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A Novel Method for Singular Value Decomposition of Polynomial Matrices and ICI Cancellation in a Frequency-Selective MIMO Channel

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Abstract

In this paper, we introduce two novel methods to cancel ICI in frequency-selective MIMO channels, which suffer from inter-channel interference (ICI) and inter-symbol interference (ISI). Both of these two methods change the multiple-input multiple-output channel to some parallel single-input single-output (SISO) channels, based on the singular value decomposition (SVD) of the polynomial matrices. In the first method, genetic algorithm (GA) is used to design near optimum precoder and equalizer to cancel ICI. In the second method, using Taylor's expansion we introduce another novel method to analytically estimate the SVD of the channel matrix, which is a polynomial matrix. The SVD estimation of the channel matrix is then used to design a precoder and an equalizer and cancel the ICI. The second method is called parametric method in this paper. The simulations results show that the both introduced methods have acceptable ability to cancel ICI. The parametric method is based on an analytical approach; therefore, it is very fast and can be used in real-time systems. Besides, the first method can be applied to cancel both destructive interferences, ICI and ISI, simultaneously.

Key Words: SVD, Frequency-selective MIMO channels, Polynomial matrices, ICI, Genetic Algorithm, Taylor's expansion.

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1. INTRODUCTION

Using new methods to achieve higher capacities is an answer to the increasing demand for mobile communications. Taking advantages of diversities in time, frequency and space can improve the mobile systems throughput. Spatial diversity and using multiple antennas at both sides of a communication link, at transmitter and receiver, is currently being extensively studied in mobile and wireless communications. Although the MIMO systems that use the spatial diversity have higher capacity, they have some disadvantages. For instance, spatial interference, which is called ICI, increases bit error rate (BER) of the system (Paulraj, A., Nabar, R., et al., 2003). Furthermore, frequency selectivity of the channel can cause another kind of interference called ISI (Simon, M. K., and Alouini, M. S., 2005).

Suppose that the channel H has N input antennas and M output antennas with a memory the length of which is L. The vector of output signals, $\mathbf{y}$, at time k is then can be written as the following convolutional form:

$$\begin{aligned}\mathbf{y}_{M\times1}(k) &= H_{M\times N} * \mathbf{s}_{N\times1}(k) + \mathbf{n}_{M\times1}(k) \\ &= \sum_{l=0}^{L-1} H^{(l)}\mathbf{s}(k-l) + \mathbf{n}(k)\end{aligned} \tag{1}$$

Where $\mathbf{s}$ and $\mathbf{n}$ are the input signals of the channel and the white Gaussian noise added to the output and H is an $M \times N$ polynomial matrix the elements of which are polynomials of z :

$$H = \begin{bmatrix} H_{11}(z) & \cdots & H_{1N}(z) \\ \vdots & \ddots & \vdots \\ H_{M1}(z) & \cdots & H_{MN}(z) \end{bmatrix} = H^{(0)} + H^{(1)} z^{-1} + H^{(2)} z^{-2} + \ldots + H^{(L-1)} z^{-L+1} \quad (2)$$

A polynomial matrix is simply a matrix with polynomial elements, however it can alternatively be thought of as a polynomial with matrix coefficients. To show the ICI and ISI we can rewrite Eqs. 1 and 2 in the following forms:

$$y_m(k) = \sum_{l=0}^{L-1} h_{mm}^{(l)} s_m(k-l) + \overbrace{\sum_{l=0}^{L-1} \sum_{\substack{n=1 \\ n \neq m}}^{N} h_{mn}^{(l)} s_n(k-l)}^{\textbf{ICI}} + n_m(k-l) \quad (3)$$

$$\mathbf{y}(k) = H^{(0)} \mathbf{s}(k) + \overbrace{\sum_{l=1}^{L-1} H^{(l)} \mathbf{s}(k-l)}^{\textbf{ISI}} + \mathbf{n}(k) \quad (4)$$

Although ICI and ISI seem to be drawbacks of a frequency-selective MIMO channel, if treated in an appropriate way, they can possibly cause constructive diversities in space and time, respectively. Hence, the performance and throughput of the system will be enhanced. In the following parts, we discuss the algorithms that are used to cancel each of these two kinds of interference when the other one is not present. Combining these algorithms helps us cancel both ICI and ISI, together.

1.1. ICI Cancellation Using SVD in a Flat MIMO Channel

There are different algorithms like maximum-likelihood, successive and BLAST family receiving methods that can be used to cancel ICI in a flat MIMO channel (Proakis, J., 2007). One of these methods is based on singular value decomposition (SVD) of a matrix, which is an efficient method to exploit the capability of MIMO systems (Zamiri-Jafarian, H., and Gulak, G., 2005)(Akhlaghi, I., and Zamiri, H., 2005)(Akhlaghi, I., and Khoshbin, H., 2007). Since our work is a generalization of the SVD-based algorithm, here, we introduce it briefly. Suppose that the MIMO flat (non-frequency-selective or ISI-free) channel is modeled with the matrix H. The singular value decomposition of the channel matrix could be defined as (Griffel, D. H., 1989):

$$H_{M \times N} = U_{M \times M} S_{M \times N} V_{N \times N}{}^{H} \quad (5)$$

$U^H U = I_{M \times M}$ and $V^H V = I_{N \times N}$

M and N are the number of antennas at the receiver and transmitter side of the link respectively. S is a diagonal matrix containing singular values of H. U and V are two orthogonal and normal (orthonormal) matrices the columns of which are respectively left and right singular vectors. Using matrices U and V as equalizer and precoder, the ICI would be omitted. The overall channel matrix and the output signal at time k become as follow (Akhlaghi, I., and Zamiri, H., 2005):

$$H' = U^H H V = U^H U S V^H V = S \tag{6}$$

$$\begin{aligned} \mathbf{z}[k] &= U^H H V \mathbf{s}[k] + U^H \mathbf{n}[k] \\ &= U^H U S V^H V \mathbf{s}[k] + U^H \mathbf{n}[k] = S\mathbf{s}[k] + \eta[k] \end{aligned} \tag{7}$$

Where, $\mathbf{n}[k]$ and $\eta[k]$ are additive Gaussian noise of the channel output and the receiver input respectively. Since the equalizer matrix U^H is an orthogonal and the equalizer input noise $\mathbf{n}[k]$ is white, $\eta[k]$ remains white, as well. Furthermore, because of the fact that power of U^H is unity, signal and noise power passing through the system remain constant. These two features make noise analysis of the system very simple. Figure 1 shows the block-diagram of a system using SVD to cancel the ICI of a flat MIMO channel (Akhlaghi, I., and Zamiri, H., 2005)(Akhlaghi, I., and Khoshbin, H., 2007).

1.2. ISI Cancellation in a Frequency-Selective SISO Channel

When there are more than one path between the transmitter and the receiver in a SISO communication system, the frequency response of the channel would not be flat, and it treats as a frequency-selective channel. In this case, the ISI appears and makes the performance worse. To prevent this, the channel frequency response may be equalized with one filter precedes and one filter follows the channel. These filters are called precoder and equalizer respectively. The precoder and equalizer filters compensate the non-ideal frequency response of the channel. There are some different methods to design these filters like Zero-Forcing method and its modified version that is called MMSE method (Proakis, J., 2007).

Another approach to prevent the ISI in a SISO channel is to divide the channel bandwidth to smaller sub-bands with almost flat frequency responses. Each of these sub-bands can be behaved like a channel with no ISI. This approach is called multi-carrier method (Wang, Z., and Giannakis, G. B., 2000). If the carriers of the sub-channels are orthogonal to each other, the system is named OFDM[1], which is currently used in some practical systems and standards like IEEE 802.11a. In the OFDM

[1] Orthogonal Frequency Division Multiplex

system, the orthogonality is applied using IFFT transform, in which each of the M successive symbols modulates the orthogonal carriers $e^{j\frac{2\pi mt}{M}}$. Hence, demodulation is done obviously using FFT transform (Pandharipande, A., 2002).

1.3. MIMO-OFDM Systems

Taking advantages of the OFDM idea in a MIMO channel will lead us to a relatively novel area, MIMO-OFDM, which has been exceedingly studied in recent years (Kyung, W. P., and Yong, S. C., 2005). Figure 2 demonstrates the structure of a MIMO-OFDM system. As it can be seen, in a MIMO-OFDM system, first, the effect of frequency-selectivity, ISI, and then ICI are cancelled (Kyung, W. P., and Yong, S. C., 2005).

MIMO-OFDM has been heralded as the solution for moving wireless systems to higher speed operation. But using this technology brings with many RF design challenges. High throughput data rates are achieved in OFDM due to precise carrier spacing and exact amplitude and phase settings for each individual carrier constellation. This is accomplished using computational modulation schemes rather than traditional analog modulations. OFDM's resistance to multi-path interference results from the increased symbol duration for each individual carrier (as compared to other modulation schemes with the same data throughput) and from the use of a cyclic prefix (guard interval) preceding each symbol. However, from the perspective of the RF section, traditional OFDM modem designs face a number of key design issues that impact system cost and performance which must be taken into account. These include power consumption, linearity, image rejection, phase distortion and phase noise (Wight, J., 2001)(Tan, M., Latinovic, Z., and Bar-Ness, Y., 2005)(Pan, L., and Bar-Ness, Y., 2005).

2. FREQUENCY-SELECTIVE MIMO CHANNELS AND SVD OF POLYNOMIAL MATRICES

For a frequency-selective MIMO channel, we should model the channel with a polynomial matrix, the elements of which are polynomials of delay variable z^{-1}. In this case, a generalized definition of SVD should be used. Where, each of the singular matrices U and V and the matrix of singular values S are polynomial matrices, themselves.

Using the generalized version of U^H and V as the equalizer and precoder, the overall channel changes to matrix S. This matrix is still a diagonal matrix the non-zero elements of which are gains of the ICI-free paths of the overall MIMO channel:

$$S = \begin{bmatrix} S_1(z) & 0 & \cdots & 0 \\ 0 & S_2(z) & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & S_N(z) \end{bmatrix} \tag{8}$$

As in Eq. 8 can be seen, each non-zero element of the diagonal matrix S is a polynomial. Hence, until now we have a MIMO channel only without ICI. However, each of these ICI-free sub-channels is still frequency selective. In order to cancel the ISI caused by frequency selectivity, temporal precoders and equalizers should be used. Obviously, each of them could be designed independently for each sub-channel. Applying these filters to the channel, guarantees that both ICI and ISI are removed. The overall channel then could be written as follows:

$$H' = W(U^H HV)G = WSG = S' = \begin{bmatrix} S_1' & 0 & \cdots & 0 \\ 0 & S_2' & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & S_N' \end{bmatrix} \quad (9)$$

In other words, we first use spatial precoder and equalizer V and U^H to remove ICI and then cancel ISI with the use of temporal precoder and equalizer G and W.

It should be mentioned that it is simply possible to combine these two spatial and temporal precoders to one ST-precoder and two spatial and temporal equalizers to one ST-equalizer. Using combined ST-precoder and ST-equalizer, both ICI and ISI are cancelled simultaneously.

2.1. SBR2 Method to Compute Eigen Structure of a Polynomial Matrix

The Sequential Best Rotation (SBR2) algorithm can be used to diagonalise any para-Hermitian polynomial matrix by applying a series of elementary paraunitary transformations (McWhirter, J. G., Baxter, P. D., et al., 2007). In particular, this algorithm can be used to decorrelate a set of convolutively mixed signals. The algorithm can therefore be thought of as an extension of the eigen value decomposition (EVD), which is applicable to scalar matrices and can be used when decorrelating a set of instantaneously mixed signals (McWhirter, J. G., Baxter, P. D., et al., 2007). The paraunitary transformations used in SBR2 algorithms are rotation and delay come after each other, until the output signals are decorrelated. Thus, the SBR2 algorithm operates in a series of iterations, where at each iteration the order of the matrix being diagonalised can increase due to the application of elementary delay matrices. In general, SBR2 algorithm can also be thought of as a generalization of the Jacobi’s method for EVD computing of a scalar matrix, which is currently being extensively used.

Although it is shown that the SBR2 algorithm can be useful to decorrelate the set of mixed signals, it has some important drawbacks, which should be considered. First of all, the developed algorithm can be applied only to para-Hermitian polynomial matrices to compute their EVD and thus cannot be used to compute the SVD of a non-para-Hermitian matrix.

The second disadvantage is that the algorithm is iterative and therefore non-optimum. To decrease the problem the iterations of the algorithm should be applied continuously and this makes the order of the diagonalised matrix increase due to the delay transformation of each iteration (McWhirter, J. G., Baxter, P. D., et al., 2007). Besides, the algorithm does not give a closed form of the eigen structure of the input para-Hermitian matrix.

2.2. Proposing Genetic Algorithm as a Solution and Its Simulation Results

Finding the optimum coefficients of the ST-precoder and ST-equalizer matrices K and P, is a problem with high complexity order. Number of the coefficients to be found is $(L_N+1)N^2$ for ST-precoder and $(L_M+1)M^2$ for ST-equalizer; in which L_N and L_M are memory length of the ST-precoder and ST-equalizer respectively.

Genetic algorithms are a relatively new optimization technique, which can be applied to various problems, including those that are NP-hard (Goldberg D. E., 1989). The technique does not ensure an optimal solution, however it usually gives good approximations in a reasonable amount of time. This, therefore, would be a good approach to try on finding coefficients of the ST-precoder and ST-equalizer matrices. Genetic algorithms are based on natural evolution and include the survival of the fittest idea into a search algorithm, which provides a method of searching which does not need to explore every possible solution in the feasible region to obtain a good result. In nature, the fittest individuals are most likely to survive and mate; therefore, the next generation should be fitter and healthier because they were bred from healthy parents. This same idea is applied to a problem by first 'guessing' solutions and then combining the fittest solutions to create a new generation of solutions which must be better than the previous generation. We also include a random mutation element to account for the occasional 'mishap' in nature.

To define a suitable fitness function, we consider three goals. First, the ST-precoder and ST-equalizer matrices must be orthonormal polynomial to have no change in the signal and noise statistical characteristics passing through the system. Then, ST-precoder and ST-equalizer matrices should decrease ICI and ISI. To achieve these goals we define the following four cost functions:

$$J_1 = mse(I_{M\times M} - U^H * U) \tag{10}$$

$$J_2 = mse(I_{N\times N} - V^H * V) \tag{11}$$

$$J_3 = \sqrt{mse(corrcoef(U^H * H * V * \mathbf{s}[k]) - I)} \tag{12}$$

$$J_4 = \sqrt{mse(\frac{R'_{yy}}{R_{yy}(0)})} \tag{13}$$

In which, $mse(.)$ is the mean square function, I is the identity matrix, $corrcoef(.)$ is the correlation between samples (rows) of some variables (columns) placed in a matrix, here variables are output of receiver antennas and $corrcoef(U^H * H * V * \mathbf{s}[k])$ denotes the ICI of the equalized channel. R_{yy} is the autocorrelation function of one of the outputs and R'_{yy} is the same as R_{yy} except that $R'_{yy}(0)$ is forced to be zero. Hence, J_4 can be considered as ISI of the equalized channel. Minimizing these cost functions guarantees that ICI and ISI have their least values and the ST-precoder and ST-equalizer matrices are near orthonormal as well. The fitness function thus is defined as the sum of these four cost functions:

$$f = J_1 + J_2 + J_3 + J_4 \tag{14}$$

Simulation results given in figures 3 and 4 indicate that the Genetic algorithm method has a quite accuracy to find the coefficients of the desired matrices. Figure 3 shows how the above cost functions converge to their minimum values through the iterations. In Figure 4, the correlation between temporal samples of the channel can be seen, without and with equalization. This figure shows that the channel output after equalization has almost zero ISI.

3. PARAMETRIC METHOD FOR CALCULATING THE SVD OF A POLYNOMIAL MATRIX

As described before, the SBR2 routine is based on the Jacobi's method, which itself is an iterative approach to the desired answer. Here, in our second proposed method we first try to find the analytical solution for the SVD of a non-polynomial matrix; and then generalize it to polynomial case. To find SVD of a matrix we should find matrices of left and right singular vectors U and V. Let's focus on the left singular vector matrix U, first. Matrix U has M^2 coefficients that should be determined. However, there are some relationships between them that reduce the number of the elements should be found. Since the matrix U is normal, sum of each column squared elements equals to one and therefore, the number of the elements to be found reduces to $M^2 - M = M(M-1)$ elements.

On the other hand, it can be simply shown that each M dimensional rotation matrix $R_{m,n}(\theta)$ in Eq. 15 is an orthogonal and normal (orthonormal) matrix. It is also clear that we can select any two rows of an $M \times M$ identical matrix to create such an M dimensional rotation matrix. Each of these $P(M,2) = \dfrac{M!}{(M-2)!} = M(M-1)$ rotation matrices rotates the multiplied vectors in a particular 2-D plane.

$$R_{m,n}(\theta) =$$

$$\begin{array}{c} 1 \\ 2 \\ \vdots \\ m \\ m+1 \\ \vdots \\ n-1 \\ n \\ n+1 \\ \vdots \\ M \end{array}
\begin{bmatrix}
1 & & & & \cdots & & & & 0 \\
 & \ddots & & & & & & & \\
 & & 1 & & & & & & \\
0 & & 0 & \cos\theta & \cdots & -\sin\theta & 0 & & 0 \\
 & & & & 1 & & & & \\
\vdots & & & \vdots & \ddots & \vdots & & & \vdots \\
 & & & & & 1 & & & \\
0 & & 0 & \sin\theta & \cdots & \cos\theta & 0 & & 0 \\
 & & & & & & 1 & & \\
 & & & & & & & \ddots & \\
0 & & & & \cdots & & & 0 & 1
\end{bmatrix} \tag{15}$$

The product of these matrices ($\hat{U} = \prod_{\substack{all\ M(M-1)\ pairs \\ of\ m\ and\ n}} R_{m,n}(\theta_i)$) is also orthonormal:

$$\hat{U}^H \hat{U} = \prod^{i} \left[R_{m,n}(\theta_i)^H R_{m,n}(\theta_i) \right] = I_{M\times M} \tag{16}$$

Therefore, our problem of finding the $M(M-1)$ elements of the orthonormal matrix U changes to finding the $M(M-1)$ simple rotation matrices. In other words, we should try to find $M(M-1)$ different θ_i rotation angles that the production of their corresponding rotation matrices become the singular vectors matrix U .

Table 1 demonstrates how the SVD decomposition of a non-polynomial matrix can be evaluated analytically for two-dimensional case. In this method, the SVD decomposition is determined directly using the elements of the matrix; so, for a polynomial matrix case, the same idea can be applied, as well. The only thing should be considered is that each element of the singular vector matrices of a polynomial matrix would be itself a polynomial of delay variable z^{-1} . The left and right singular vector matrices calculated using this method could diagonalise the polynomial matrix.

Unfortunately, the elements of the optimum singular vector matrices have a complex relationship with the delay variable z^{-1} and therefore, implementing them practically is quite difficult and in many cases impossible. To overcome this major problem, we propose a novel method, which will be shown that has a good performance.

3.1. Using Taylor's Expansion to Convert an IIR Filter to an FIR Filter

The Taylor's expansion of a given function about a point is a representation of the function as a polynomial of its variable. Coefficients of the Taylor's expansion are calculated according to the following formula (Thomas, G. B., and Finney, R. L., 1996)(Greenberg, M., 1998):

$$\begin{aligned} f(x) = f(x_0+\Delta x) = f(x_0) + \frac{1}{1!}\Delta x f'(x_0) + \frac{1}{2!}\Delta x^2 f''(x_0) + \ldots + \\ + \ldots + \frac{1}{n!}\Delta x^n f^{(n)}(x_0) + \ldots \end{aligned} \tag{17}$$

If the number of the terms in the right hand of the equation 17 is limited, the polynomial is only an estimation of the function about the central point. This may be regarded as the limit of the Taylor polynomials. Here, we use this technique to convert the complex IIR filter created by the parametric method, described above, to a simple FIR filter.

Each element of the optimum S-equalizer and S-precoder matrices is a complex function of delay variable z^{-1}. To convert these elements to constructible functions of z^{-1}, we expand each of these elements about zero. The answer would be a polynomial of z^{-1}, which can be simply constructed using FIR filter design methods. For example, the i,j element of the matrix U, therefore is expanded as follows:

$$\begin{aligned} u_{i,j}(z^{-1}) \cong u_{i,j}(0) + \frac{1}{1!}z^{-1}.u_{i,j}'(0) \\ + \frac{1}{2!}z^{-2}.u_{i,j}''(0) + \ldots + \frac{1}{n!}z^{-n}.u_{i,j}^{(n)}(0) \end{aligned} \tag{18}$$

To show the performance of this method, suppose that $H(z) = \dfrac{z^{-3}+1.2z^{-2}+0.3z^{-1}+1}{z^{-2}+1}$. Using the Taylor's expansion, we convert this IIR filter to FIR case. The solid curve in figure 5 demonstrates the power spectral density of the original IIR filter and dotted ones show the power spectral density of the FIR estimation of the original filter. Note that, here $H(z)$ is a rational function of z^{-1} and there exist several methods to implement such a transform function in practice. However, for our case the elements of the optimum S-equalizer and S-precoder do not have such a simple relationship with z^{-1}, as the case of the example described before. Hence, the need of IIR to FIR conversion using the proposed method is obviously felt.

Here, we explain why we choose zero as the central point in Taylor's expansion. In Taylor's expansion of equation 17 only when the difference variable Δx is small enough, the limited number of terms could be used to have a quite precise estimation. In our case, the variable of the functions is delay variable z^{-1} and it is related to the frequency variable ω according to $z^{-1} = r^{-1}e^{-j\omega}$ and $z^{-1} = e^{-j\omega}$ when $r = 1$ (on the unit circle). In filter design all possible values of ω are quite important. Therefore, if we choose a central point close to $z_0^{-1} = r^{-1}e^{-j\omega_0}$, our estimation has an acceptable accuracy only for the frequency ω_0 and for the far frequencies the estimation is not acceptable. Hence, to have at least an equal accuracy for all frequencies, we choose a point with equal distance for all possible z^{-1} and all frequencies ω. The point with equal distance to all points on the unit circle is zero.

It should be noticed that this distance for all points is one and since we have the coefficient $\frac{1}{n!}$ in the Taylor's expansion the series would converge. Therefore, in the case of converting an IIR filter to FIR, each element should be expanded about zero. Simulation results of the figure 6 confirm this issue. In this figure, the normalized MSE of the three FIR estimations of the former $H(z)$ for different central point between zero and 0.1 are sketched.

3.2. The Algorithm

Now we can summarize the algorithm of the parametric method as follows:

- The look-up table is generated according to the proposed parametric method.
- Using the look-up table, the optimum elements of the S-equalizer U and S-precoder V matrices are calculated.
- Taylor's expansion about zero is finally applied to each of these elements to produce the FIR estimation of the optimum S-equalizer and S-precoder matrices.

Simulation results show that the summarized algorithm has a less complexity order than the proposed Genetic Algorithm and SBR2 methods. Simulation time for these three methods are given in table 2 on the same hardware and software platforms.

4. ICI CANCELLATION IN A PRACTICAL SIMULATION

Here, our proposed algorithm is applied to a practical case as the example. Suppose that the frequency-selective MIMO channel matrix is as follows:

$$H = \frac{1}{2}\begin{bmatrix} 2 + 2z^{-1} & z^{-2} \\ 1 & z^{-1} \end{bmatrix} \tag{19}$$

Using the first row of the look-up table of the table 1 and substituting the parametric elements $h_{m,n}$ with the actual gains of the channel, the optimum S-equalizer and S-precoder matrices U and V become:

$$U = \begin{bmatrix} \frac{1}{2}\frac{4z^{-4}+7z^{-2}+8z^{-1}+4+A}{2z^{-3}+3z^{-1}+2} & 1 \\ \frac{1}{2}\frac{4z^{-4}+7z^{-2}+8z^{-1}+4-A}{2z^{-3}+3z^{-1}+2} & 1 \end{bmatrix}^T \quad (20)$$

And

$$V = \begin{bmatrix} -\frac{1}{2}\frac{4z^{-4}+z^{-2}-8z^{-1}-4+A}{4z^{-3}+4z^{-2}+3z^{-1}+2} & 1 \\ -\frac{1}{2}\frac{4z^{-4}+z^{-2}-8z^{-1}-4+A}{4z^{-3}+4z^{-2}+3z^{-1}+2} & 1 \end{bmatrix}^T \quad (21)$$

where:

$$A = \sqrt{(4z^{-4}+9z^{-2}+12z^{-1}+4)(4z^{-4}+9z^{-2}+4z^{-1}+8)} \quad (22)$$

As it can be seen, the optimum matrices of U and V are not rational functions of z^{-1}. Hence, they may not be implemented using the ordinary methods of IIR filter implementing. Applying the Taylor's expansion to each element of U and V we will have:

$$U_{FIR} = \begin{bmatrix} 2.41+.85z^{-1}+1.75z^{-2}-4.10z^{-3}+6.92z^{-4}-12.67z^{-5} & 1 \\ -.41+.15z^{-1}+.25z^{-2}-.90z^{-3}+1.58z^{-4}-2.08z^{-5} & 1 \end{bmatrix}^T \quad (23)$$

And

$$V_{FIR} = \begin{bmatrix} 2.41+.85z^{-1}-5.08z^{-2}+2.02z^{-3}+5.04z^{-4}-1.98z^{-5} & 1 \\ -.41+.15z^{-1}-.92z^{-2}+.98z^{-3}-1.54z^{-4}+2.73z^{-5} & 1 \end{bmatrix}^T \quad (24)$$

These FIR representations of the optimum S-equalizer and S-precoder can be simply put into practice. Simulation results demonstrate that the correlation coefficient (ρ) of the output signals (for different receiving antennas) changes from 0.797 when there is no S-equalizer and S-precoder used, to a value less than 0.1 when these FIR filters have at least 25 taps. Figure 7 shows these results and shows how the S-equalizer and the S-precoder decorrelate the correlated signals.

Conclusion

In this paper two different methods are proposed to cancel the ICI of the frequency-selective MIMO channels. Both of them take the advantage of the SVD of the polynomial matrices to change a MIMO channel to some SISO parallel channels. The first method uses a genetic algorithm to calculate a near optimum SVD estimation of the channel. In the second method, parametric method, we use the Taylor's expansion to generalize the ordinary approach of calculating the SVD of matrices for polynomial case. Both of these two proposed algorithms have higher performance in comparison with the only alternate method, SBR2. SVD estimation of the channel matrix is then used to design a precoder and an equalizer to cancel the ICI of the channel. Simulation results show that applying these two filters to the channel, ICI would be cancelled. Although the GA is an iterative method and hence the first method is not fast enough to be used in real-time applications, it can be applied with a little change to cancel the ICI and ISI simultaneously. The parametric method, however, is analytical and very fast and the results support this fact.

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Table 1. Parametric results of the left and right singular matrices of a given 2×2 matrix

$H = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
$U = \begin{bmatrix} \dfrac{-c^2 + b^2 - d^2 + a^2 + \sqrt{A}}{2(ac+bd)} & 1 \\ \dfrac{-c^2 + b^2 - d^2 + a^2 - \sqrt{A}}{2(ac+bd)} & 1 \end{bmatrix}^T$
$V = \begin{bmatrix} \dfrac{c^2 - b^2 - d^2 + a^2 + \sqrt{A}}{2(ab+cd)} & 1 \\ \dfrac{c^2 - b^2 - d^2 + a^2 - \sqrt{A}}{2(ab+cd)} & 1 \end{bmatrix}^T$
Where: $A = (b^2 + a^2 - 2cb + c^2 + 2ad + d^2) \cdot (b^2 + a^2 + 2cb + c^2 - 2ad + d^2)$

Table 2. Taken time in seconds for 50 taps

Proposed method	GA method	SBR2
2	>300	180

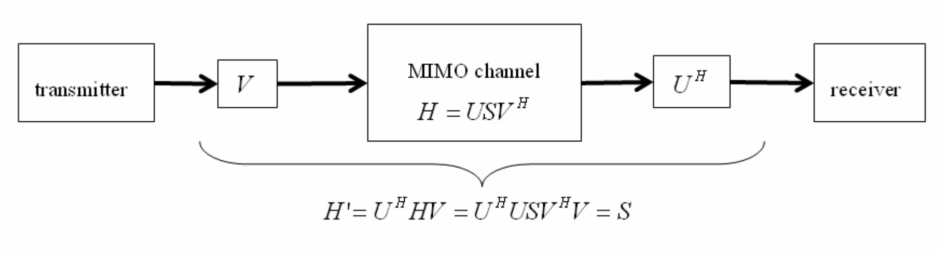

Figure 1. ICI cancellation using SVD in flat MIMO channels

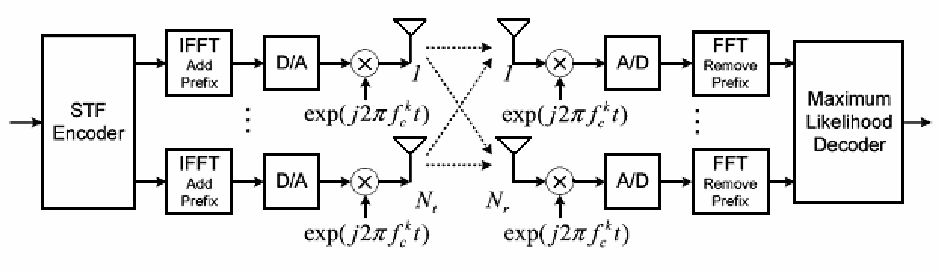

Figure 2. MIMO-OFDM scheme

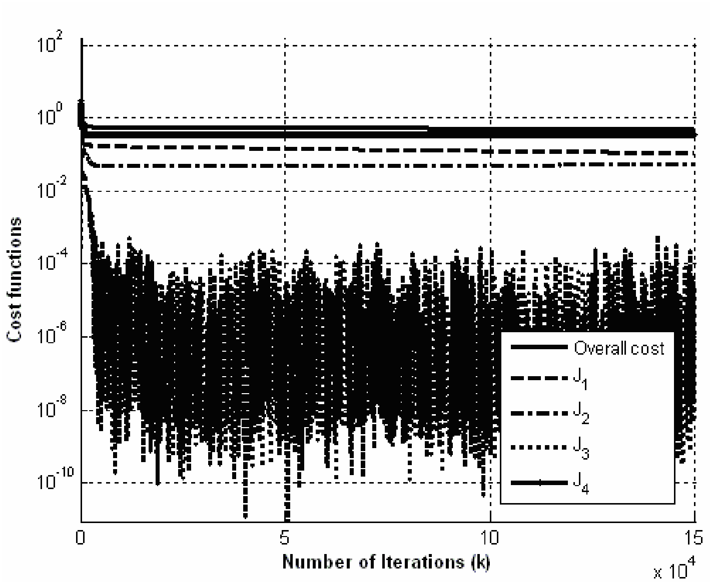

Figure 3. Convergence of the cost functions and fitness function used in the GA

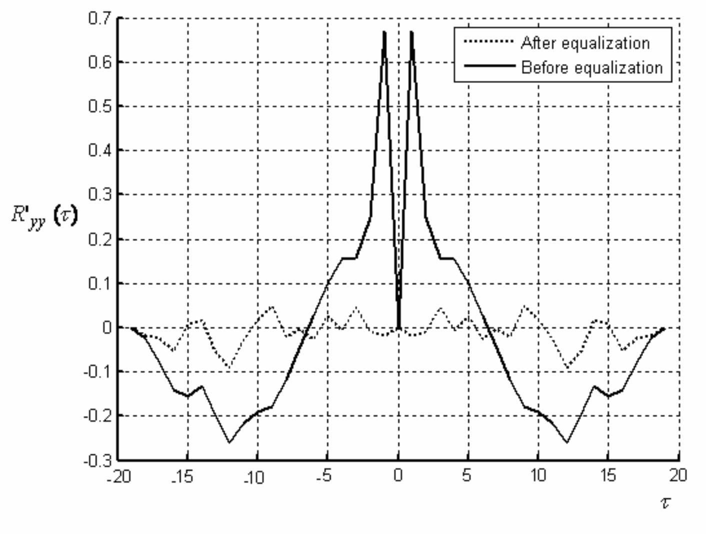

Figure 4. Correlation between temporal samples before and after equalization using Genetic Algorithm method

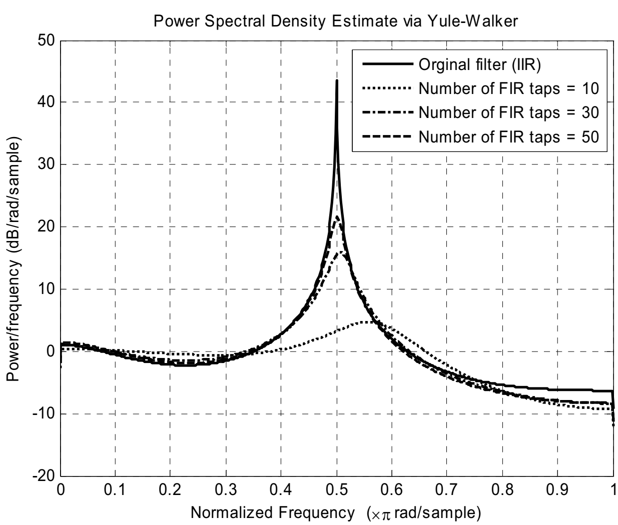

Figure 5. Frequency response of the original IIR filter and its FIR filter counterparts

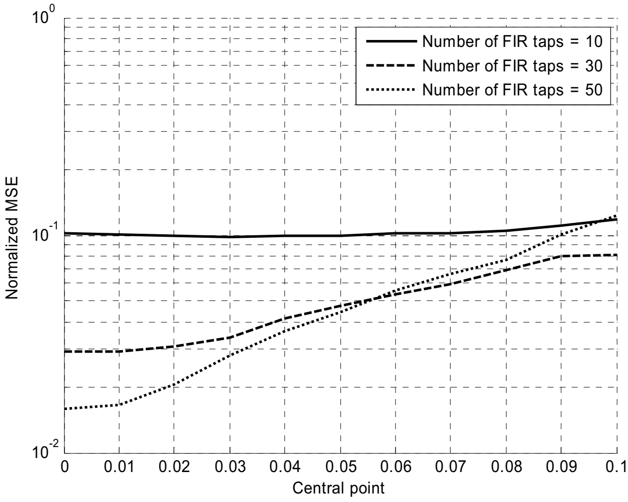

Figure 6. The effect of central point on Taylor's expansion

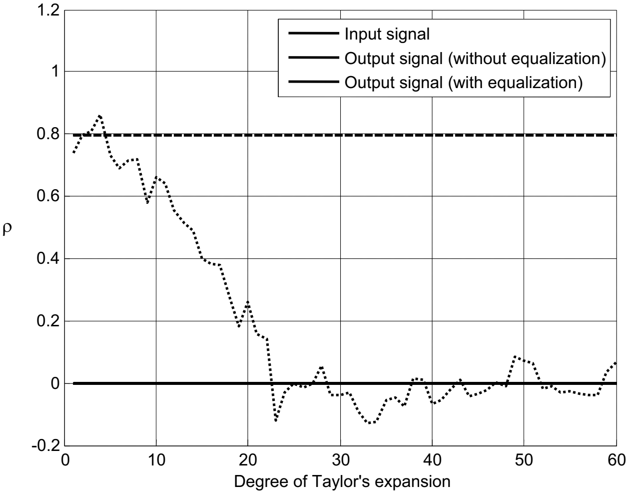

Figure 7. Correlation coefficient between output signals