Wavelet based estimation of the derivatives of a density for a ρ^*-mixing process

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1 Introduction

Wavelet analysis is a comparatively new mathematical tool for probability and statistics research. In early years of the past decade, many wavelets results have been introduced to statistical research fields and many interesting results may be found in periodicals and books. The reader may refer to Härdle *et al.*(1998) and Vidakovic (1999) for a detailed coverage of wavelet theory in statistics ants valuable robust properties and to Prakasa Rao (1999) for a recent comprehensive review and application of these and other methods of nonparametric functional asestimation. In this paper, our purpose is to extend the results in Prakasa Rao (1996) for estimating the derivatives of a density using wavelets to the case of a ρ^*-mixing sequence along the lines in Prakasa Rao (2003).
Definition: Let $S, T \subset N$ be nonempty and define $F_s = \sigma(X_k, k \in S)$, and the maximal correlation coefficient $\rho_n^* = \sup corr(f, g)$ where the supremum is taken over all (S, T) with $\text{dist}(S, T) \geq n$ and all $f \in L_2(F_s), g \in L_2(F_s)$, where $dist(S, T) = \inf_{x \in S, y \in T} |X - Y|$. A sequence of random variables $\{X\}_{n \geq 1}$ on a probability space $\{\Omega, F, P\}$ is said to be ρ^*-mixing if

$$\lim_{n \to \infty} \rho_n^* < 1$$

In the setup considered by Prakasa Rao (1996), we assume that ϕ is a scaling function generating an $r-$regular multiresolution analysis and $f^{(d)} \in \mathbf{L}_2(\mathbf{R})$, for some $r \geq (d+1)$. Furthermore, we assume that there exists $C_m \geq 0$ and $\beta_m \geq 0$ such that

$$|f^{(m)}(x)| \leq C_m(1 + |x|)^{-\beta_m}, 0 \leq m \leq r. \tag{1}$$

Prakasa Rao (1996) showed that the projection of $f^{(d)}$ on V_{j_0} is

$$f_{j_0}^{(d)}(x) = \sum_{k \in K_{j_0}} a_{j_0,k} \phi_{j_0,k}(x),$$

where

$$a_{j_0,k} = (-1)^d \int \phi_{j_0,k}^{(d)}(x) f(x) dx.$$

So its estimator is

$$\hat{f}_{j_0}^{(d)}(x) = \sum_{k \in K_{j_0}} \hat{a}_{j_0,k} \phi_{j_0,k}(x), \tag{2}$$

where

$$\hat{a}_{j_0,k} = \frac{(-1)^d}{n} \sum_{i=1}^{n} \phi_{j_0,k}^{(d)}(X_i).$$

The estimator in Eq. (2) will be used as an estimator for $f^{(d)}(x)$.

2 Main Results

First, we consider the sequence of random variables $\{X_i, i = 1, \ldots, n\}$ and extend the results of Prakasa Rao (1996) to integrated squared error, when the error is measured in $p-$norm. Therefore, one obtains his result by letting $p = 2$. Also, by considering $d = 0$, we obtain the results obtained in Kerkyacharian and Picard (1992), Leblanc (1996) and Tribouley (1995). Next, we consider the case of sequences with ρ^*-mixing condition and obtain similar results.

Before we discuss the main theorem of this paper, we state the following results that will be required in subsequent proofs, which are readily obtained by using the results of Sergey and Peligrad (2003):

For positive numbers $q \geq 2$ and $0 \leq r < 1$, there exists a positive constant $D = D(q, r)$ such that if $\{X_n\}_{n \geq 1}$ is a sequence of ρ^*-mixing centered random variables with finite absolute moments of order q and with $\rho^* \leq 1$, then for all $n \geq 1$,

$$E|\sum_{i=1}^{k} X_i|^q = D(\sum_{i=1}^{n} EX_i^q + (\sum_{i=1}^{n} EX_i^2)^{q/2})$$

Theorem Let $f^{(d)}(x) \in F_{s,p,q}$ with $s \geq \max(1/p, d), p \geq 1$, and $q \geq 1$. Consider the linear wavelet density estimator in Eq. (2.4) for an *i.i.d.* sequence of random variables $X_1, ..., X_n$. Then for $p' \geq max(2, p)$, there exists a constant C such that

$$\mathbf{E}\|\hat{f}_{j_0}^{(d)}(x) - f^{(d)}(x)\|_{p'}^2 \leq C\ n^{-\frac{2(s'-d)}{1+2s'}}$$

where $s' = s + 1/p' - 1/p$ and $2^{j_0} = n^{\frac{1}{1+2s'}}$.

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