Capacity Region for a More General Class of Broadcast Channels

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Abstract- We can categorize the broadcast channels with known capacity into two groups: One group consists of broadcast channels where the receivers have different capabilities (degraded, less noisy and more capable channels); the other group consists of broadcast channels with specialized message sets (broadcast channels with degraded message sets, semi-deterministic and deterministic broadcast channels). We combine these two groups into a more general class named extended less noisy broadcast channels and determine its capacity region using Marton inner bound and the outer bound considered by Nair and El Gamal. The capacity region for this class of channels (i) includes as its sub-regions those of most previous classes and (ii) is the first capacity region for broadcast channels which involves three auxiliary random variables.

Key words: Broadcast channel, Capacity region, More general class

I. INTRODUCTION

Broadcast channel. The two receiver discrete and memoryless broadcast channel $(\mathcal{X}, p(y,z|x), \mathcal{Y}\times\mathcal{Z})$ consists of three finite sets $\mathcal{X}$ (input alphabet), $\mathcal{Y},\mathcal{Z}$ (output alphabets) and a collection of probability distributions $p(y,z|x)$ on $\mathcal{Y}\times\mathcal{Z}$, one for each $x\in\mathcal{X}$. Without loss of generality, it is assumed that the channel components are independent, i.e. , given an input letter x, two output letters y and z are generated independently of each other. The nth extension of the broadcast channel $(\mathcal{X}, p(y,z|x), \mathcal{Y}\times\mathcal{Z})$ is the broadcast channel $\left(\mathcal{X}^n, p\left(\underline{y},\underline{z}\middle|\underline{x}\right), \mathcal{Y}^n\times\mathcal{Z}^n\right)$, where $\underline{x}$, $\underline{y}$ and $\underline{z}$ indicate the vectors $\underline{x}=(x_1,\cdots,x_n)$, $\underline{y}=(y_1,\cdots,y_n)$ and $\underline{z}=(z_1,\cdots,z_n)$ respectively and $p\left(\underline{y},\underline{z}\middle|\underline{x}\right)=\prod_{i=1}^{n}p(y_i,z_i|x_i)$.

Three sources M_0,M_1,M_2 (random variables) are defined on the set of integers $\mathcal{M}_0=[1, 2^{nR_0}]$, $\mathcal{M}_1=[1, 2^{nR_1}]$, $\mathcal{M}_2=[1, 2^{nR_2}]$, respectively, where $m_1\in\mathcal{M}_1$ and $m_2\in\mathcal{M}_2$ are private messages and $m_0\in\mathcal{M}_0$ is a common message. We assume that M_0,M_1,M_2 are generated independently and equiprobably over $\mathcal{M}_0,\mathcal{M}_1,\mathcal{M}_2$, respectively.

A $\left((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n\right)$ code for a broadcast channel consists of three sets of integers $\mathcal{M}_0=[1, 2^{nR_0}]$, $\mathcal{M}_1=[1, 2^{nR_1}]$, $\mathcal{M}_2=[1, 2^{nR_2}]$, an encoding function $\underline{x}: \mathcal{M}_0\times\mathcal{M}_1\times\mathcal{M}_2\longrightarrow\mathcal{X}^n$
and two decoding functions

$g_1: \mathcal{Y}^n\longrightarrow\mathcal{M}_0\times\mathcal{M}_1$; $g_1(\underline{Y})=(\widehat{M_0},\widehat{M_1})$

$g_2: \mathcal{Z}^n\longrightarrow\mathcal{M}_0\times\mathcal{M}_2$; $g_2(\underline{Z})=\left(\widehat{\widehat{M_0}},\widehat{M_2}\right)$.

The set $\{\underline{x}(m_0,m_1,m_2) : (m_0,m_1,m_2)\in\mathcal{M}_0\times\mathcal{M}_1\times\mathcal{M}_2\}$ is called the set of codewords. Assuming a uniform distribution on the set of messages $\mathcal{M}_0\times\mathcal{M}_1\times\mathcal{M}_2$. we can define $P_{e_1}^n$ and $P_{e_2}^n$ to be the average probabilities of error at the decoders g_1 and g_2 respectively:

$$P_{e_1}^n=\frac{1}{2^{n(R_0+R_1+R_2)}}\sum_{(m_0,m_1,m_2)\in\mathcal{M}_0\times\mathcal{M}_1\times\mathcal{M}_2}P\left(g_1(\underline{Y})\neq(m_0,m_1)\middle|(m_0,m_1,m_2)\ sent\right)$$

$$P_{e_2}^n=\frac{1}{2^{n(R_0+R_1+R_2)}}\sum_{(m_0,m_1,m_2)\in\mathcal{M}_0\times\mathcal{M}_1\times\mathcal{M}_2}P\left(g_2(\underline{Z})\neq(m_0,m_2)\middle|(m_0,m_1,m_2)\ sent\right)$$

The rate (R_0, R_1, R_2) is said to be achievabe by a broadcast channel if, for any $\varepsilon>0$, there exists for all sufficiently large n, a $\left((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n\right)$ code such that max $\{P_{e_1}^n, P_{e_2}^n\}<\varepsilon$.

The capacity region C for the broadcast channel is the closure of the set of all achievable rates (R_0, R_1, R_2) .

Background. The broadcast channel was first introduced by Cover in his pioneering work [1]. Since then the problem of determining the capacity region has been investigated extensively and complete solutions have been characterized for special classes, including the degraded broadcast channels [2],[3],[4], broadcast channels with degraded message sets [5], less noisy [6], more capable [7], deterministic with no common message [8],[9], semi-deterministic [10], deterministic with common massage [11] channels, the sum and product of two reversely degraded broadcast channels [24] and Gaussian multiple-input multiple-output broadcast channels with $R_0=0$ [25].General inner bounds [12],[13],[14],[22] and outer bounds [14],[15],[22] have been established. The outer bound in [16] is strictly tighter than other outer bounds. Recently developed new outer bound in [26] does'not have clear improvement on other bounds.

Additionally, broadcast channels with confidential messages [17], broadcast channels with cooperating decoders [18], wide-band broadcast channels [19], [20], broadcast channels with relay [21], [22] and broadcast channels with correlated sources and receiver side information [27] have been studied. In [23] a region of achievable rate triplets for arbitrarily varying general broadcast channels is given.

Our work. In this paper, we define a more general class of broadcast channels named Extended Less Noisy Broadcast Channels (ELNBC) and determine its capacity region using Marton inner bound [14] and Nair, El Gamal outer bound [16]. Then, we show that this capacity region includes the capacity regions for broadcast channels with degraded message sets, semi-deterministic, deterministic with and without common message, degraded, less noisy and (special) more capable broadcast channels.

Paper organization.The paper is organized as follows. In section II we refer to existing capacity regions and then recall the known inner and outer bounds of broadcast channels. In section III we define Extended Less Noisy Broadcast Channels (ELNBC) and determine its capacity region using Marton inner bound and recently developed Nair and El Gamal outer bound. In section IV we prove that the capacity region for this new class includes the capacity regions for broadcast channels with degraded message sets, semi-deterministic, deterministic with and without common message, less noisy and (special) more capable broadcast channels. Finally, a conclusion is prepared.

II. EXISTING CAPACITY REGIONS, INNER BOUNDS AND OUTER BOUNDS FOR BROADCAST CHANNELS

Here, we remind broadcast channels with known capacity region and recall general inner bounds and outer bounds for broadcast channels. In the remainder of the paper,

- We use the auxiliary random variable W for bounding the rate R_0 of common message M_0 and the auxiliary random variables U, V for bounding the rates R_1, R_2 of private messages M_1, M_2, respectively.
- We discuss about every broadcast channel by considering Fig.1and general joint distributions of the form $p(uvwx)\ p(yz|x)$, from where for the random variables $(WUVXYZ)$, we have the following Markov chain:

$$WUV \longrightarrow X \longrightarrow YZ \qquad (5)$$

- Therefore, a broadcast channel $(\mathcal{X}\ , p(y,z|x)\ , \mathcal{Y} \times \mathcal{Z})$ can be shown by the Fig.1 .

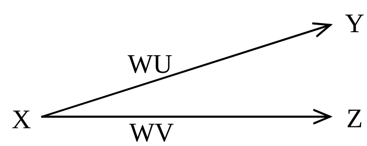

Fig.1 Broadcast channel

II.1 Broadcast channels with known capacity: Special broadcast channels with known capacity can be categorized into two groups: One group consists of degraded [2]-[4] and capability degraded broadcast channels [6],[7]; the other group consists of broadcast channels with specialized message sets, namely broadcast channels with degraded message sets [5], deterministic [8],[9],[11] and semi-deterministic [10] broadcast channels.

II.2 The inner bound theorem: Here, we recall a more general theorem from Marton with dependent W, U, V,which is a generalization of the inner bound theorem with independent W, U, V(Cover, Van der Meulen, Hajek and Pursley)[14,theorem 1].

The inner bound theorem (Marton) [14, theorem 2] : For the broadcast channel in Fig.1, the set of rate triples $(R_0\ , R_1\ , R_2)$ satisfying :

$$\begin{cases} R_y = R_0 + R_1 \leq I(WU;Y) & \text{(6-a)} \\ R_z = R_0 + R_2 \leq I(WV;Z) & \text{(6-b)} \\ R_y + R_z = R_0 + R_1 + R_2 \leq min\,\{I(W;Y)\ , I(W;Z)\} + I(U;Y|W) + I(V;Z|W) - I(U;V|W) = A & \text{(6-c)} \end{cases}$$

for some $p(uvwx)\ p(y,z|x)$, is achievable.

II.3 The outer bound theorem : The below outer bound from Nair-El Gamal with common message is a general one and is contained in Korner-Marton outer bound [14, theorem 5 and its remarks].

Nair-El Gamal outer bound [16, theorems 2.1 and 3.1] : The set of rate triples $(R_0\ , R_1\ , R_2)$ satisfying :

$$\begin{cases} R_0 \leq \min\{I(W;Y)\ , I(W;Z)\} & \\ R_0 + R_1 \leq I(UW;Y) & \text{(7-a)} \\ R_0 + R_2 \leq I(WV;Z) & \text{(7-b)} \\ R_0 + R_1 + R_2 \leq min\,\{\boldsymbol{B = I(UW;Y) + I(V;Z|UW)\ , C = I(VW;Z) + I(U;Y|VW)}\} = D & \text{(7-c)} \end{cases}$$

for some joint distribution of the form

$$\begin{cases} p(u,v,w,x)\ p(y,z|x) = p(u)\ p(v)\ p(w|v,u)\ p(x|\mathrm{u},v,w)\ p(y,z|x) \quad , \quad \mathcal{W} \neq \emptyset & \text{(8)} \\ p(u,v,x)\ p(y,z|x) = p(u,v)\ p(x|u,v)\ p(y,z|x) \quad , \quad \mathcal{W} = \emptyset & \text{(9)} \end{cases}$$

constitutes an outer bound to the capacity region for the broadcast channel.

III. EXTENDED LESS NOISY BROADCAST CHANNEL AND ITS CAPACITY REGION

In this section we define a more general class of broadcast channels having specialized message sets and the property of less noisiness and then show that the coincidence of Marton general inner bound and Nair-El Gamal outer bound results in its capacity region which is the first capacity region with three auxiliary random variables for broadcast channels.

Definition : A broadcast channel $(\mathcal{X}\ , p(y,z|x)\ , \mathcal{Y} \times \mathcal{Z})$ with auxiliary random variables W, U, V is said to be the Extended

Less Noisy Broadcast channel (ELNBC) if there exists joint distribution $p(u,v,w,x)$ such that:

$$\begin{cases} V \longrightarrow WZ \longrightarrow U & \text{(10-a)},\\ I(W;Y) \le I(W;Z) & \text{(10-b)}, \end{cases}$$

where

$$\begin{cases} p(u,v,w,x)\,p(y,z|x) = p(u)\,p(v)\,p(w|v,u)\,p(x|u,v,w)\,p(y,z|x) \quad , \quad \mathcal{W} \ne \emptyset & \text{(10-c)}\\ p(u,v,x)\,p(y,z|x) = p(u,v)\,p(x|u,v)\,p(y,z|x) \quad , \quad \mathcal{W} = \emptyset \;\; , \;\; V \ne U & \text{(10-d)} \end{cases}$$

Interpretation: Specialized message sets are seen in (10-a) and the relation (10-b) indicates the less noisiness.

Theorem : The set of rate triples (R_0 , R_1 , R_2) satisfying:

$$\begin{cases} R_y = R_0 + R_1 \le I(WU;Y) & \text{(11-a)}\\ R_z = R_0 + R_2 \le I(WV;Z) & \text{(11-b)}\\ R_y + R_z = R_0 + R_1 + R_2 \le I(WU;Y) + I(V;Z|UW) = B & \text{(11-c)} \end{cases}$$

constitutes the capacity region for the ELNBC.

Remark : The relations (10-a,b) can be replaced by (12-a,b) below ($V \rightleftarrows U$ and $Y \rightleftarrows Z$) :

$$\begin{cases} U \longrightarrow WY \longrightarrow V & \text{(12-a)}\\ I(W;Z) \le I(W;Y) & \text{(12-b)} \end{cases}$$

Then, from (11-a,b,c) by the replacements ($V \rightleftarrows U$, $Y \rightleftarrows Z$, $R_1 \rightleftarrows R_2$), the corresponding capacity region (13-a,b,c) is obtained:

$$\begin{cases} R_y = R_0 + R_1 \le I(WU;Y) & \text{(13-a)}\\ R_z = R_0 + R_2 \le I(WV;Z) & \text{(13-b)}\\ R_y + R_z = R_0 + R_1 + R_2 \le C = I(VW;Z) + I(U;Y|VW) & \text{(13-c)} \end{cases}$$

<u>The proof of theorem</u> :

Achievability : The relations (11-a,b) are trivial from (6-a,b). And (11-c) is obtained from (6-c) :

$R_0 + R_1 + R_2 \le A = min\,\{I(W;Y)\,, I(W;Z)\} + I(U;Y|W) + I(V;Z|W) - I(U;V|W) \overset{\text{(10-b)}}{=} I(UW;Y) + I(V;Z|W) - I(U;V|W) = I(UW;Y) - H(V|WZ) + H(V|UW) \overset{\text{(10-a)}}{=} I(UW;Y) - H(V|WZU) + H(V|UW) = I(UW;Y) + I(V;Z|UW) = B$,

therefore,

$$R_0 + R_1 + R_2 \le A = B = I(UW;Y) + I(V;Z|UW) \qquad \text{(14)}$$

Converse : The relations (11-a,b) are trivial from (7-a,b).

And from (7-c) we have :

$R_0 + R_1 + R_2 \le D = min\,\{B, C\} \overset{(14)}{=} min\,\{A, C\} \le A = B$, that is, the inner bound (6-c) coincides with the outer bound (7-c). Then the proof is complete.

Remark : For a broadcast channel with (12-a,b), the capacity region is (13-a,b,c), instead of (11-a,b,c).

IV. EXTENDED LESS NOISY BROADCAST CHANNELS AND OTHER KNOWN CLASSES

Here, we want to compare our general class, namely extended less noisy broadcast channels (ELNBC) and previous classes. At first, we show that broadcast channels with degraded message sets, semi-deterministic and deterministic (with and without common message) broadcast channels are all special cases of ELNBC and their capacity regions are derived from the capacity region of ELNBC. Then, we demonstrate that ELNBC is an improved and extended definition of less noisy channel(special ELNBC is a less noisy channel and vice versa) and also ELNBC is a more capable channel in a special case.

IV.1 ELNBC and broadcast channel with degraded message sets

For every broadcast channel with degraded message sets we have one of two states.

State I. $I(W;Y) \le I(W;Z)$ where (10-b) is satisfied. By $U = \emptyset$, (10-a) is true and Fig.1 indicates a broadcast channel with degraded message sets and $R_1 = 0$. By $WV = X$, we have $p(u,v,w,x)\,p(y,z|x) = p(x)\,p(y,z|x)$. So, broadcast channel with degraded message sets is a ELNBC and (11-a,b,c) give its capacity region:

(11-a) $\Longrightarrow$ $R_0 \le I(W;Y)$ (15-a)

(11-b) $\Longrightarrow$ $R_0 + R_2 \le I(WV;Z) = I(X;Z)$ (15-b)

(11-c) $\Longrightarrow$ $R_0 + R_2 \le I(W;Y) + I(V;Z|W) \le I(W;Z) + I(V;Z|W) = I(WV;Z) = I(X;Z)$ (15-c)

(15-a,c) $\Longrightarrow$ $R_2 \le I(V;Z|W) = I(X;Z|W)$ (15-d)

That is, (15-a,b,d) show the capacity region for the broadcast channel with degraded message sets as in [5].

State II. $I(W;Z) \le I(W;Y)$ where (12-b) is satisfied. By $V = \emptyset$, (12-a) is true and Fig.1 indicates a broadcast channel with degraded message sets and $R_2 = 0$. By $WU = X$, we have $p(u,v,w,x)\,p(y,z|x) = p(x)\,p(y,z|x)$. So, broadcast channel with degraded message sets is a ELNBC and (13-a,b,c) give its capacity region (the details are omitted).

IV.2 ELNBC and semi-deterministic broadcast channel

In a semi-deterministic broadcast channel, If $Z = g(X)$(generally $g(X)$ is non-invertible), in the definition we can choose $W = \emptyset$ and $V = g(X) = Z$, thereby $p(u,v,w,x)\,p(y,z|x) = p(u,x)\,p(y,z|x)$ and (10-a,b) are satisfied and semi-deterministic broadcast channel becomes a ELNBC. Then, (11-a,b,c) give its capacity region as in [10] :

(11-a) $\Longrightarrow$ $R_y \leq I(U;Y)$
(11-b) $\Longrightarrow$ $R_z \leq H(Z)$
(11-c) $\Longrightarrow$ $R_y + R_z \leq I(U;Y) + H(Z|U)$

- If $Y = f(X)$, by $Y = U$, $W = \emptyset$, the conditions (12-a,b) are satisfied and the corresponding capacity region (13-a,b,c) gives the capacity for semi-deterministic broadcast channel :

$$R_y \leq H(Y)\ , R_z \leq I(V;Z)\ , R_y + R_z \leq I(V;Z) + H(Y|V)$$

IV.3 ELNBC and deterministic broadcast channel without common message

If $Y = f(X)$ and $Z = g(X)$, by $W = \emptyset$, $V = Z$, $U = Y$, (10-a,b) are true and deterministic channel is a ELNBC, therefore, (11-a,b,c) give the capacity for deterministic broadcast channel without common message :

$$R_y \leq H(Y)\ , R_z \leq H(Z)\ , R_y + R_z \leq H(Y;Z)$$

IV.4 ELNBC and deterministic channel with common message

In this case $W \neq \emptyset$, $Y = f(X)$, $Z = g(X)$, generally $f(X)$, $g(X)$ are non-invertible functions and we have one of two states.

State I. $I(W;Y) \leq I(W;Z)$ where (10-b) is satisfied. By $V = g(X) = Z$, (10-a) is true, then deterministic channel is a ELNBC and by noting to $U = f(X) = Y$, (11-a,b,c) give its capacity as follows.

$$R_0 + R_1 \leq I(W;Y) + H(Y|W) \qquad (16\text{-a})$$

$$R_0 + R_2 \leq I(W;Z) + H(Z|W)$$

From $I(W;Y) \leq I(W;Z)$, it is sufficient to have $R_0 + R_2 \leq I(W;Y) + H(Z|W)$ (16-b), and

$$R_0 + R_1 + R_2 \leq I(W;Y) + H(YZ|W) \qquad (16\text{-c})$$

State II. $I(W;Z) \leq I(W;Y)$ where (12-b) is satisfied. By $U = f(X) = Y$, (12-a) is true, then deterministic channel is a ELNBC and by noting to $V = g(X) = Z$, (13-a,b,c) give its capacity in this state as follows.

$$R_0 + R_1 \leq I(W;Z) + H(Y|W) \qquad (17\text{-a})$$

$$R_0 + R_2 \leq I(W;Z) + H(Z|W) \qquad (17\text{-b}),$$

$$R_0 + R_1 + R_2 \leq I(W;Z) + H(YZ|W) \qquad (17\text{-c})$$

Therefore, from (16 and 17a,b,c) for deterministic channel having common message we have as in [11]:

$$R_y = R_0 + R_1 \leq min\,\{I(W;Y), I(W;Z)\} + H(Y|W)\ , R_z = R_0 + R_2 \leq min\,\{I(W;Y), I(W;Z)\} + H(Z|W)\ ,$$

$$R_y + R_z = R_0 + R_1 + R_2 \leq min\,\{I(W;Y), I(W;Z)\} + H(YZ|W)$$

IV.5 ELNBC and less noisy channels (note that every degraded channel is a less noisy channel and hence need not be considered here.)

- ELNBC in a special case is a less noisy channel. We can prove this claim in both cases $W = \emptyset$ and $W \neq \emptyset$.

(a) $W = \emptyset$: By $V = X$, we have :

(10-a) $\Longrightarrow$ $V = X \longrightarrow Z \longrightarrow U$ (18),

and (18) and ((5) $\longrightarrow$ $(YZ \longrightarrow X \longrightarrow U)$) $\Longrightarrow$ $I(U;X) = I(U;Z)$ (19),

and ((5) $\longrightarrow$ $(Y \longrightarrow X \longrightarrow U)$) $\Longrightarrow$ $I(U;Y) \leq I(U;X)$ (20),

and (19, 20) $\Longrightarrow$ $I(U;Y) \leq I(U;Z)$ $\Longrightarrow$ channel is a less noisy channel [6],

and then

(11-a) $\Longrightarrow$ $R_1 \leq I(U;Y)$
(11-b) $\Longrightarrow$ $R_2 \leq I(V;Z) = I(X;Z)$
(11-c) $\Longrightarrow$ $R_1 + R_2 \leq I(U;Y) + I(X;Z|U) \leq I(U;Z) + I(X;Z|U) = I(X;Z)$

So, we have: $R_1 \leq I(U;Y)\ , R_2 \leq I(X;Z|U)$.

(b) $W \neq \emptyset$: By $U = W$, $VW = VU = X$, we have :

(10-b) $\Longrightarrow$ $I(U;Y) \leq I(U;Z)$ $\Longrightarrow$ channel is a less noisy channel [6],

- Less noisy channel is a special ELNBC.

If $W \neq \emptyset$, $U = W$, $VW = VU = X$, we have : $p(u,v,w,x)\,p(y,z|x) = p(u)\,p(x|u)\,p(y,z|x)$ which shows the less noisy channel's distribution and

less noisy channel $\Longrightarrow$ $I(U = W;Y) \leq I(U = W;Z)$ $\Longrightarrow$ (10-b) and

$U = W$ $\Longrightarrow$ (10-a), then, (10-a,b) $\Longrightarrow$ ELNBC

IV.6 ELNBC and more capable channel

- For every ELNBC with $W \neq \emptyset$, $U = W = V = X$, (10-b) gives $I(X;Y) \leq I(X;Z)$ for every $p(u,v,w,x)\,p(y,z|x) = p(x)p(y,z|x)$, that is, ELNBC is a more capable channel. But, the inverse is not true because X is the input variable and $I(X;Y) \leq I(X;Z)$ does not necessarily lead to (10-a,b) having arbitrary auxiliary random variables.
- Special more capable broadcast channel is a ELNBC

If Z is more capable than Y, then every private message to Y is always a common message for both Y and Z. Therefore, we can put $U = \emptyset$, $WV = X$ in Fig.1.Now consider a special more capable channel ($U = \emptyset$, $WV = X$, $V \longrightarrow W \longrightarrow Y$) in which $I(X;Y) \leq I(X;Z) \Longrightarrow I(X;Z) = I(W;Z) + I(V;Z|W) \geq I(X;Y) = I(W;Y) + I(V;Y|W)\ , (V \longrightarrow W \longrightarrow Y) \Longrightarrow$

$I(V;Z|W) \geq I(V;Y|W)$, hence $I(W;Z) - I(W;Y) \geq$ negative number $\Longrightarrow I(W;Z) \geq I(W;Y)$ or $I(W;Z) \leq I(W;Y)$, i.e., (10-a,b) are satisfied and the channel is a ELNBC and (11-a,b,c) give the capacity region for special more capable channel.

So, the class of ELNBC with three random variables W, U, V and hence general distribution $p(u,v,w,x)\, p(y,z|x)$ is a more general class and its capacity region includes as its sub-regions those of most previous classes.

V. CONCLUSION

A more general class of broadcast channels with general distribution $p(u,v,w,x)\, p(y,z|x)$ was defined the capacity region of which (i) is obtained from the coincidence of Marton general inner bound and recently developed Nair-El Gamal tight outer bound ,(ii) includes the capacity region for broadcast channels with degraded message sets, semi-deterministic, deterministic with and without common message, less noisy and (special) more capable broadcast channels and (iii) is the first capacity region with three auxiliary random variables.

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