, \rho_i, G_i))}{E(f_i(\rho_m, \rho_i, G_i))} \quad (16)$$

R_i is the risk measure in step i.

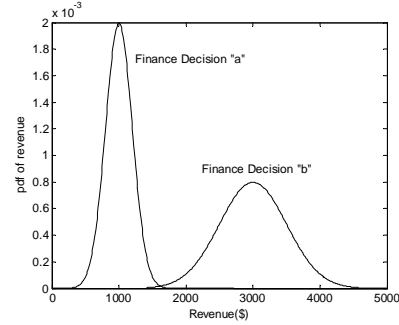


Figure 4: Two pdf for two finance options

The risk index can be considered as a penalty term in bidding decision making problem. Therefore we have:

$$Max \quad f(\rho_m, \bar{\rho}, \bar{G}) = \sum_{n=1}^N f_i - \sum_{n=1}^N \omega_i R_i \quad (17)$$

ω_i is a weighting factor that represents the importance of risk index in comparison with expected profit in step i. ω_i can change within $[0, \infty]$.

In single step bidding, eq.17 will be as follows:

$$Max \quad f(\rho_m, \rho, g) = f - \omega R = (\rho - c)G(1 - (1 + \omega)F_{\rho_m}(\rho)) \quad (18)$$

To obtain the optimal solution we have:

$$\frac{d}{d\rho}(f(\rho_m, \rho, G)) = 0 \Rightarrow \rho^* - c = \frac{1 - (1 + \omega)F_{\rho_m}(\rho^*)}{f_{\rho_m}(\rho^*)} \quad (19)$$

This equation is similar to eq.8 for different values of ω , we have different optimal solutions. Consequently, ω is a parameter which can represent the degree of risk of a GenCo.

$\omega = 0$ is refer to a risk neutral GenCo. If the function $(1-F_{\rho_m}(\rho))/f_{\rho_m}(\rho)$ is a descending function, increasing ω lead to decreasing the optimal bid price because of decreasing the right side of eq.19 for a specific ρ (fig. 5).

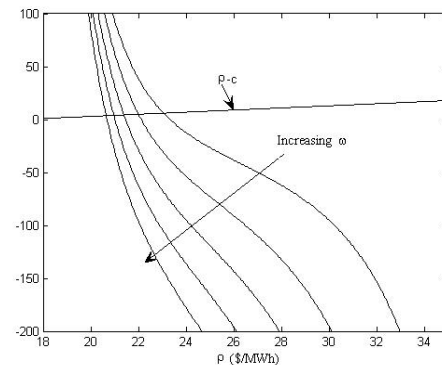


Figure 5: Different solutions for eq.19 relevant to different ω for a typical normal pdf($N \sim (30, 16)$) (intersection of left and right side of eq.19)

If function $(1-F_{\rho_m}(\rho))/f_{\rho_m}(\rho)$ is an ascending function increasing of ω cause to optimal price increasing. Therefore, selection of ω in order to risk management depend on the behavior of $(1-F_{\rho_m}(\rho))/f_{\rho_m}(\rho)$ function.

4. NUMERICAL EXAMPLES

Assume a GenCo with one generation unit and single step bidding strategy. To achieve on maximum profit, GenCo will offer maximum power to market. Let maximum power of generator be 250MW with 15\$/MWh as the average cost of production. In addition, the pdf of MCP is assumed to be normal.

In order to consider the variation of MCP's parameters in the calculation process, different cases are taken into account as table 1.

$\mu_m = cte$	$N\sim(30,9)$	$N\sim(30,16)$	$N\sim(30,36)$
$\sigma_m^2 = cte$	$N\sim(27,16)$	$N\sim(30,16)$	$N\sim(35,16)$

Table 1: Different constant variable for pdf

4.1. Optimal bidding without considering the risk

GenCo profit as a function of bid price is:

$$profit = f(\rho) = 250(\rho - 15)(1 - F_{\rho_m}(\rho)) \quad (20)$$

GenCo profit is shown in figs. 6 and 7 as a function of bid price for different values of μ_m and σ_m . The bid price correspond to the maximum profit is the optimal bid price. According to these figures, the optimal bid price, the probability of acceptance and the maximum profit are presented in tables 2 and 3.

Second order derivative of expected profit is shown in fig. 8 for $\mu_m=30$. It is clear that, for different σ_m , the prices less than μ_m are maximized the objective function.

From figs. 6 and 7, it can be seen that in the PAB auction, optimal bid price is not equal to generation average cost contrary to the UP auction. Since GenCo's profit is determined by its bid price, the proposed price should as high as possible. However, greater price has lower acceptance probability. Consequently, GenCo must be sought a judicious compromise between bid acceptance probability and expected profit.

$\mu_m=30$	$\sigma_m=3$	$\sigma_m=4$	$\sigma_m=6$
Optimal price	26.709	26.570	26.917
Acceptance Probability	0.8637	0.8043	0.6963
Expected profit	2528.2	2326.7	2074.5

Table 2: Optimal price property for different σ_m and constant μ_m

$\sigma_m=4$	$\mu_m=27$	$\mu_m=30$	$\mu_m=35$
Optimal price	24.341	26.587	30.658
Acceptance Probability	0.7469	0.8043	0.8611
Expected profit	1744.2	2326.7	3370.1

Table 3: Optimal price quantity different μ_m and constant σ_m

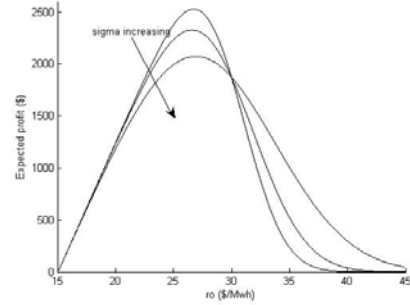


Figure 6: Expected profit for different σ_m and constant μ_m

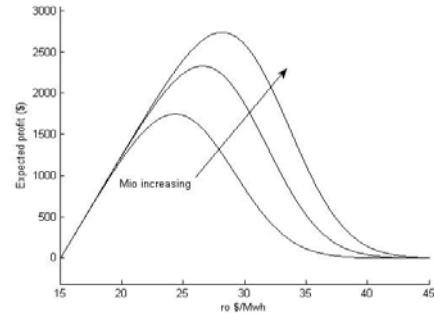


Figure 7: Expected profit for different μ_m and constant σ_m

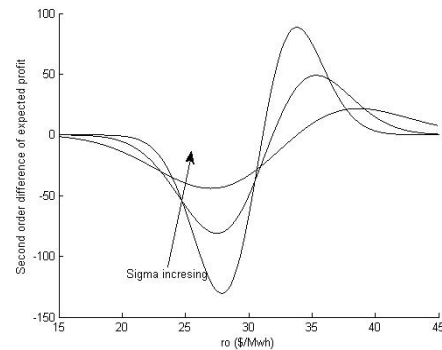


Figure 8: Second order derivative of expected profit function for different σ_m and $\mu_m=30$

According to fig. 6, increasing σ_m that is equivalent to increasing uncertainty in the market, while keeping μ_m constant, constitutes a decrease in the acceptance probability of optimal price and finally a decrease in the maximum expected profit.

It is evident from fig.7 that the optimal bid price and the maximum profit increase due to increase in μ_m . Since for two different μ_m , the acceptance probability for larger μ_m is greater in each price. Note that by increasing expected profit, acceptance probability is increasing too. This is the result of lower uncertainty.

4.2. Introducing the risk: method I

In method I that was proposed for risk consideration, at first α^* must be calculated. For different conditions α^* is calculated and presented in tables 4 and 5. According to the presented results in table 4, it can be seen that increase in uncertainty (i.e. increase in σ_m), leading to decrease in α^* . Increasing μ_m while keeping σ_m constant,

μ_m/σ_m is increased and consequently uncertainty is decreased relatively. Thus α^* will be increased (table 5).

$\mu_m=30$	$\sigma_m=3$	$\sigma_m=4$	$\sigma_m=6$
α^*	0.8637	0.8043	0.7745

Table 4: α^* for different σ_m and constant μ_m

$\sigma_m=4$	$\mu_m=27$	$\mu_m=30$	$\mu_m=35$
α^*	0.7469	0.8043	0.8308

Table 5: α^* for different μ_m and constant σ_m

For a GenCo, α is determined according to risk profile (risk averse, risk lover and risk neutral). Using eq. 10 and comparing the α and α^* the optimal bid price (ρ^*) will be obtained.

The maximum expected profit as a function of α is drawn in fig. 9 ($\sigma_m=4, \mu_m=30$). It is clear that when α is less than α^* , the optimal bid price is constant and equal to ρ^* and when α is greater than α^* the optimal bid price is equal to $F_{\rho_m}^{-1}(1 - \alpha)$ that is depend on α .

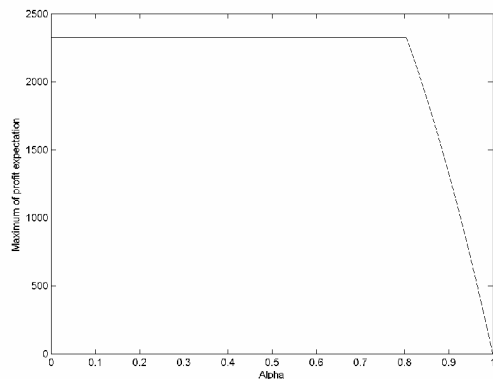


Figure 9: Maximum expected profit as a function of α . ($\sigma_m=4, \mu_m=30$)

4.3. Introducing the risk: method II

Objective function of GenCo with introducing the risk using method II is rewritten as below.

$$\text{Max } f(\rho, G) = 250(\rho - 15)(1 - (1 + \omega)F_{\rho_m}(\rho)) \quad (21)$$

Because of similarity between eqs. 17 and 16, for a constant ω , effect of market parameters (σ_m, μ_m) variations on optimization process is similar too.

Fig. 10 shows the objective function as a function of bid price for different value of ω . It is obvious that with increasing ω , optimal bid price decreases. Since increasing ω , in a specific bid price (ρ), result in decreasing $(1 - (1 + \omega)F_{\rho_m}(\rho))$ and consequently $f(\rho, G)$ decreases too.

It can be shown that with increasing ω GenCo is forced to offer the price which is near or equal to the average cost of power generation. Consequently in a PAB auction, the profit of GenCo is diminished.

Therefore, risk averse GenCos must select greater ω . GenCo can manage the risk with choosing a suitable ω and obtain the optimal bid price from maximization of eq. 21

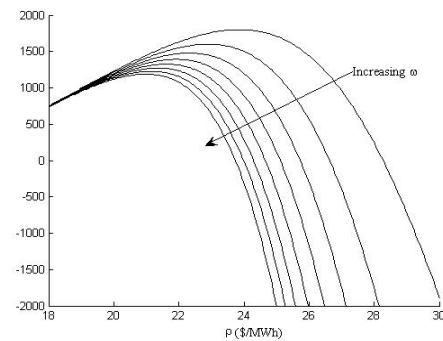


Figure 10: $f(\rho, G)$ for different value of ω

5. CONCLUSION

In this research, bidding decision making problem in a Pay-as-Bid electricity auction is formulated from the supplier viewpoint. The profit of a GenCo is modeled as a random variable and its expected value is considered as the objective function of bidding decision making problem. When considering a risk neutral GenCo, the optimal bid price is derived from the maximization of expected profit considering single and multi step bidding protocols. The effect of variations of MCP's parameters (mean and variance) on the expected profit of a GenCo is also studied. The results show that the optimal bid price is more sensitive to mean value of MCP than its variance. Therefore, forecasting the mean value of MCP is a critical issue in electricity PAB auction.

Risk concept is taken into account in the bidding problem using two proposed methods. In the first method risk is considered as a constraint in the optimization problem. The numerical example shows the area of permissible bidding price is limited through this risk constraint. In the second method, according to the managerial finance theory the risk index is defined as the ratio of variance to expected value of the profit and added to the problem as a penalty. The numerical examples illustrate this method is more flexible and the risk can be continuously managed.

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