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A new approach for cost allocation and reactive power pricing in a deregulated environment

S. Hasanpour · R. Ghazi · M. H. Javidi

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Abstract Power industry has been facing restructuring problems during the past decade. Appropriate management of reactive power is very essential for supporting power system security. Reactive power has dominant effects on real energy transfer. Furthermore, it can support the secure operation of the system as an ancillary service. However, most researches have been focused on active power as the main good transacted in electricity markets. On the other hand, while reactive power production cost is highly dependent on real power output, it is mainly confined to local consumption. As a result, to avoid market power and to maintain the secure operation of the system, a fair cost allocation method seems to be very essential. Appropriate pricing of reactive power as an ancillary service has been a challenging problem during the past decade. However, most methods proposed so far for reactive power pricing are essentially based on empirical approximations. In this paper, a new method for reactive power cost allocation is proposed. The method is based on calculation of the accurate cost which will be imposed on generators due to supporting reactive power. The proposed method is fair, accurate and realistic and it can be formulated very easily. Furthermore, a new approach based on tracing algorithm is proposed for pricing of reactive power which considers the cost of both active and reactive losses allocated to each generator. Application of the proposed method on IEEE 9-bus standard network confirms its validity and effectiveness.

Keywords Reactive power pricing · Cost allocation · Tracing algorithm · Restructuring

S. Hasanpour (✉) · R. Ghazi · M. H. Javidi
Department of Electrical Engineering,
Ferdowsi University of Mashhad, Mashhad, Iran
e-mail: so_ha73@stu-mail.um.ac.ir; hasanpour_s@yahoo.com

1 Introduction

Provision of reactive power is very essential for secure and reliable operation of power systems. In vertically integrated electricity industry, reactive power support is considered as part of system operator's activities and its cost which should be recovered is usually calculated based on approximate methods. In some systems, reactive power cost is included in the price of active power. In some other systems, the power factor is used for calculation of a penalty factor for the price to compensate the cost.

In a restructured environment, in spite of the fact that the cost of reactive power may be dominantly linked with the price of active energy as well as other services, it is considered as an ancillary service which is priced separately. On the other hand, it is well known that a fair pricing of such a service can lead to market liquidity which in turn results in approaching the optimal condition.

Many investigations have been carried out for appropriate pricing of reactive power [1–10]. Some of these methods utilize various search techniques such as genetic and ant colony algorithms for pricing [4], others have focused on formulating reactive power pricing [5,6]. Muchayi [7] have presented a survey on some of the reactive pricing algorithms. Dona and Paredes [8] have proposed a pricing technique based on minimization of the operation cost as well as the transmission losses using decoupled OPF. Cost allocation of reactive power using modified Y-bus matrix method has been reported by Chu and Chen [9]. Ro [10] has presented the reactive charging scheme composed of recovering capital cost and operational cost. Pricing of real and reactive power as bundled products in synchronous machine has been investigated in [11]. Rider and Paucar [12] have proposed a nonlinear reactive power pricing method. They have presented the total cost of reactive power production as a nonlinear model which is solved by modified predictor-corrector interior-point method. Active and reactive pricing using interior point nonlinear optimization method has been demonstrated by Xie [13]. Chung et al. [14] have presented a method for cost-based reactive power pricing in which the cost of reactive power production by generators and capacitors are minimized. Also a methodology for calculation of cost of reactive power by generators, synchronous condenser and static reactive power sources has been reported by Deksnys and Staniulis [15].

The cost of reactive power produced by a generator is essentially composed of two components: fixed costs or investment costs and variable costs. Variable costs, in turn, consist of operating costs (including fuel and maintenance cost) and the opportunity cost which is imposed on the generator resulting from reduction of its active power generation.

Some of the above-mentioned methods consider only the opportunity cost [4,14] while others consider only the operation and investment costs [5,13]. Therefore, such approaches lead to approximate reactive pricing techniques.

In this paper, we have proposed a new method for reactive power pricing in a pool-based power market. Our method utilizes the accurate relation between active and reactive power to assign an accurate function for the cost of reactive power production. Various components of reactive power cost have been considered in our proposed approach.

While, in some of reactive power pricing methods, the cost of active losses is attributed only to one plant (the one at slack bus) in the optimization problem [14], our method not only considers the cost of active power losses but also considers the cost of reactive power losses. Furthermore, the contribution of each generator in covering the active and reactive losses is determined using tracing method. To show the credibility of the proposed approach, it has been applied to IEEE 9-bus system.

The presented paper is organized in five sections. The procedure of the proposed reactive cost allocation method is discussed in Sect. 2. In Sect. 3, the analysis of cost for reactive power support and reactive power pricing are discussed. The simulation results as well as their comparison with other methods are presented in Sect. 4. The conclusions that can be drawn from this paper are presented in Sect. 5.

2 Reactive support cost allocation

Conventional cost methods for active and reactive power support are based on empirical methods in which active and reactive costs of generators are defined in quadratic forms as below

$$\mathrm{Cost}(P) = a_p P^2 + b_p P + c_p \tag{1}$$

$$\mathrm{Cost}(Q) = 0.05 b_p Q^2 \tag{2}$$

It should be noted that the active and reactive power generated by each generator are essentially bundled with each other. This bundling property highly depends on the operating point of generator. Therefore, while Eq. 1 provides an almost accurate value for the cost of active power, Eq. 2 results in a rough estimation for the value of the cost of reactive power. Furthermore, Eq. 2 considers only the operational cost of reactive power generation and the cost for covering the investment for reactive power generation is not included in this equation.

To overcome inaccuracies associated with conventional cost methods, Song, Irving and Zhau proposed a method for cost evaluation of reactive power which is based on the triangular relationship between active and reactive power [5]. In this triangular approach, the cost of reactive power is formulated as below:

$$\mathrm{Cost}(Q) = a'' Q^2 + b'' Q + c'' \tag{3}$$

where, a'', b'', c'' are constants depending on power factor ($\cos\theta$) and are calculated as follows:

$$\begin{aligned} a'' &= a_p \sin^2\theta \\ b'' &= b_p \sin\theta \\ c'' &= c_p \end{aligned} \tag{4}$$

This method of reactive power cost calculation is essentially based on the formulation for active power cost, in which the active power is replaced by reactive power using the triangular relationship. However, as the investment for generators is essentially based on the optimal solution for active power generation, employing the same formula for the cost of reactive power will lead to calculation of wrong fixed costs for reactive power. Xie [13] in another approach, have used a second-order polynomial for the cost of reactive power in which a, b and c constants are approximated to be one-tenth of those for the cost of active power. Furthermore, such an equation normally is valid for a special range of reactive power production.

The present paper proposes a new method for the formulation of reactive cost allocation. In the proposed approach, all the investment, operation and opportunity costs due to reactive power support are taken into account. Attempt has been made to formulate the equation by a quadratic function as below.

In generator, the relationship between active and reactive power is:

$$S^2 = P^2 + Q^2 \tag{5}$$

If a generator produces its maximum active power ($P_{\max}$), then its cost for generating active power equals cost ($P_{\max}$). In such a situation, no reactive power is produced and therefore, S equals $P_{\max}$. Reactive power production itself does not seem to impose any fuel cost on generator except for the cost imposed for losses. However, reactive power production by a generator will reduce its capability to produce active power. Hence, provision of reactive power by generator will result in reduction of its active power production.

To generate reactive power Q_i by generator i which has been operating at its nominal power ($P_{\max}$), it is required to reduce its active power to P_i such that

$$P_i = \sqrt{P_{\max}^2 - Q_i^2}$$
$$\Delta P = P_{\max} - P_i \tag{6}$$

where, ΔP represents the amount of active power that will be reduced as a result of generating reactive power.

To accurately calculate the cost of reactive power Q_i, we should include all the costs imposed on generator as below:

$\mathrm{Cost}(P_{\max})$: cost of producing active power equal to $P_{\max}$ in one hour.
$\mathrm{Cost}(P_{\max} - \Delta P)$: cost of generator when producing both active and reactive power with the amounts P_i and Q_i, respectively.
$\mathrm{Cost}(P_{\max}) - \mathrm{Cost}(P_{\max} - \Delta P)$: Reduction in the cost of active power due to compulsory reduction in active power generation (ΔP) which happens due to generating reactive power with the amount of Q_i.

This represents the cost of reactive power production while the operating point of generator is moved from point 1 to point 2 (Fig. 1) as below:

$$\mathrm{Cost}(P_{\max}) - \mathrm{Cost}(P_{\max} - \Delta P) = \mathrm{Cost}(Q_i) + \frac{\Delta P}{P_{\max}}\mathrm{Cost}(P_{\max}) \tag{7}$$

where, $\frac{\Delta P}{P_{\max}}\mathrm{Cost}(P_{\max})$ is related to the change of operating point (In fact this represents the cost of ΔP Mwh energy when the generator is generating its nominal power). Therefore, from the above equation it can be concluded

$$\mathrm{Cost}(Q_i) = \mathrm{Cost}(P_{\max}) - \mathrm{Cost}(P_{\max} - P_i) - \frac{\Delta P}{P_{\max}}\mathrm{Cost}(P_{\max})$$
$$\mathrm{Cost}(Q_i) = \frac{P_{\max} - \Delta P}{P_{\max}}\mathrm{Cost}(P_{\max}) - \mathrm{Cost}(P_{\max} - P_i) \tag{8}$$

Now, we should express $\mathrm{Cost}(Q)$ as a function of Q. Assuming we will use the full potential of generator capability, we may conclude that its operating point will always be such that its current will be equal to its nominal value and we will be able to write Q as a function of P (Eq. 6). Therefore, considering Q as variable and using Eqs. 6 and 8, its production cost can be calculated for different values of Q. The results, interpolated by using Newton–Gregory polynomial, confirm that they can accurately be fitted into a quadratic polynomial form as below:

$$\mathrm{Cost}(Q) = a_q Q^2 + b_q Q + c_q \tag{9}$$

This equation is very simple and as it is extracted from the power cost function of the generator, it is more realistic and can provide accurate results in reactive power pricing as compared with conventional empirical approximate method. The proposed cost function, as compared with previously used methods, not only considers the operational cost imposed to the system due to reactive power support, but also the opportunity cost is taken into account. Furthermore, investment cost in this equation is accurately included.

Figure 2 shows the plotted cost curves for active power and the proposed reactive power formulation. From the figure, it can be observed that both cost curves show similar characteristics. However, as it should be, the reactive cost is much smaller than the active cost.

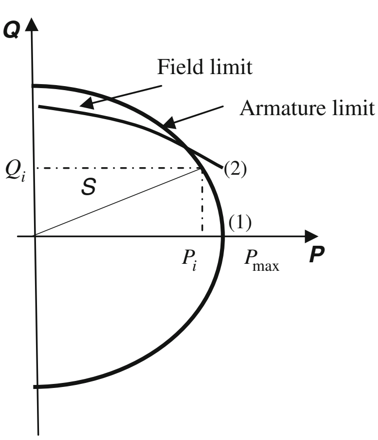

Fig. 1 Capability curve of generator

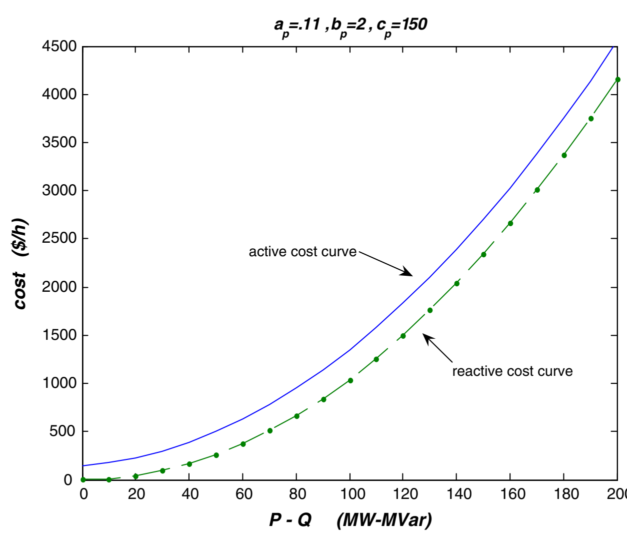

Fig. 2 Active cost curve and proposed reactive cost curve

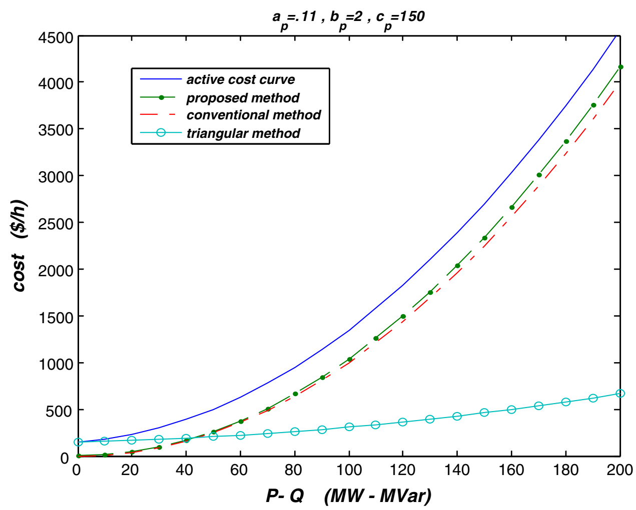

Fig. 3 Active and reactive power cost function in three methods

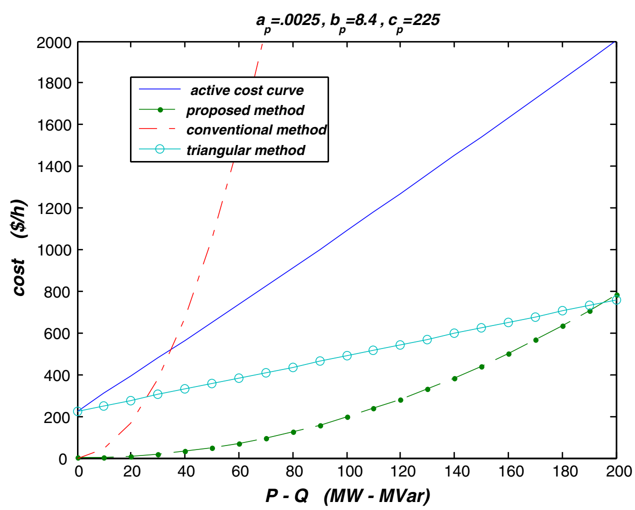

Fig. 4 Comparison of the new method with conventional and triangular methods

In Figs. 3 and 4, the cost allocated to reactive power, obtained by using the proposed method is compared with those obtained using conventional and triangular methods for two different cases in which a_p, b_p and c_p parameters are different. While, for both cases, the triangular method is almost compatible with our proposed method, it can be easily observed that the conventional cost method may not be reasonable (Fig. 4). This is mainly due to the fact that investment cost is not at all included in the pricing of reactive power for the conventional method. Therefore, depending on the values of a_p, b_p and c_p parameters for the generator, the results obtained by the conventional method may differ significantly from the actual cost of reactive power production imposed on generator.

3 Reactive power pricing

Active and reactive marginal prices are normally obtained through solving the optimal power flow in which an objective function subject to a set of equality and inequality constraints is minimized. In this paper, we also propose a new frame for reactive power cost allocation which covers all costs associated with reactive power generation in objective function of optimization problem. In our approach, both active and reactive losses allocated to each generator are included in the proposed objective function. Therefore, it guarantees a more accurate and non-discriminative pricing scheme for active and reactive power. In this section, first the proposed cost formulation for reactive power is used in the proposed objective function of OPF. Then, to specify the cost allocated to each generator, the results are applied to tracing algorithm. In order to demonstrate inaccuracies of the conventional method, the obtained results are compared with the results of proposed approach.

3.1 Objective function

So far, different objective functions have been used for OPF; the following formulations (a and b) are more common:

(a) Summation of active and reactive power production costs,

$$C_{\text{total}} = \sum_{i=1}^{N_g} [\text{Cost}(P_{G_i}) + \text{Cost}(Q_{G_i})] \tag{10}$$

where,

$\text{Cost}(P_{G_i})$: Active power cost function of generator i,
$\text{Cost}(Q_{G_i})$: Reactive power cost function of generator i,
N_g : Number of generators.

While the above-mentioned method considers the total costs of active and reactive power produced by generators, it may not be accurate, because all active and reactive losses are assumed to be generated at slack bus. Therefore, this approach may not be fair, especially when the marginal cost at slack bus significantly differs from those in other buses.

(b) Summation of active and reactive power production costs and the cost of active and reactive losses,

$$C_{\text{total}} = \sum_{i=1}^{N_g} [\text{Cost }(P_{G_i}) + \text{Cost }(Q_{G_i})] + P_{\text{loss}}\alpha + Q_{\text{loss}}\beta \tag{11}$$

where,

P_{loss} : Active power losses,
Q_{loss} : Reactive power losses,
α : Price of active power losses,
β : Price of reactive power losses,

where, α and β are mean marginal price values for active and reactive power generated by different generators. Therefore, we can write

$$\alpha = \sum_{i=1}^{N_g} \lambda_{p_i}/N_g \qquad \beta = \sum_{i=1}^{N_g} \lambda_{Q_i}/N_g \tag{12}$$

where,

λ_{P_i} : Active power price in generator i,
λ_{Q_i} : Reactive power price in generator i.

While this approach considers the effect of marginal price of various generators, losses are accounted for twice in this formulation. In fact, losses are taken into account by addition of third and fourth terms, while they are also included in the first and second terms.

To overcome the deficiencies in previously proposed methods, we have proposed a new formulation for the objective function, in which all generators contribute in active and reactive power losses. To accurately include the effect of marginal cost of different generator on the total cost imposed by losses, we should first evaluate the amount of active and reactive power losses attributed to each generator. This is achieved by using a well-known tracing algorithm. As a result, the cost of losses assigned to each generator can be fairly calculated. To do this, we have formulated the objective function as the summation of cost functions for pure consumed active and reactive power as well as the cost functions for losses. In fact, we have formulated the total cost imposed on each generator in four different terms including costs for active and reactive power supplied to customers and costs for active and reactive losses. Therefore, we can write the objective function as below:

$$C_{\text{total}} = \sum_{i=1}^{N_g} [\text{Cost}(P_{G_i} - \Delta P_{G_i}) + \text{Cost}(Q_{G_i} - \Delta Q_{G_h}) + \Delta P_{G_i}.\lambda_{P_i} + \Delta Q_{G_i}.\lambda_{Q_i}] \tag{13}$$

where,

ΔP_{G_i} : Active power losses allocated to generator i,
ΔQ_{G_i} : Reactive power losses allocated to generator i,
$P_{G_i} - \Delta P_{G_i}$: Active power production by generator i without considering active losses,
$Q_{G_i} - \Delta Q_{G_i}$: Reactive power production by generator i without considering reactive losses.

In the above formulation, ΔP_{G_i} and ΔQ_{G_i} are calculated using a tracing based loss allocation algorithm [16].

3.2 Constraints

The constraints, considered in this problem, are the standard set of equality and inequality constraints which are normally considered in OPF. In fact, the set of equality constraints represent the standard power flow equations for active and reactive power and the set of inequality constraints represent the physical and security limits of the system as below:

$$\sum_{i=1}^{N_g} P_{G_i} - \sum_{i=1}^{N} P_{D_i} - P_{\text{loss}} = 0$$
$$\sum_{i=1}^{N_g} Q_{G_i} - \sum_{i=1}^{N} Q_{D_i} - Q_{\text{loss}} = 0$$

where,

$$P_{\text{loss}} = \sum_{i=1}^{N} |V_i||V_j||Y_{ij}|\cos(\theta_{ij} + \delta_j - \delta_i)$$
$$Q_{\text{loss}} = \sum_{i=1}^{N} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} + \delta_j - \delta_i)$$

and,

$$\begin{aligned} &P_{G_{\min}} \le P_{G_i} \le P_{G_{\max}} \\ &Q_{G_{\min}} \le Q_{G_i} \le Q_{G_{\max}} \\ &P_{G_i}^2 + Q_{G_i}^2 \le S_{G_i,\max} \quad i = 1, \ldots, N_g \\ &|V_i|_{\min} \le |V_i| \le |V_i|_{\max} \quad i = 1, \ldots, N \end{aligned} \tag{14}$$

In the above formulas different variables are defined as below:

N: Number of buses of the network
P_{G_i}, Q_{G_i} : Supply of active and reactive power in ith bus
P_{D_i}, Q_{D_i} : Active and reactive demand in ith bus
$S_{G_i,\max}$: Maximum apparent power in bus i
$V_i = |V_i|\angle\delta_i$: Voltage phasor in bus i
$Y_{ij}\angle\theta_{ij}$: The ijth element of admittance matrix

4 Case study

To investigate the validity of the proposed algorithm, it has been applied to IEEE 9-bus system (Fig. 5) with a typical daily load as shown in Fig. 6. Tables 1 and 2 show the parameters of this system.

To be able to make an analytical comparison between the proposed method and previous algorithms, two different

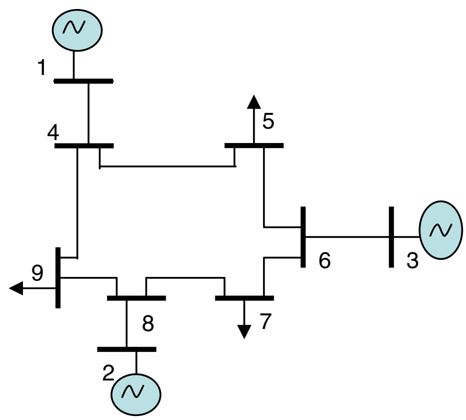

Fig. 5 IEEE 9-bus system

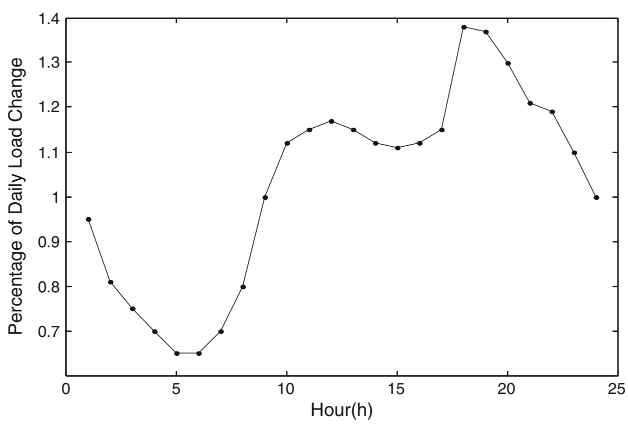

Fig. 6 Percentage of load daily change

Table 1 Generators characteristics

No. bus	a_p	b_p	c_p	$P_{\max}$ MW	$P_{\min}$ MW	$Q_{\max}$ MVAR	$Q_{\min}$ MVAR
1	0.11	5	150	250	10	300	−300
2	0.08	1.2	600	300	10	300	−300
3	0.12	1	335	270	10	300	−300

Table 2 Load characteristics

No. bus	Active power (MW)	Reactive power (MVAR)
5	90	30
7	100	35
9	125	50

scenarios have been analyzed. In the first scenario, network losses and its effect on cost function are neglected. In the second approach, we have not only accurately considered network losses in the objective function but also included their cost.

Scenario No. 1

For this scenario, we have simulated three different cases as below:

(1) In case 1, only the cost for active power produced by generators is considered in the objective function.
(2) In case 2, the costs for both active and reactive power are considered in the objective function. In this case, the cost function has been modeled based on the conventional reactive cost formulation.
(3) In case 3, while the costs for both active and reactive power are included in the objective function, the cost function for reactive power has been modeled according to our proposed formulation.

Table 3 shows the simulation results for three above-mentioned cases. The active and reactive marginal prices during 24 of the typical day for generator 1 are shown in Fig. 7. Comparing the results for these three cases, it can be easily concluded that

(1) Comparing the costs for cases 2 and 3 with that for case one it can be concluded that irrespective of the cost function modeled for reactive power, the cost of reactive power and consequently its price is much lower than that of the active power.
(2) Our proposed method for reactive power cost allocation is based on a more accurate modeling in comparison with approximate conventional methods. On the other hand, comparing the results of case 2 and case 3, it can be easily observed that the cost allocated for reactive power in our formulation (case 3) may be significantly different from that of conventional method (case 2). It should be emphasized that these differences arise from that fact that in conventional models, investment and opportunity costs are not considered. As a result, it can be easily concluded that reactive power pricing based on conventional scheme is not fair.
(3) As it can be seen, the cost dedicated to reactive power in our model is much greater than that of conventional ones, which in turn, may imply a positive signal for investors to think about investment on reactive power supplies. This will result in a more secure operation of the system in the future specially in restructured power systems. On the other hand, in power markets where the reactive power is priced based on conventional methods, there will not be any motivation for expansion of reactive power suppliers. It should be emphasized that in spite of the fact that reactive power is very important for enhancement of secure operation of the system, its cost is not compared with that of active power.

Table 3 Analysis results for different cases of scenario no. 1

	No. bus	P_G (MW)	Q_G (MVAR)	λ_p (\$/MW)	λ_Q (\$/MVAR)	cost_Q (\$)	$\text{Cost}_{\text{total}}$ (\$)
Case1	1	86.5645	54.3637	24.0442	–	–	5.216036E+3
	2	134.3774	82.6430	24.0442	–	–	
	3	94.0581	−20.5583	24.0442	–	–	
Case2	1	86.5675	11.3053	24.0449	5.6226	31.98	5.541202E+3
	2	134.3773	47.1471	24.0441	5.6577	133.283	
	3	94.0551	56.5476	24.0435	5.6548	159.934	
Case3	1	86.5714	34.3719	24.0457	8.2555	141.82	5.690612E+3
	2	134.3834	47.4364	24.0452	8.2560	195.79	
	3	94.0452	33.1917	24.0411	8.2562	137.01	

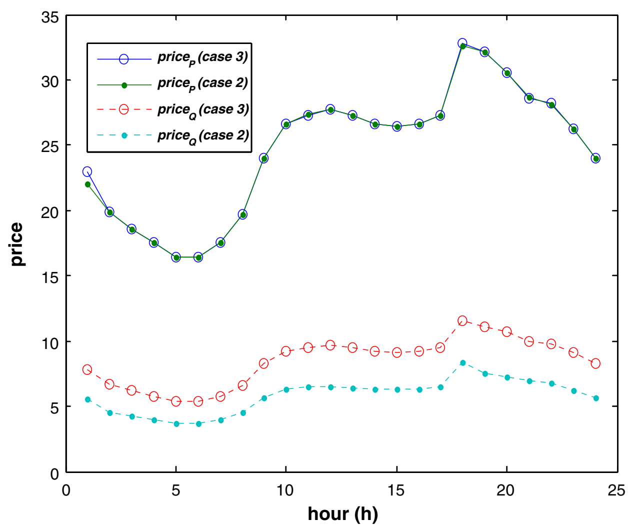

Fig. 7 Active and reactive marginal prices in conventional and proposed methods

Scenario No. 2

In this scenario, we have emphasized on analysis and the effects of allocating network losses to all generators. For this scenario, two different cases have been considered. Similar to case 3 of scenario 1, in both cases of scenario 2, the cost of reactive power is modeled and calculated according to our proposed formulation. Therefore, the results for cases 1 and 2 of scenario 2 will only be compared with the results for case 3 in scenario 1. In case 1 of scenario 2, the costs of active and reactive losses are computed based on the average values of their marginal prices using Eq. 11. In case 2 of this scenario, the portion of losses produced by each generator is determined based on tracing algorithm and its cost is accurately calculated using our proposed formulation (Eq. 13). In fact, in the second approach the cost of losses associated with each generator is computed using the cost function for the same generator.

The simulations results, for these two cases, are shown in Table 4. The results confirm that

(1) It can be observed that although the total costs for case 2 of this scenario are smaller than the total costs for case 1, the total costs for both of these cases of scenario are not significantly different from each other. This should be due to the fact that the average value for marginal prices of different generators has been taken as the price for losses which is not necessarily correct. It should also be mentioned that in large networks, where the tie lines may be congested, the difference will be more pronounced.

(2) As our tracing-based proposed method allocates the active and reactive losses to different generators and their costs are evaluated accurately, it is more compatible with open access networks. Therefore, it will not

Table 4 Analysis results for different cases of scenario No. 2

	No. bus	P_G (MW)	Q_G (MVAR)	λ_p (\$/MW)	λ_Q (\$/MVAR)	$\text{Cost}_{\text{total}}$ (\$)
Mean value of marginal price method	1	86.56246	34.3712	24.0442	8.25559	5.690592E+3
	2	134.37377	47.4368	24.0442	8.2560	
	3	94.0576	33.1923	24.0441	8.2562	
Proposed method	1	86.4012	34.3388	24.0083	8.2529	5.690421E+3
	2	134.6350	47.4408	24.0880	8.2619	
	3	93.96338	33.1704	24.0211	8.2506	

mislead to an unfair and wrong signal for generators. However, it should be emphasized that the costs for active and reactive losses imposed on the system due to reactive power support is very small in comparison with the cost of reactive power support itself.

Comparing the results in Tables 3 and 4, it can be concluded that

(3) The total costs in both cases of scenario 2 are smaller than that in case 3 of scenario 1. This is just due to the fact that in case 3 of scenario 1, the cost is not valued accurately and all active and reactive losses are assumed to be generated at slack bus.

(4) Comparing the costs of reactive power support using our proposed method (cases 1 and 2 of scenario 2 and case 3 of scenario 1) with the costs of reactive power support using previously used conventional approaches (cases 1 and 2 of scenario 1), it can be concluded that the costs of reactive power obtained through our proposed approach which is based on the accurate calculation is much higher than costs obtained through approximate methods.

Finally, as our method is based on the accurate relation between active and reactive power and takes the actual generation capabilities into account, the market clearing will result in a feasible solution.

5 Conclusion

In this paper a new method for reactive power pricing and reactive power cost allocation has been proposed. The proposed method at first utilizes the accurate relation between active and reactive power to assign an accurate quadratic function for cost function of reactive power support. Then, in reactive power pricing, using tracing algorithm and thorough an optimization technique, active and reactive losses are fairly allocated to different generators. Finally, the costs of these losses are calculated based on the accurate cost function of generators. The obtained results show that approximate conventional reactive power cost allocation techniques may result in wrong signals for market participants. This, in turn, may result in threatening the secure operation of the system as well. While our proposed reactive pricing method is simple and flexible, it is accurate and fair as compared with conventional methods. Therefore, it is more compatible with non-discriminatory philosophy of open access deregulated systems.

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