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# A New Method for Fast Computation of Maximum Loading Margin Utilizing the Weak Area of the System

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**Abstract** – *Voltage stability has become a very important issue for operating power systems close to their limits due to the continuous load increase and economic constraints. This paper discusses some important aspects related to the maximum loading margin in voltage stability studies and proposes a new method for fast and accurate calculation for maximum loading margin. The proposed method is based on an optimization technique in which the genetic algorithm is utilized to solve the parameterized nonlinear load flow equations. To reduce the computational burden, the optimization problem is examined in the weak area of the system as a subsystem which is defined by modal analysis. The proposed method is simple, efficient and quite fast for determining the maximum loading margin. Therefore it will be a suitable approach for online assessment of voltage stability problem. Simulation results for IEEE 14, 30 and 57 bus systems are shown to validate the proposed method.* 

***Keywords:*** *Genetic Algorithm, Maximum Loading Margin, Modal Analysis, Saddle Node Bifurcation, Voltage Stability, Weak Area*

## Nomenclature

| | |
|---|---|
| $J$ | Jacobian matrix |
| $J_R$ | Reduced Jacobian matrix |
| $N$ | Number of buses |
| $N_{GA}$ | Number of variables in GA algorithm |
| $N_{VW}$ | Number of buses in weak area |
| $P_G$ | Active power generation |
| $P_D$ | Active power demand |
| $Q_D$ | Reactive power demand |
| $X$ | System variable |
| $\lambda$ | Loading parameter |

## I. Introduction

Voltage stability refers to the ability of a power system to maintain acceptable voltage at all buses after being subjected to a disturbance. Power flow methods are very important tools for analyzing the voltage stability problem in power systems. However, at voltage collapse point the Jacobian matrix becomes singular with zero eigenvalue. Therefore, at this point, no solution for power flow equations is obtained. Substantial researches have been carried out to analyze this phenomenon and consequently various methods have been developed for determining the maximum loading margin.

The continuation power flow (CPF) is an accurate method for estimating the maximum loading margin. However, it is a very time consuming approach especially for large networks [1]. So, it can not be suitable for online voltage stability assessment. In general, reducing the computational time is an important problem in voltage collapse studies. Therefore, developing fast and accurate methods for estimating the saddle node bifurcation or maximum loading margin still deserve special attention.

Several investigations have been carried out to propose appropriate voltage stability indices to identify load margin and critical buses. Load margin is a measure of proximity to the bifurcation-related instability, which is defined as the distance of the operating point from the saddle node bifurcation point. Kessel et al. in [2] have proposed L-index as the voltage stability index based on the solution of power flow equations. L-index can be varied in a range between 0 (no load) and 1 (voltage collapse). The voltage stability $\Gamma$-index has been developed by Kim et al. in [3]. This index is defined the ratio of the equivalent generator bus voltage to the load bus voltage and its minimum and threshold value is 1 for no load and 2 for voltage collapse. Comparison between voltage stability indices has been made in [4] and some techniques such as P-V curves, L-index, Modal analysis and line stability indices have been analyzed. Yuan et al. have presented comparison of load margin analysis such as CPF and interior point methods for steady state voltage stability [5].

In an online approach, in order to asses the voltage instability problem arising from bifurcation, fast and reliable approaches are needed. Therefore, this problem

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can be an important issue from a practical point of view. Fast decoupled continuation power flow has been proposed by Mohn and de Souza in [6]. The process of the voltage collapse point calculation is accelerated because of the constant matrices used during the convergence process. Alves et al. have proposed the parameterized fast decoupled power flow for obtaining the maximum loading point (MLP) of power systems [7]. Fast computation of voltage security margin based on sensitivity analysis has been developed by Zarate et al. in [8]. The method utilizes two approximating functions (quadratic and linear functions) to determine the load increment. Karbalaie et al. have proposed a new method for fast computation of saddle node bifurcation by solving an optimization problem [9]. Fast prediction of loadability margin using neural network has been suggested by Canizares in [10]. The method selects a group of loading directions and trained back propagation neural network to approximate the security boundary considering the static and oscillatory voltage stability. A fast guided continuation method to predict a reliable index for voltage collapse has been developed by Chen et al. in [11]. In this paper, the continuation step size is guided by reactive power limit index that can pinpoint the limit reaching points. Zhou et al. in [12] have presented a fast framework based on different ional manifold approaches that identifies saddle node and Hopf bifurcation point without calculating any eigenvalues. Zhong et al. have proposed a new method to identify the critical instability area by C-means fuzzy clustering method for real time operation [13]. Bedoya et al. in [14] have presented a fast method to compute the voltage stability margin and the respective load increase direction using an optimization problem.

In the presented paper, we have proposed a new method for fast computation of voltage collapse point. The method is based on an optimization problem for identifying the maximum loading margin. The new method utilizes the genetic algorithm to solve the parameterized nonlinear load flow equations. Therefore, the algorithm can be speeded up. In order to further speed up the computational burden and to get faster responses, the optimization problem is examined only in weak area of the power system. From the voltage stability viewpoint, this area is determined by modal analysis. To increase the efficiency of this new method, it is combined with the conventional Newton-Raphson method, in which the maximum loading margin is defined through two stages as described in the paper. To show the credibility of the proposed approach it has been applied to IEEE 14, 30 and 57 bus systems.

The presented paper is organized in 5 sections. Section 2 explains the theoretical review and sets out power flow equations. The procedure of the proposed method is discussed in details in section 3. The simulation results of the proposed method are presented in section 4. The conclusions that can be drawn from this paper are presented in section 5.

## II. Theoretical Review

### *II.1. Power Flow Equation*

The operating condition of a power system is represented by a set of nonlinear algebraic equations namely load flow equations as below:

$$f(X)=0 \tag{1}$$

where $X$ represents the system variables including voltage magnitudes and phase angles of various buses in a power system. The set of parameterized load flow equations for voltage stability evaluation can be obtained as:

$$f(X,\lambda)=0 \tag{2}$$

where $X$ and $\lambda$ represent the system state variables and loading parameter, respectively. The loading parameter is used to move the system from one equilibrium point to another. This type of model has already been employed for voltage collapse studies in which $\lambda$ is being considered as the system load/generation increase factor as bellow:

$$\begin{aligned} P_{G_i} &= P_{G_{i0}}\left(1+K_{PG}\Delta\lambda\right)\\ P_{D_i} &= P_{D_{i0}}\left(1+K_{PD}\Delta\lambda\right)\\ Q_{D_i} &= Q_{D_{i0}}\left(1+K_{QD}\Delta\lambda\right) \end{aligned} \tag{3}$$

where $P_{D_{i0}}$ and $Q_{D_{i0}}$ are the initial active and reactive powers of load at bus $i$, $P_{G_{i0}}$ is the initial active power generation at bus $i$ and $\Delta\lambda$ is the step size of the loading parameter. Constants $K_{PD}$, $K_{QD}$ and $K_{PG}$ show the direction of increased active and reactive powers. In typical bifurcation diagrams, voltages are plotted as functions of $\lambda$ and called P-V curves.

### *II.2. Continuation Power Flow (CPF)*

Continuation power flow method can be used to trace the path of a power system from a steady state equilibrium point to a bifurcation point according to the load increase.

In this method, we can move along the bifurcation path by taking the following two steps: Predictor step realized by the computation of the tangent vector and corrector step that can be obtained by perpendicular intersection.

The shortcoming of this method is the computational time involved, especially when large power systems are considered.

## III. Proposed Method

This paper presents an accurate method based on optimization technique for fast estimation of bifurcation point. In this new approach, we have used genetic algorithm (GA) for the solution of nonlinear parameterized load flow equations. Furthermore, to speed up the computation of maximum loading margin, weak area has been used in the optimization process. To identify the weak area of a power system, the modal analysis technique is employed. The main goal is to introduce an efficient optimization-based technique to increase the speed of algorithm.

### *III.1. Modal Analysis*

Kundur in ref. [1] has derived a smallest eigenvalue and the related eigenvector of the reduced Jacobian matrix using the steady state system mode for voltage stability study of a power system.

The power flow equations can be presented as bellow:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \qquad (4)$$

where $J_{P\theta}$, $J_{PV}$, $J_{Q\theta}$ and $J_{QV}$ are sub matrices of load flow Jacobian, $\theta$ and $V$ are the state variables (angle and voltage magnitudes, respectively), $P$ and $Q$ are active and reactive powers injected to each bus.

The reduced Jacobian matrix $J_R$ is determined as:

$$\begin{gathered} \Delta Q = J_R \, \Delta V \\ J_R = \left[ J_{QV} - J_{Q\theta}\, J_{P\theta}^{-1}\, J_{PV} \right] \end{gathered} \qquad (5)$$

If all eigenvalues of the reduced Jacobian matrix are positive, the system is voltage stable. A zero eigenvalue means that the system is on the border of voltage collapse and the smallest magnitude of eigenvalue of the reduced Jacobian matrix determines the critical bus or weakest bus in a power system. So, weak area can be obtained by identifying those buses having small eigenvalues of the reduced Jacobian matrix using modal analysis. These buses which are located in the weak area are subjected to voltage collapse faster than the other buses.

### *III.2. Genetic Algorithm for Solving Nonlinear Equations*

A nonlinear set of equations is defined as:

$$F(X) = \begin{bmatrix} f_1(X) \\ f_2(X) \\ \vdots \\ f_n(X) \end{bmatrix} \qquad (6)$$

where $f_1, f_2, \ldots f_n$ are nonlinear functions and $X = [x_1, x_2, \ldots, x_n]$ is the variable of nonlinear system.

Now, we transform the nonlinear equations to a multi-objective optimization problem as below:

$$\begin{cases} \text{optimize} & f(X) = (f_1(X), \ldots f_n(X)) \\ \text{subject to} & X \in \Omega \end{cases}$$

where, $\Omega$ is the search space.

Genetic algorithm has been applied to solve the above mentioned multi-objective problem. The fitness function which should be minimized in the following forms may be formulated as:

1) $g = max\left[abs(f_i)\right] \qquad i = 1,2,\ldots n$
2) $g = \left[w_1 f_1(X) + w_2 f_2(X) + \ldots w_n f_n(X)\right]$

where $g$ is the fitness function to be minimized and $w_1, w_2, \ldots w_n$ are weight coefficients such that:

$$w_1 + w_2 + \ldots + w_n = 1$$

In cases in which the number of variables is limited, Genetic algorithm is able to provide an efficient and fast approach in solving the nonlinear equations in compare with classical methods [15].

### *III.3. New Methodology*

As mentioned, the new methodology of this paper for estimating the maximum loading margin in voltage collapse study utilizes genetic algorithm for solving the nonlinear load flow equations.

The fitness function to be minimized by the genetic algorithm can be presented as bellow:

$$g = max\left[abs\left(f_i(X)\right)\right] \quad i = 1,2,\ldots,n \qquad (7)$$

where $f_i$ represents the ith equation in the set of nonlinear power flow equations and x stands for the set of variables of the optimization problem including voltage phase, voltage magnitudes and loading parameter $\lambda$. Number of variables in the optimization problem can be calculated as bellow:

$$N_{GA} = 2 \times N + 1 \qquad (8)$$

where $N_{GA}$ represents the number of variables in optimization problem and $N$ represents the number of buses in the power system.

In this approach to reduce the number of variables of the nonlinear system, genetic algorithm is applied only in weak area which is a subsystem and defined by modal analysis. In this situation the voltage of all buses in weak area and the loading parameter $\lambda$ are considered as the variables of the optimization problem. So, the number of variables in optimization problem can be reduced to:

$$N_{GA} = 2 \times N_{VW} + 1 \quad (9)$$

where $N_{VW}$ is the number of buses in voltage weak area. Hence, the variables of nonlinear equations in genetic algorithm are significantly reduced and consequently the computational time is greatly decreased.

To increase the efficiency of this new method, it is combined with the conventional Newton-Raphson method. In this hybrid method, the maximum loading margin is defined through two following stages:

1) In the first stage, the system load/generation is increased via loading parameter $\lambda$ and the resulting nonlinear load flow equations are solved by conventional Newton-Raphson method. This procedure is continued so long as the Newton-Raphson method can be converged. The final solution of conventional method is used in the second stage. It is clear that according to the selected step size, the number of iterations is limited in the stage.
2) In the second stage, the step size of loading parameter $\Delta\lambda$ is reduced and voltage collapse study is examined using the proposed method. In this stage the variables of optimization problem in genetic algorithm are angle and voltage magnitude of buses in weak area and loading parameter $\lambda$. Voltage magnitudes as well as phase angles at other buses of power system are kept the same as those obtained in the previous stage. Then the maximum loading margin in weak area is computed. This computed value may be considered as the maximum loading margin of the whole system.

The flowchart in Fig. 1 indicates the basic steps of proposed method. The new method is fast, robust and accurate in determining the voltage security margin of power systems.

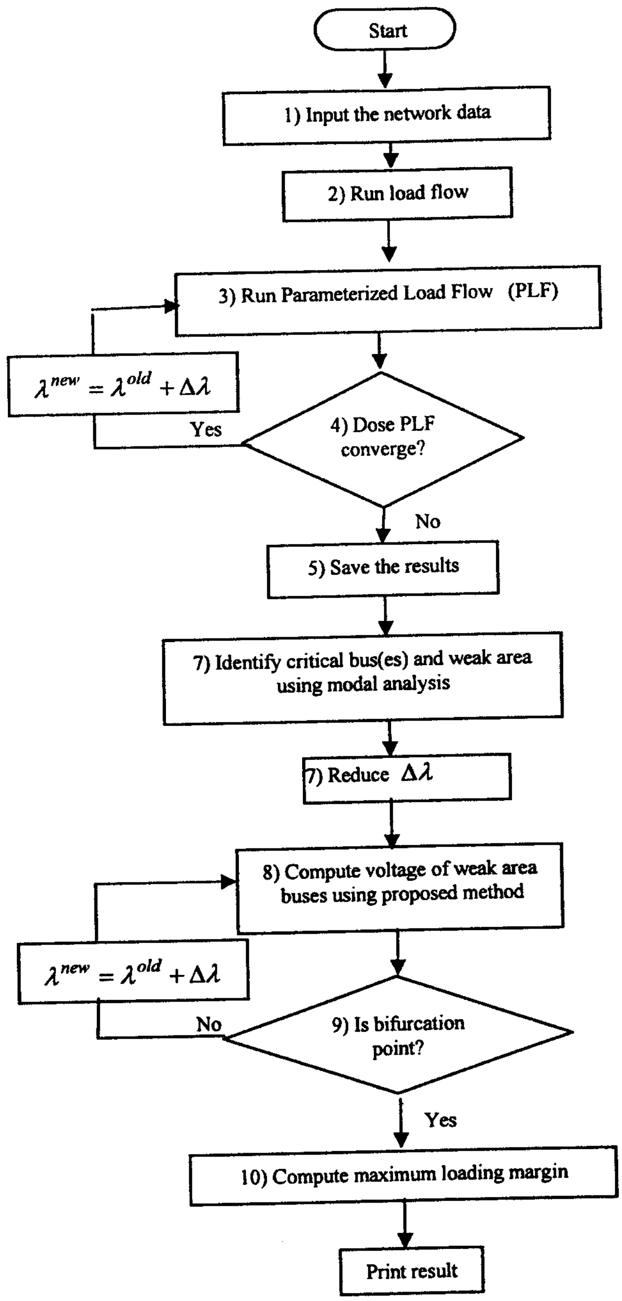


Fig. 1. Flowchart of proposed method

## IV. Case Study

To investigate the validity of the proposed algorithm, it has been applied to three systems, namely IEEE 14, 30 and 57 buses. The required data is found in [16].

The IEEE-14 bus system is shown in Fig. 2. This system has 5 generator buses, 9 load buses and 20 interconnected branches. The base load is presented in Table I.

As in the implementation of the proposed method, the critical bus(es) and weak area must be defined, hence before making any attempt to do so, at first the procedure of determination of such aims is described in section A. For theses objectives the IEEE-14 bus system and base load is considered.

### *IV.1. Determination of Weak Area with the Aid of Modal Analysis*

For base case, all eigenvalues of reduced reactive Jacobian matrix are positive, indicating the system is voltage stable at this operating point. Fig. 3 shows the eigenvalues of reduced reactive Jacobian matrix of power system. The smallest eigenvalue is 2.6813. This value is considered as the least stable mode for the critical point and is applied to determine the bus participation factor [1]. Fig. 4 shows the bus participation factors in the least stable mode for the base case.

From this figure, it can be observed that the weakest or the critical bus of this system is bus 12 having the highest participation factor. From Fig. 4, it can be easily observed that buses 10, 11, 12, 13 and 14, with high participation factors constitute the weak area of the system.

Therefore, for base case, it is assumed that these buses contribute more to the voltage collapse phenomenon.

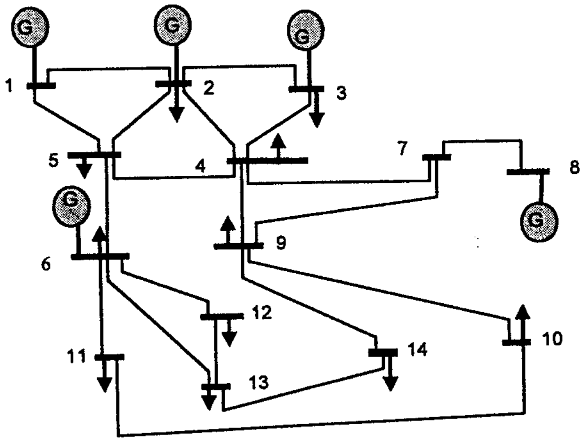


Fig. 2. IEEE 14 bus test system

TABLE I
BASE LOAD OF IEEE 14-BUS STANDARD

| No. load buses | $P_D$ [MW] | $Q_D$ [MVAr] |
|---|---|---|
| **4** | 47.8 | 3.9 |
| **5** | 7.6 | 1.6 |
| **7** | 0 | 0 |
| **9** | 29.5 | 16.6 |
| **10** | 9 | 5.8 |
| **11** | 3.5 | 1.8 |
| **12** | 16.1 | 11.6 |
| **13** | 13.5 | 15.8 |
| **14** | 14.9 | 5 |

### *IV.2. Computation of Voltage Stability Margin Using the Proposed Method*

According to the flowchart of Fig. 1, the following steps to be taken in order to compute the voltage stability margin:

1) First, the parameterized nonlinear load flow equations are solved by conventional power flow method for all buses of power system. Fig. 5 shows the V-$\lambda$ curve only for bus 12 as the loading factor increases. This stage is continued until the method is diverged, so the solution is terminated at point A in Fig. 5, before reaching the fold point. Therefore, this point of curve (point A) is obtained using conventional power flow through some iteration.

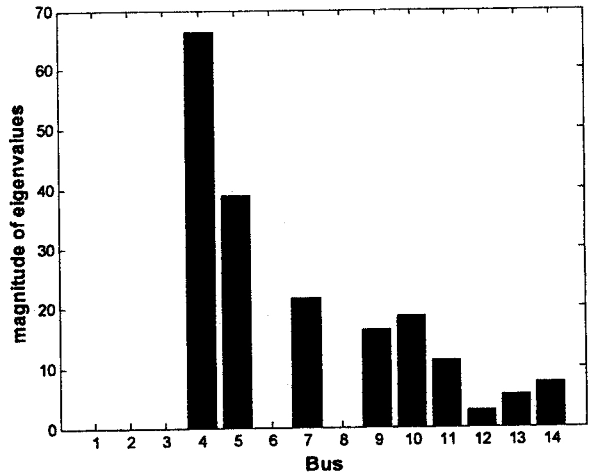


Fig. 3. Eigenvalue of test system for base case

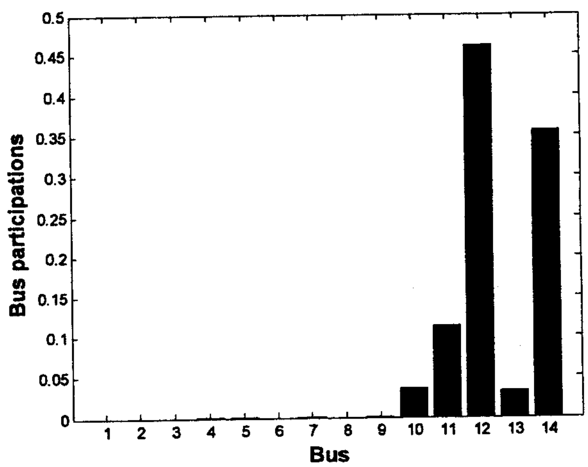


Fig. 4. Bus participation factors in the least stable mode for base case

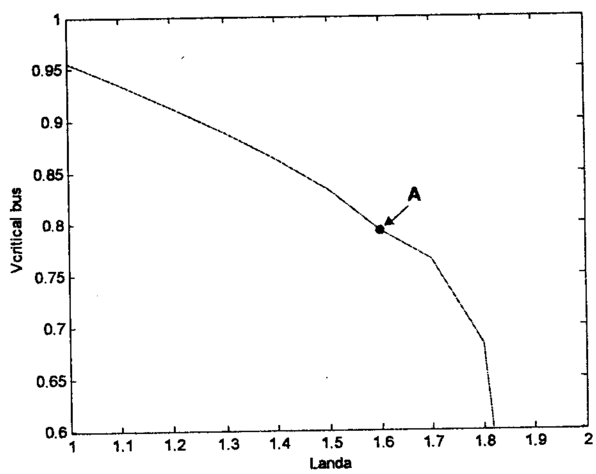


Fig. 5. V-$\lambda$ curve for bus No.12

2) Second, the critical bus(es) and weak area are determined using the results obtained in the first stage. With the aid of modal analysis, which is described in

section A, the critical bus is bus No.12 and the weak area contains buses 11, 12, 13 and 14 $(N_{VW} = 4)$.

3) Now, we apply the proposed method for computation of maximum loading margin in weak area. Variables of optimization problem in genetic algorithm are voltage magnitudes as well as phase angles of buses in weak area and loading parameter $\lambda$ $(N_{GA} = 2\times 4+1 = 9)$. Here, as the number of variables is decreased, genetic algorithm can be an efficient and fast approach in solving nonlinear power flow equations. Consequently, the computation time is considerably reduced.

Fig. 5 shows the complete V- $\lambda$ curve for the weakest bus (No. 12). As shown in this figure, bifurcation occurs when $\lambda$ is equal to 1.823 and the maximum loading margin of this system is 823%. Fig. 6 shows the complete V- $\lambda$ curves for weak area of the system.

In order to show the performance of the proposed method it is also applied to IEEE 30 and 57 bus systems. Figs. 7 and 8 show the V- $\lambda$ curves for weak area of the 30 and 57 test systems, respectively.

For the purpose of comparison, the maximum loading margin for the test systems was also obtained using standard continuation power flow (CPF) method for the base load. Table II shows the comparison between the CPF and the proposed methods. It can be seen that the computation time is greatly reduced. The amount or reduction for 57 bus system is about 43.2%.

In the CPF method, the computational time depends on the number of load flow iterations. In any iteration, numerous equations should be solved which is a computational burden. However, in our proposed method, by consideration of weak buses and using genetic algorithm for solving nonlinear load flow equations with reduced variables, a faster response is obtained. Furthermore, the proposed method provides accurate results within a short computational time, especially for large systems.

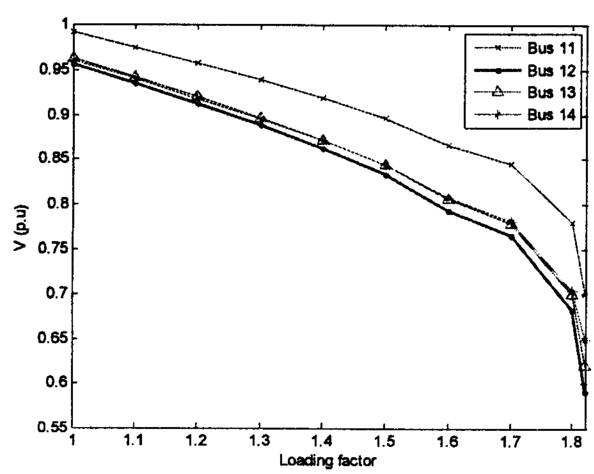


Fig. 6. V- $\lambda$ curve for weak buses for the 14-bus system

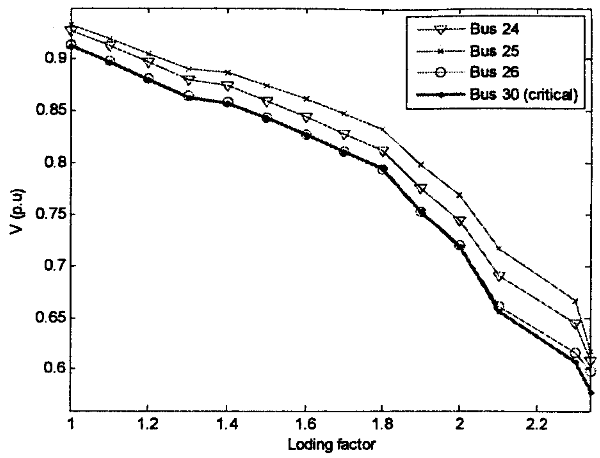


Fig. 7. V- $\lambda$ curve for weak buses for the 30-bus system

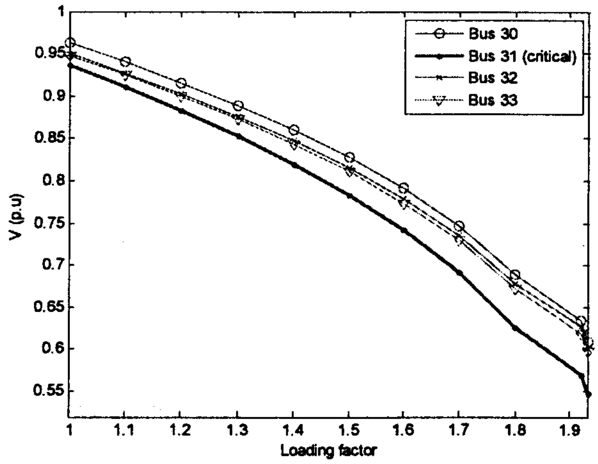


Fig. 8. V- $\lambda$ curve for weak buses for the 57-bus system

TABLE II
COMPARISON BETWEEN THE PROPOSED METHOD AND THE CPF METHOD

| System | Proposed method | | CPF method | |
|---|---|---|---|---|
| | $\lambda_{cr}$ | t (s) | $\lambda_{cr}$ | t (s) |
| **14 bus** | 1.82 | 2.2 | 1.841 | 2.4 |
| **30 bus** | 2.32 | 3.1 | 2.312 | 3.45 |
| **57 bus** | 1.93 | 7.35 | 1.943 | 12.95 |

## V. Conclusion

This paper proposes a new method for fast and accurate calculation of voltage collapse point in a power system. The method is based on the optimization problem in which the genetic algorithm is utilized to solve the parameterized nonlinear load flow equations associated with the weak area. The weak area is considered as a subsystem which is defined by modal analysis. In order to keep accuracy, a hybrid methodology is used whereby the maximum loading margin and voltage collapse point are determined in two stages. As our investigation is restricted only to the weak area along with reduced number of variables, the computation time is greatly reduced as compared with the standard CPF method. Therefore, this method is suitable for online voltage stability studies and can be comfortably applied to large power systems, without losing accuracy.

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