On the Devroye–Mitran–Tarokh Rate Region for the Cognitive Radio Channel

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***Abstract*—An achievable rate region for the genie-aided cognitive radio channel is obtained using the celebrated Han-Kobayashi jointly decoding strategy for the interference channel and the Gel'fand-Pinsker coding scheme for channels with side information known at the transmitter. The achievable rate region is then simplified by Fourier-Motzkin elimination. The obtained achievable rate region (i) extends the Chong-Motani-Garg region for the interference channel to the cognitive channel and (ii) is a simplified description of the Devroye-Mitran-Tarokh rate region for the genie-aided cognitive radio channel.**

***Index Terms*—Interference channel, cognitive radio channel, rate region, simplified description.**

I. Introduction

A Genie-aided cognitive radio channel (Fig.1) is defined to be an interference channel (Fig.2) in which two senders TX1 and TX2 can transmit simultaneously over a common channel to two independent receivers RX1 and RX2 , when TX2 is non-causally aware of the message to be sent by TX1 (in the figures 1,2 and 3 the solid black lines show the wireless channels and the dotted lines denote the interference).

A. Interference Channel

The interference channel is a common channel between several pairs of sender-receivers, where each sender communicates with its respective receiver interfering with communications of the other sender-receivers. The study of this channel was initiated by Shannon [1], and furthered by Ahlswede [2]. Sato [3] obtained various inner and outer bounds by considering the associated multiple access and broadcast sub-channels in the interference channel. Carleial [4] established an improved achievable rate region (with one auxiliary random variable for every sender in Fig.2) by using sequential decoding and convex hull operation based on the superposition coding of Cover [5]. Han and Kobayashi [6], [7] generalized Cover's superposition coding to the many variable case, applied jointly or simultaneous decoding strategy (instead of sequential decoding) and the time-sharing formulation (instead of convex hull operation) for the general interference channel, thereby establishing the best achievable rate region to date. Chong, Motani, Garg and H. El Gamal [8], by slightly modifying the decoding error definition and

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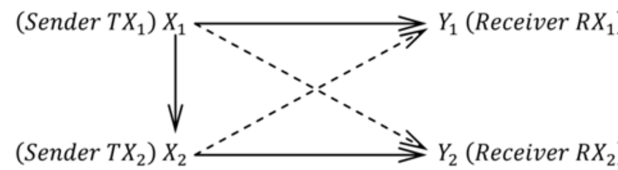

Fig. 1. Cognitive radio channel.

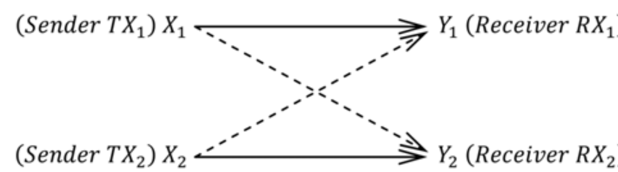

Fig. 2. Interference channel.

reducing the number of auxiliary random variables, derived a simplified description of the Han-Kobayashi rate region. The problem of determining the capacity region for the general interference channel is still open and has been solved for some very special cases (Han and Kobayashi [6], Carleial [9], Sato [10], Chung and Cioffi [11], Liu and Ulukus [12], Benzel [13], A. El Gamal and Costa [14], [15], Maric,Yates and Kramer [16]).

B. Cognitive Radio Channel

The idea of cognitive radio is a novel approach in wireless communications and was first presented by Mitola [17]. In cognitive radio either a network or a wireless node changes its transmission or reception parameters (every possible parameter or only the radio frequency spectrum) to efficiently communicate with licensed or unlicensed users.

Despite the explosive growth of interest in cognitive radios, many of the fundamental theoretical questions on the limits of such technology remain unanswered. The information theory for cognitive radio has been studied in [18] – [20]. Specifically, Devroye, Mitran and Tarokh [21] obtained an achievable rate region, in the information –theoretic sense, for cognitive radio channel by using the Han-Kobayashi jointly decoding strategy [6] (with four auxiliary random variables) and the Gel'fand-Pinsker coding [22]. Also, Jiang and Xin [23] derived a new achievable rate region (including several previously known rate regions) for cognitive radio channel.

C. Definitions

We denote random variables by $X,\ Y,\ W_1, \cdots$ with values $x, y, w_1, \cdots$ in finite sets $\mathcal{X}, \mathcal{Y}, \mathcal{W}_1, \cdots$; n-tuple vectors of $X_1, X_2, W_1, \cdots$ are denoted with $\mathbf{x}_1, \mathbf{x}_2, \mathbf{w}_1, \cdots$. We use the

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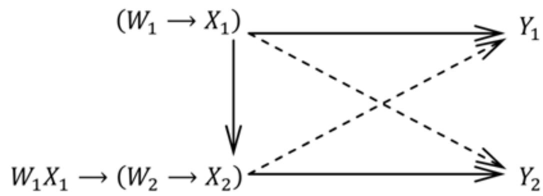

Fig. 3. Modified cognitive radio channel.

symbol $A_\epsilon^n(X_1, X_2, \cdots, X_l)$ to indicate the set of ϵ- typical n-sequences $(\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_l)$ [24].

In the discrete memoryless interference channel (Fig.2) and the cognitive radio channel (Fig.1), random variables X_1, X_2 are the inputs to the channels characterized by the conditional probabilities $p(y_1 \mid x_1x_2), p(y_2 \mid x_1x_2)$ with output random variables Y_1 , Y_2.

As in [6], we introduce a modified genie-aided cognitive radio channel (Fig.3) with auxiliary random variables W_1 and W_2 , representing the public information to be sent from TX1 to (RX1, RX2) with the rate T_1 and from TX2 to (RX1, RX2) with the rate T_2, respectively; and also serving as cloud centers that can be decoded by both receivers. The private information to be sent from TX1 to RX1 (with the rate S_1) and from TX2 to RX2 (with the rate S_2) is included in the random variables X_1 and X_2, respectively. Also, as in [6], $Q \in \mathcal{Q}$ is a time-sharing random variable whose n-sequences $\mathbf{q} = (q_1, q_2, \cdots, q_n)$ are generated independently of the messages. The n-sequences $\mathbf{q}$ are given to both senders and receivers.

For the modified genie-aided cognitive radio channel with four auxiliary random variables [21], the code and the achievable rate pair (R_1, R_2) are defined the same as in [21], but for the same channel with two auxiliary random variables, we can define the code as follows.

An $\left(n, \lfloor 2^{nT_1} \rfloor, \lfloor 2^{nS_1} \rfloor, \lfloor 2^{nt_2} \rfloor, \lfloor 2^{ns_2} \rfloor, \epsilon\right)$ code for the modified genie-aided cognitive radio channel (Fig.3) consists of $\lfloor 2^{nT_1} \rfloor$ codewords $\mathbf{w}_1(j)$, $\lfloor 2^{nS_1} \rfloor$ codewords $\mathbf{x}_1(j,k)$ for TX1 ; and $\lfloor 2^{nt_2} \rfloor$ codewords $\mathbf{w}_2(l)$, randomly thrown into $\lfloor 2^{nT_2} \rfloor$ bins, $\lfloor 2^{ns_2} \rfloor$ codewords $\mathbf{x}_2(b,l)$, randomly thrown into $\lfloor 2^{nS_2} \rfloor$ bins, for TX2 ; $j \in \{1, 2, \cdots, \lfloor 2^{nT_1} \rfloor\}$, $k \in \{1, 2, \cdots, \lfloor 2^{nS_1} \rfloor\}$, $l \in \{1, 2, \cdots, \lfloor 2^{nt_2} \rfloor\}$, $b \in \{1, 2, \cdots, \lfloor 2^{ns_2} \rfloor\}$, such that the average probability of decoding error is less than ϵ .

A quadruple (T_1, S_1, T_2, S_2) of nonnegative real numbers is achievable if there exists a sequence of codes such that the average error probabilities under some decoding scheme are less than ϵ. An achievable region for the modified genie-aided cognitive radio channel is the closure of a subset of the positive region of R^4 of achievable rate quadruples (T_1, S_1, T_2, S_2).

II. The Main Result

In this paper, first, we establish an achievable quadruple (T_1, S_1, T_2, S_2) for the genie-aided cognitive radio channel (theorem 1), and then describe the quadruple as the set of rate pairs (R_1, R_2) in theorem 2 which (i) includes the achievable rate pairs of Chong-Motani-Garg for the interference channel [8], and (ii) presents a simple description for Devroye-Mitran-Tarokh achievable rates in [21].

Theorem 1: For the modified genie-aided cognitive radio channel (Fig.3), let $Z = (QW_1W_2X_1X_2Y_1Y_2)$ and let $\mathcal{P}$ be the set of distributions on Z that can be decomposed into the general form:

$$p(qw_1w_2x_1x_2y_1y_2) = p(q)\ p(x_1w_1 \mid q)\, p(x_2w_2 \mid qx_1w_1) \cdot p(y_1y_2 \mid x_1x_2) \qquad \text{(i)}$$

For any $Z \in \mathcal{P}$ let $S(Z)$ be the set of all quadruples (T_1, S_1, T_2, S_2) of nonnegative real numbers such that

$$\begin{aligned}
S_1 &\le I(W_2; X_1 \mid W_1Q) + I(X_1; Y_1 \mid W_1W_2Q) \\
&= a_1 && (a) \\
S_1 + T_2 &\le I(Y_1; X_1W_2 \mid W_1Q) - I(W_2; X_1W_1 \mid Q) \\
&= b_1 && (b) \\
S_1 + T_1 &\le I(W_2; X_1 \mid Q) + I(Y_1; X_1 \mid QW_2) \\
&= c_1 && (c) \\
S_1 + T_1 + T_2 &\le I(Y_1; W_2X_1 \mid Q) - I(W_2; X_1W_1 \mid Q) \\
&= d_1 && (d) \\
S_2 &\le I(W_2; X_2 \mid W_1Q) + I(Y_2; X_2 \mid W_1W_2Q) \\
&\quad - I(X_2; X_1W_1 \mid Q) = a_2 && (e) \\
S_2 + T_1 &\le I(W_2; X_2W_1 \mid Q) + I(Y_2; X_2W_1 \mid QW_2) \\
&\quad - I(X_2; X_1W_1 \mid Q) = b_2 && (f) \\
S_2 + T_2 &\le I(Y_2; X_2 \mid W_1Q) - I(W_2; X_1W_1 \mid Q) \\
&\quad - I(X_2; X_1W_1 \mid Q) = c_2 && (g) \\
S_2 + T_2 + T_1 &\le I(Y_2; X_2W_1 \mid Q) - I(W_2; X_1W_1 \mid Q) \\
&\quad - I(X_2; X_1W_1 \mid Q) = d_2 && (h) \\
S_1 &\ge 0\ ,\ S_2 \ge 0\ ,\ T_1 \ge 0\ ,\ T_2 \ge 0 && (j)
\end{aligned}$$

Let S be the closure of $\bigcup_{Z \in \mathcal{P}} S(Z)$. Then any element of S is achievable.

Proof: See the Appendix A.

Corollaries:

Corollary 1: The quadruple (T_1, S_1, T_2, S_2) for the interference channel in [8, the lemma 3] is readily obtained from the quadruple (T_1, S_1, T_2, S_2) in theorem 1 if we consider the distribution (i) in the following special case as in [8]:

$$p(qw_1w_2x_1x_2y_1y_2) = p(q)\ p(x_1w_1 \mid q)\, p(x_2w_2 \mid q) \cdot p(y_1y_2 \mid x_1x_2) \qquad \text{(ii)}$$

Corollary 2: The quadruple (T_1, S_1, T_2, S_2), with two auxiliary random variables satisfying eight constraints in theorem 1, is a simplified description of the achievable rates with four auxiliary random variables satisfying sixteen constraints in [21, theorem 1,the constraints 6-21] for the genie-aided cognitive radio channel.

Theorem 2: For a fixed $P \in \mathcal{P}$ in theorem 1, let $\mathcal{R}(P)$ be the set of rate pairs $(R_1 = S_1 + T_1\ ,\ R_2 = S_2 + T_2)$ satisfying

$$
\begin{aligned}
&R_1 \le c_1 \;\;,\;\; R_1 \le a_1 + b_2 \;\;,\;\; R_2 \le c_2 \\
&R_2 \le a_2 + b_1 \;\;,\;\; R_1 + R_2 \le a_2 + d_1 \\
&R_1 + R_2 \le a_1 + d_2 \;\;,\;\; R_1 + R_2 \le b_1 + b_2 \\
&R_1 + 2R_2 \le a_2 + d_2 + b_1 \\
&2R_1 + R_2 \le a_1 + d_1 + b_2 \\
&R_1 \le d_1 \;,\; R_1 + R_2 \le b_1 + d_2 \\
&R_1 \ge 0 \;\;,\;\; R_2 \ge 0 \;\;.
\end{aligned}
$$

where a_i , b_i , c_i , d_i , $i = 1,2$ are all the same as in theorem 1. Then, $R = \bigcup_{P\in\mathcal{P}} \mathcal{R}(P)$ is an achievable rate region for the genie-aided cognitive radio channel.

Proof: See the Appendix B.

Corollaries:

Corollary 1: It can be shown that by considering the special distribution (ii) instead of (i), theorem 2 is reduced to the lemma 4 in [8].

Corollary 2: As mentioned before (corollary 2 to theorem 1), theorem 1 gives a simple description of the achievable rates in [21] with the constraints on (T_1, S_1, T_2, S_2). Therefore, theorem 2 as a consequence of theorem 1 demonstrates the corresponding simplified description on the rate pair (R_1, R_2).

APPENDIX A
THE PROOF OF THEOREM 1

It is sufficient to show that any element of $S(Z)$ for each $Z \in \mathcal{P}$ is achievable. So, fix $Z = (QW_1W_2X_1X_2Y_1Y_2)$ and take any (T_1, S_1, T_2, S_2) satisfying the constraints of the theorem.

Codebook generation: Consider $n > 0$, some distribution of the form (i) and

$$
\begin{aligned}
p(w_2 \mid q) &= \sum_{x_1,w_1} p(w_1 \mid q)\; p(x_1 \mid qw_1)\; p(w_2 \mid x_1w_1q) \\
p(x_2w_2 \mid q) &= p(w_2 \mid q)\; p(x_2 \mid qw_2) \\
&= \sum_{x_1,w_1} p(w_1 \mid q)\; p(x_1 \mid qw_1)\; p(x_2w_2 \mid x_1w_1q).
\end{aligned}
$$

Therefore, by using random binning we can generate the sequences of $\mathbf{w}_2$ and $\mathbf{x}_2$ independently of $\mathbf{w}_1$ and $\mathbf{x}_1$. So,

1. generate a n-sequence $\mathbf{q}$, i.i.d. according to $\prod_{i=1}^{n} p(q_i)$,
2. for the codeword $\mathbf{q}$, generate $\lfloor 2^{nT_1} \rfloor$ conditionally independent codewords $\mathbf{w}_1(j)$, $j \in \{1, 2, \cdots, \lfloor 2^{nT_1} \rfloor\}$ according to $\prod_{i=1}^{n} p(w_{1i} \mid q_i)$,
3. for the codeword $\mathbf{q}$ and each of the codewords $\mathbf{w}_1(j)$, generate $\lfloor 2^{nS_1} \rfloor$ n-sequence $\mathbf{x}_1(j,k)$, $k \in \{1, 2, \cdots, \lfloor 2^{nS_1} \rfloor\}$, i.i.d. according to $\prod_{i=1}^{n} p(x_{1i} \mid w_{1i}(j), q_i)$,
4. for the codeword $\mathbf{q}$, generate $\lfloor 2^{nt_2} \rfloor$ n-sequence $\mathbf{w}_2(l)$, $l \in \{1, 2, \cdots, \lfloor 2^{nt_2} \rfloor\}$, i.i.d. according to $\prod_{i=1}^{n} p(w_{2i} \mid q_i)$, and throw them randomly into $\lfloor 2^{nT_2} \rfloor$ bins such that the sequence $\mathbf{w}_2(l)$ in bin s_{21} is denoted as $\mathbf{w}_2(s_{21}, l)$, $s_{21} \in \{1, 2, \cdots, \lfloor 2^{nT_2} \rfloor\}$,
5. for the codeword $\mathbf{q}$ and each of the codewords $\mathbf{w}_2(s_{21}, l)$, generate $\lfloor 2^{ns_2} \rfloor$ n-sequences $\mathbf{x}_2(b,l)$, $b \in \{1, 2, \cdots, \lfloor 2^{ns_2} \rfloor\}$, i.i.d. according to $\prod_{i=1}^{n} p(x_{2i} \mid w_{2i}(l), q_i)$, and throw them randomly into $\lfloor 2^{nS_2} \rfloor$ bins such that the sequence $\mathbf{x}_2(b,l)$ in bin s_{22} is denoted as $\mathbf{x}_2(s_{22}, b, l)$, $s_{22} \in \{1, 2, \cdots, \lfloor 2^{nS_2} \rfloor\}$.

Encoding: The aim is to send a four dimensional message (j, k, s_{21}, s_{22}) whose first two components j and k are message indices and whose last two components s_{21} and s_{22} are bin indices. The messages actually sent over the genie-aided cognitive radio channel are $\mathbf{x}_1$ and $\mathbf{x}_2$. The message and bin indices are mapped into $\mathbf{x}_1$ and $\mathbf{x}_2$ as follows.

The sender TX1 to send j and k looks for $\mathbf{w}_1(j)$, $\mathbf{x}_1(j,k)$ and sends $\mathbf{x}_1(j,k)$.

The cognitive sender TX2 knowing $\mathbf{w}_1(j)$, $\mathbf{x}_1(j,k)$ non-causally and $\mathbf{q}$, to send (s_{21}, s_{22}) finds a sequence $\mathbf{w}_2(l)$ in bin s_{21} such that $(\mathbf{q}\;,\; \mathbf{w}_1(j)\;,\; \mathbf{x}_1(j,k)\;,\; \mathbf{w}_2(l)) \in A_\epsilon^n$ and then finds a sequence $\mathbf{x}_2(b,l)$ in bin s_{22} such that $(\mathbf{q}\;,\; \mathbf{w}_1(j)\;,\; \mathbf{x}_1(j,k)\;,\; \mathbf{w}_2(l)\;,\; \mathbf{x}_2(b,l)) \in A_\epsilon^n$ and sends $\mathbf{x}_2(s_{22}, b, l)$.

Decoding and analysis of error probability: The receivers RX1 and RX2 decode the corresponding messages independently, based on strong joint typicality [6]. The inputs $\mathbf{x}_1$ and $\mathbf{x}_2$, to the genie-aided cognitive radio channel are received at the receivers as $\mathbf{y}_1$ and $\mathbf{y}_2$, according to the conditional distributions $p(y_1 \mid x_1x_2)$ and $p(y_2 \mid x_1x_2)$, respectively. It is assumed that all messages are equiprobable. Without loss of generality it is assumed that $(j = 1, k = 1; s_{21} = 1, s_{22} = 1)$ is sent with the codeword $\mathbf{q}$, known to both receivers and senders. Notice that the first two components, j and k, are message indices, whereas the last two components, s_{21} and s_{22}, are bin indices. Now, we can find the constraints in theorem 1 such that the average probability of error $P_e^{(n)} \longrightarrow 0$ as the block length $n \longrightarrow \infty$.

The receiver RX_1 , by receiving $\mathbf{y}_1$ and knowing $\mathbf{q}$, decodes $j = 1, k = 1; s_{21} = 1$ or $jk(s_{21}, l) = 11(1, l)$ simultaneously [6]. Therefore, we can define the event $E_{jk(s_{21},l)}$ and $P_e^{(n)}$ as follows, thereby establishing the constraints that lead to ($P_e^{(n)} \longrightarrow 0$ as the block length $n \longrightarrow \infty$). (The state $j = k = 1$; $s_{21} \ne 1$ is not considered as error).

$$
\begin{aligned}
&E_{jk(s_{21},l)} = \{(\mathbf{q}, \mathbf{w}_1(j), \mathbf{x}_1(j,k), \mathbf{w}_2(s_{21}, l), \mathbf{y}_1) \in A_\epsilon^n\} \\
&P_e^{(n)} = P\left\{E_{11(1,l)}^c \cup E_{jk(s_{21},l)\ne 11(1,\hat{l})}\right\} \\
&\le P\left(E_{11(1,l)}^c\right) + \sum_{jk(s_{21},l)\ne 11(1,l)} P\left(E_{jk(s_{21},l)}\right) \\
&\le \epsilon + \overbrace{\sum_{j\ne1,k=s_{21}=1}\cdots}^{\boxed{1}} + \overbrace{\sum_{j=s_{21}=1,k\ne1}\cdots}^{\boxed{2}} + \overbrace{\sum_{j\ne1,k\ne1,s_{21}=1}\cdots}^{\boxed{3}} \\
&+ \overbrace{\sum_{j\ne1,s_{21}\ne1,k=1}\cdots}^{\boxed{4}} + \overbrace{\sum_{k\ne1,s_{21}\ne1,j=1}\cdots}^{\boxed{5}} + \overbrace{\sum_{j\ne1,k\ne1,s_{21}\ne1}\cdots}^{\boxed{6}} \;,
\end{aligned}
$$

from where in order to ($P_e^{(n)} \longrightarrow 0$ as the block length $n \longrightarrow \infty$), in accordance with the codebook it is necessary and sufficient that:

$$S_1 \le I(Y_1W_2; X_1 \mid QW_1) \tag{1}$$

$$S_1 + t_2 \le I(Y_1; W_2X_1 \mid W_1Q) \tag{2}$$

$$S_1 + T_1 \leq I\left(Y_1 W_2; X_1 W_1 \mid Q\right) \tag{3}$$
$$S_1 + T_1 + t_2 \leq I\left(Y_1; X_1 W_2 \mid Q\right) \tag{4}$$

Similarly, for the receiver RX2 that receives $\mathbf{y}_2$, knows $\mathbf{q}$ and decodes $j = 1\ ,\ s_{21} = 1\ ,\ s_{22} = 1$ or $j\left(s_{22}, b\right)\left(s_{21}, l\right) = 1\left(1, b\right)\left(1, l\right)$ simultaneously [6], we can define the similar event and $P_e^{(n)}$ as follows: (The state $j \neq 1\ ;\ s_{21} = s_{22} = 1$ is not considered as error).

$$\begin{aligned}
&E_{j(s_{22},b)(s_{21},l)} \\
&\qquad = \left\{\left(\mathbf{q}, \mathbf{w}_1\left(j\right), \mathbf{x}_2\left(s_{22}, b, l\right), \mathbf{w}_2\left(s_{21}, l\right), \mathbf{y}_2\right) \in A_\epsilon^n\right\} \\
&P_e^{(n)} \leq \epsilon + \sum_{j(s_{22},b)(s_{21},l) \neq 1(1,b)(1,l)} P\left(E_{j(s_{22},b)(s_{21},l)} \middle| \cdots\right) \\
&\qquad \leq \epsilon + \sum_{j(s_{22},b)(s_{21},l) \neq 1(1,b)(1,l)} \cdots \\
&\qquad = \epsilon + \overset{\boxed{1}}{\sum_{j=1=s_{21}, s_{22}\neq 1}} \cdots + \overset{\boxed{2}}{\sum_{j=1=s_{22}, s_{21}\neq 1}} \cdots \\
&\qquad + \overset{\boxed{3}}{\sum_{j\neq 1, s_{22}\neq 1, s_{21}=1}} \cdots + \overset{\boxed{4}}{\sum_{j=1, s_{22}\neq 1, s_{21}\neq 1}} \cdots \\
&\qquad + \overset{\boxed{5}}{\sum_{j\neq 1, s_{22}=1, s_{21}\neq 1}} \cdots + \overset{\boxed{6}}{\sum_{j\neq 1, s_{21}\neq 1, s_{22}\neq 1}} \cdots\ ,
\end{aligned}$$

from where in order to ($P_e^{(n)} \longrightarrow 0$ as the block length $n \longrightarrow \infty$), in accordance with the codebook it is necessary and sufficient that:

$$s_2 \leq I\left(Y_2 W_2; X_2 \mid W_1 Q\right) \tag{5}$$
$$T_1 + s_2 \leq I\left(Y_2 W_2; W_1 X_2 \mid Q\right) \tag{6}$$
$$s_2 + t_2 \leq I\left(Y_2; X_2 \mid W_1 Q\right) \tag{7}$$
$$s_2 + t_2 + T_1 \leq I\left(Y_2; X_2 W_1 \mid Q\right) \tag{8}$$

On the other hand, in accordance with the used codebook and Gel'fand-Pinsker coding [22], we have:

$$\begin{aligned}
&I\left(W_2; X_1 W_1 \mid Q\right) \leq t_2 - T_2 \\
&\qquad \Longrightarrow T_2 - t_2 \leq -I\left(W_2; X_1 W_1 \mid Q\right) && (9) \\
&I\left(X_2; X_1 W_1 \mid Q\right) \leq s_2 - S_2 \\
&\qquad \Longrightarrow S_2 - s_2 \leq -I\left(X_2; X_1 W_1 \mid Q\right) && (10)
\end{aligned}$$

Then, the constraints (a)-(h) in theorem 1 are obtained by eliminating t_2 and s_2 from (1)-(10) as follows:

$$\begin{aligned}
&(1) \Longrightarrow (a)\ ;\ (2)\ \text{and}\ (9) \Longrightarrow (b)\ ;\ (3) \Longrightarrow (c) \\
&(4)\ \text{and}\ (9) \Longrightarrow (d)\ ;\ (5)\ \text{and}\ (10) \Longrightarrow (e) \\
&(6)\ \text{and}\ (10) \Longrightarrow (f)\ ;\ (9), (7)\ \text{and}\ (10) \Longrightarrow (g) \\
&(8), (9)\ \text{and}\ (10) \Longrightarrow (h)\ .
\end{aligned}$$

APPENDIX B
THE PROOF OF THEOREM 2

We put $S_1 = R_1 - T_1$ and $S_2 = R_2 - T_2$ in all of the relations (a)-(j) in theorem 1.Then, by using Fourier-Motzkin elimination technique as in [7] and eliminating redundant relations, we reach to the constraints in theorem 2 (for brevity the details are omitted).

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