NUMERICAL SIMULATION OF PARTIAL CAVITATION OVER AXISYMETRIC BODIES USING THE BOUNDARY ELEMENT METHOD BASED ON POTENTIAL

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ABSTRACT

In this paper, partial cavitation over axisymmetric bodies have been solved, using the Boundary Element Method (BEM), based on potential. In this method, the cavity and the wetted surface of the body will be estimated by panels. Then, the cavitation will be modeled, by means of Green's third identity integral. For this purpose, the rings of the sources are distributed on the cavity surface, and the ring of the dipoles are distributed on the body and the cavity surface. The high speed and also proper accuracy in calculating the geometry of the cavity and the drag coefficient are considerable advantages of this method.
Keywords: Partial cavitation, Boundary element method, Axisymetric bodies

INTRODUCTION

Cavitation is recognized as an inadvisable problem in most phenomena, but in some circumstances, cavitation is remarked as a beneficial problem. The most important example is the submerged projectiles, in which cavitation is desired because of intense decrease in drag force. The dimensionless parameter which is represented for introducing cavitation is the cavitation number, which is defined as below:

$$\sigma = \frac{P_\infty - P_v}{\frac{1}{2}\rho V_\infty^2} \tag{1}$$

Where p_∞ is atmosphere pressure, p_v is vapor pressure, ρ is the fluid density, and V_∞ is the far field fluid velocity. If bodies move with relatively high velocities inside liquids, cavitation starts at a point in which its local pressure reaches fluid vapor pressure. In low velocities or in high cavitation numbers, cavity is closed on the body and is called partial cavitation. With increase in velocity and decrease in cavitation number, cavity grows and covers all the body, which is called supercavitation[1].

Early studies of cavitation were performed by Efors (1946) and Tulin (1953), using theoretical methods. Cavitation stream can also be solved with boundary element method (BEM). In this method, a distribution of potential flow elements (vortex, source, sink, doublet and dipole) is located over the boundary of the flow. In 1993, Fine and Kinnas devised a nonlinear boundary element method based on potential for solving partial cavitation flow over a hydrofoil [2]. Partial cavitation flow over torpedoes was conducted by Uhlman et al [2], using BEM method, and source and dipole distribution over body surface and cavity in 2003. All of the performed studies are limited to the specific geometries, but in this paper, partial cavitation and supercavitation have been studied, using the boundary element method on different bodies.

BOUNDARY ELEMENT METHOD (BEM)

This section explains the BEM cavitation model based on the potential flow theory.

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Mathematical Formulation

The potential flow model presented here is based on Green's third identity formulation [3]. Applying this formulation to the axisymmetric disturbance velocity potential, φ, results in:

$$2\pi\varphi(r,x)= \oiint_S \left\{ \frac{\partial\varphi}{\partial n} G(x,r;\xi,R) - \varphi(r,x)\frac{\partial G(x,r;\xi,R)}{\partial n} \right\} R\, d\varphi\, ds \tag{2}$$

Where n is the normal vector directed outward from the solid-body surface and the cavity interface, s is the arclength along a meridian, and x and r are the components of the axisymmetric coordinate system. $G(x,r;\xi,R)$ is the potential function related to the fluid sources distributed along a ring of radius R located on the axis at $x=\zeta$ (see Figure 1,2). The potential function is defined as:

$$G(x,r;\xi,R)= \int_{-\pi}^{+\pi} \frac{\rho\, d\varphi}{\sqrt{(x-\xi)^2 + r^2 + R^2 - 2rR\cos(\varphi)}} = R J_1^0(A,B) \tag{3}$$

Where

$$\begin{aligned} A &= r^2 + R^2 + (x-\xi)^2 \\ B &= 2rR \end{aligned} \tag{4}$$

And

$$\begin{aligned} J_1^0(A,B) &= \frac{4}{\sqrt{A+B}} K(k) \\ K(k) &= \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1-k^2\sin^2(\varphi)}} \\ k^2 &= \frac{2B}{A+B} \end{aligned} \tag{5}$$

The total and disturbance potentials are related by:

$$\phi = x + \varphi \tag{6}$$

Where all quantities have been made dimensionless with respect to ρ, U_∞ and d. The boundary conditions are kinematic condition on the solid-body surface, and both the kinematic and dynamic conditions on the cavity interface. These conditions are mathematically formulated as:

$$\frac{\partial\phi}{\partial n} = 0 \text{ on } S_b \cup S_c \tag{7}$$

$$\frac{\partial\phi}{\partial s} = \sqrt{1+\sigma} \text{ on } S_c \tag{8}$$

Where S_b and S_c are the areas of the solid-body surface and the cavity interface, respectively. These boundary conditions are equal to:

$$\frac{\partial\varphi}{\partial n} = -x_n \text{ on } S_b \cup S_c \tag{9}$$

And

$$\frac{\partial \varphi}{\partial s} = \sqrt{1+\sigma} - x_s \text{ on } S_c \tag{10}$$

Where n and s are unit vectors normal and tangent to the solid-body/cavity boundary, respectively. The last boundary condition may be integrated to yield:

$$\varphi = \varphi_0 + \sqrt{1+\sigma}(s - s_0) - (x - x_0) \text{ on } S_c \tag{11}$$

Where φ_0 , is the potential at the detachment point of the cavity on the solid body.

Governing Integral Equation

Placing the unknowns on the left-hand side and the knowns on the right-hand side, Green's third identity is written as:

$$\begin{aligned} &2\pi\varphi + \iint_{S_b} \varphi \frac{\partial G}{\partial n} dS - \iint_{S_c} \frac{\partial \varphi}{\partial n} G dS = \\ &\iint_{S_b} \frac{\partial \varphi}{\partial n} G dS - \iint_{S_c} \varphi \frac{\partial G}{\partial n} dS \end{aligned} \tag{12}$$

On the wetted portion of the solid-body/cavity boundary and on the cavity interface:

$$\begin{aligned} &\iint_{S_b} \varphi \frac{\partial G}{\partial n} dS - \iint_{S_c} \frac{\partial \varphi}{\partial n} G dS = \\ &\iint_{S_b} \frac{\partial \varphi}{\partial n} G dS - 2\pi\varphi - \iint_{S_c} \varphi \frac{\partial G}{\partial n} dS \end{aligned} \tag{13}$$

Implementing the above boundary conditions Eq. (12) can be written as:

$$\begin{aligned} &2\pi\varphi + \iint_{S_b} \varphi \frac{\partial G}{\partial n} dS - \iint_{S_c} \frac{\partial \varphi}{\partial n} G dS \\ &+ \varphi_0 \iint_{S_c} \frac{\partial G}{\partial n} dS + \sqrt{1+\sigma}\left[\iint_{S_c} (s - s_0) \frac{\partial G}{\partial n} dS\right] \\ &= -\iint_{S_b} x_n G dS + \iint_{S_c} (x - x_0) \frac{\partial G}{\partial n} dS \end{aligned} \tag{14}$$

On the solid body and Eq. (13) becomes

$$\begin{aligned} &\iint_{S_b} \varphi \frac{\partial G}{\partial n} dS - \iint_{S_c} \frac{\partial \varphi}{\partial n} G dS + \varphi_0 \left[2\pi + \iint_{S_c} \frac{\partial G}{\partial n} dS\right] \\ &+ \sqrt{1+\sigma}\left[2\pi(s - s_0) + \iint_{S_c} (s - s_0) \frac{\partial G}{\partial n} dS\right] \\ &= -\iint_{S_b} x_n G dS + \left[2\pi(x - x_0) + \iint_{S_c} (x - x_0) \frac{\partial G}{\partial n} dS\right] \end{aligned} \tag{15}$$

On the cavity interface.

In addition to these equations, an auxiliary condition is required for which we impose the condition that the net source strength is equal to the flux through the jet, which may be expressed as:

$$\iint_{S_c} \frac{\partial \varphi}{\partial n} dS = \iint_{S_b} x_n dS \tag{16}$$

RESULTS

Partial cavitation for a blunt cylinder and a cylinder with a spherical head is considered in this study. Figure 3 shows a 3D view of the cylinder with spherical head and it's cavity shape for $\sigma = 0.2$. Also Figure 4 shows a 3D view of a blunt cylinder and it's cavity shape for $\sigma = 0.3$. Figure 5 shows the pressure coefficient (C_p) versus non-dimensional length for a cylinder with spherical head for a cavitation number of 0.2. As it is observed, BEM accurately predict the experimentally measured cavity length. At the end of the cavity region where the cavity is closed on the cylinder body (3.5 < L/D < 4), the C_p variation shows an overshoot which may be attributed to the simple model used in this method for the cavity termination on the solid body. The variation of C_p along the cavity for a blunt cylinder for $\sigma = 0.3$ is displayed in Figure 6. Similar to previous case, the BEM predictions show inaccuracies at the closure region. The method for the rest of the cylinder predicts the measurements very well.

Dimensionless cavity diameter versus cavitation number for both a blunt cylinder and cylinder with spherical head are shown in Figure 7. For both geometries, the cavity diameter is increased as the cavitation number is reduced. Also Dimensionless cavity length versus cavitation number for both a blunt cylinder and a cylinder with spherical head are shown in Figure 8. For both geometries, the cavity length is increased as the cavitation number is reduced. At a constant cavitation number, a longer cavity length is provided by the blunt cylinder. It can be concluded that the spherical head generates a smaller cavity region. To represent the capability of boundary element method in modeling the cavitation over more complex geometries, the partial cavitation on a type of torpedo with a non–dimensional length of 24 and two different cavity lengths are shown in figures 9 and 10. As it is shown in figure 11, the cavity length reduces when the cavitation number increases.

CONCLUSION

In this paper the partial cavitations over axisymmetric bodies have been studied using BEM method. The results for two cases: a blunt cylinder and a cylinder with a spherical head are presented. The implemented BEM method led to very good results compared to the experiments, for pressure and the cavity shape. However, an overshoot at the end of the cavity region is predicted. It seems that the existing re-entrant jet at the end of the cavity should be considered particularly and accurately. The main advantage of the method is the needing a very short time (a few seconds) to reach the desirable convergence.

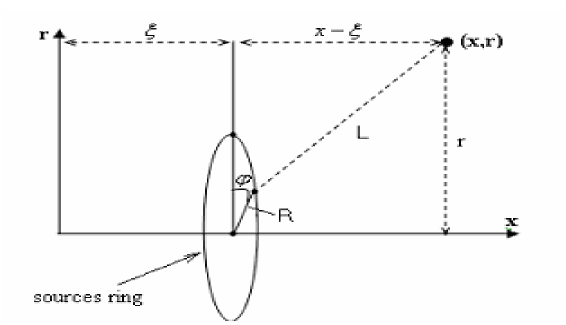

Figure 1. *Source ring in a cylindrical coordinate.*

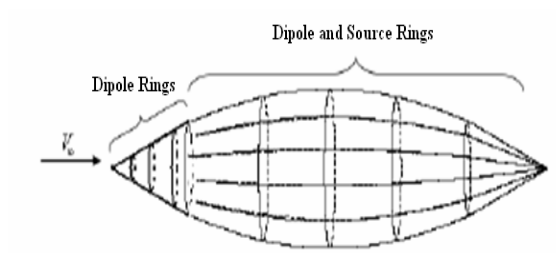

Figure 2. *Applying superposition of the free stream, with distributions of the dipoles and sources rings on the interface of the body and the cavity to solve cavitation.*

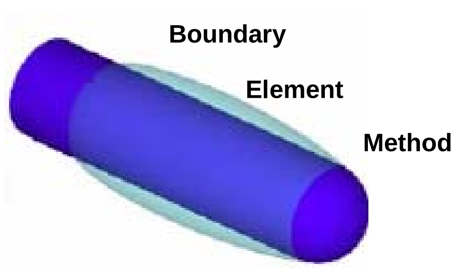

Figure 3. *The steady shape of the cavity using BEM for a cylinder with a spherical head ($\sigma = 0.2$).*

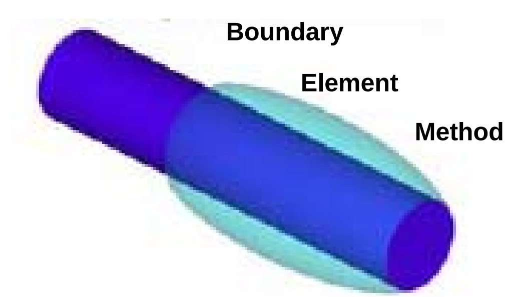

Figure 4. *The steady shape of the cavity using BEM for a blunt cylinder ($\sigma = 0.3$).*

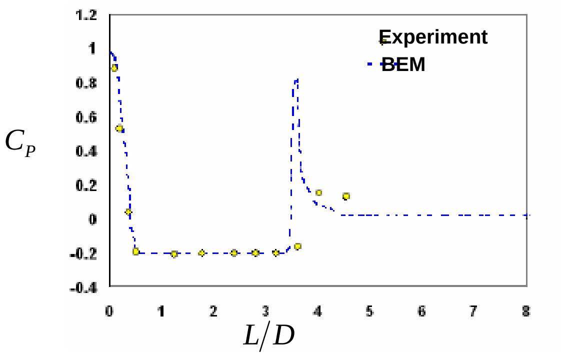

Figure 5. *Pressure coefficient vs. non-dimensional length for a cylinder with a spherical head for* $\sigma = 0.2$ *from experiments [4].*

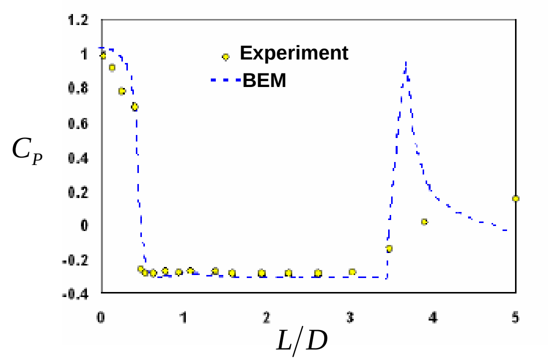

Figure 6. *Pressure coefficient vs. non-dimensional length for a blunt cylinder for cavitation number* $\sigma = 0.3$ *from experiments [4].*

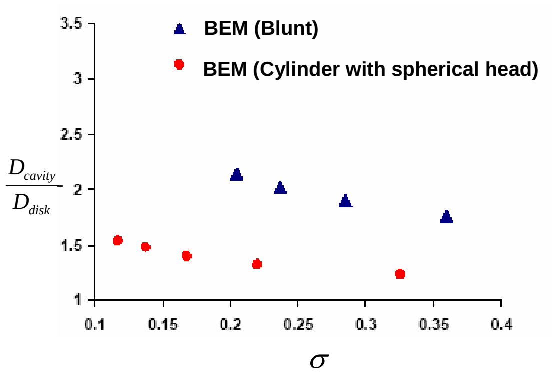

Figure 7. *Dimensionless cavity diameter vs. cavitation number for a blunt cylinder and a cylinder with spherical head using BEM model.*

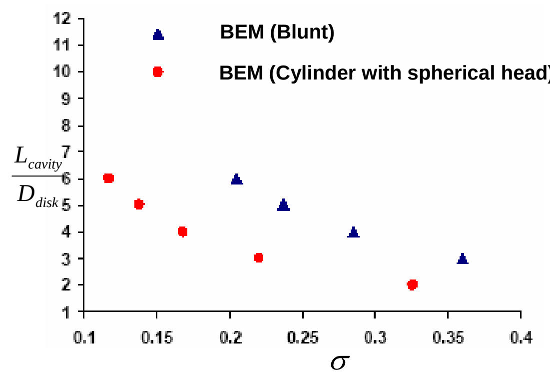

Figure 8. *Dimensionless cavity length vs. cavitation number for a blunt cylinder and a cylinder with spherical head using BEM model.*

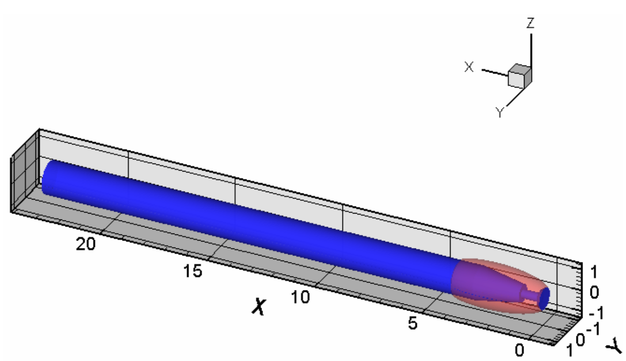

Figure 9. *The partial cavitation over a torpedo with a non–dimensional torpedo length of 24, cavity length of 4 and* $\sigma = 0.246$.

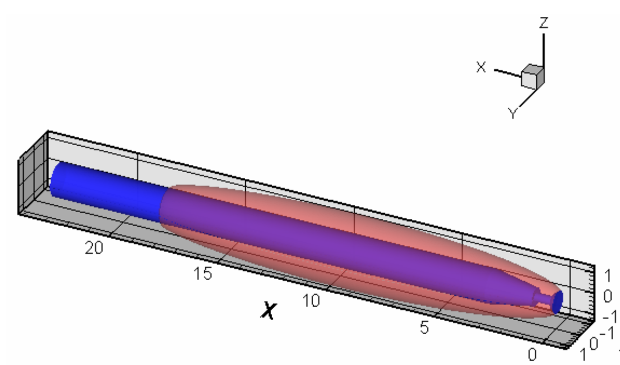

Figure 10. *The partial cavitation over a torpedo with a non–dimensional length of 24, cavity length of 18 and* $\sigma = 0.0769$.

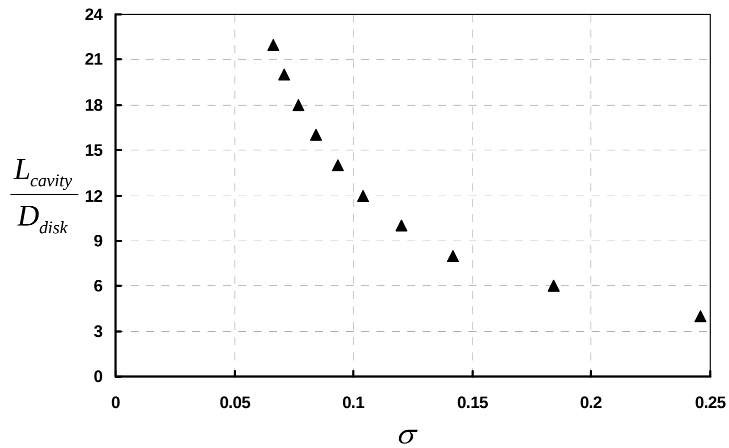

Figure 11. *The change of cavity length versus the cavitations number for the partial cavitation on a type of torpedo in the non–dimensional length of 24.*

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