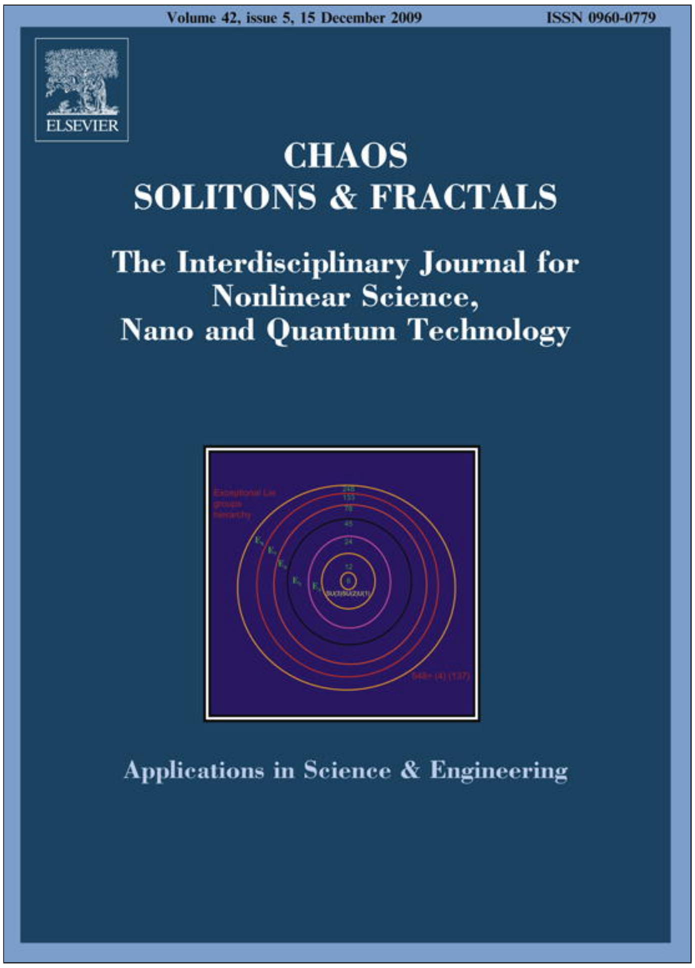
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Solitary waves in dusty plasmas with variable dust charge and two temperature ions

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ABSTRACT

Propagation of nonlinear waves in dusty plasmas with variable dust charge and two temperature ions is analyzed. The Kadomtsev–Petviashivili (KP) equation is derived by using the reductive perturbation theory. A Sagdeev potential for this system has been proposed. This potential is used to study the stability conditions and existence of solitonic solutions. Also, it is shown that a rarefactive soliton can be propagates in most of the cases. The soliton energy has been calculated and a linear dispersion relation has been obtained using the standard normal-modes analysis. The effects of variable dust charge on the amplitude, width and energy of the soliton and its effects on the angular frequency of linear wave are discussed too. It is shown that the amplitude of solitary waves of KP equation diverges at critical values of plasma parameters. Solitonic solutions of modified KP equation with finite amplitude in this situation are derived.

1. Introduction

Solitary waves and solitons represent one of the interesting and famous aspects of nonlinear phenomena in spatially extended systems. They appear as specific types of localized solutions of various nonlinear partial differential equations and possess several important properties.

Dusty plasmas are ideal medium for creating solitary waves and solitons. Such these environments have been observed in the earth's magnetosphere, cometary tail, planetary rings and so on [1–3]. Moreover study of dusty plasmas is very attractive because of theoretical features and also their applications.

The low frequency oscillations in dusty plasmas have been studied in [4,5]. The effects of dust temperature have been investigated in [6] and the normal-modes of plasmas because of the existence of heavy dust particles have been modified in [7]. In most investigations reductive perturbation method has been used for deriving the KdV or modified-KdV equations in one-dimensional case [8–10] and also for KP equation in higher dimensions [11]. Lin and Duan have investigated dusty plasmas with two-ions in [12]. They have shown that solitary solutions of the KP, modified KP and also coupled KP equations can be propagated in these type of plasmas. The effects of nonthermal distributed ions on the behaviour of dust acoustic solitary waves have been studied by Lin and Duan too [13]. Hot dust plasmas have been investigated by Duan in [14]. He has also derived the KP equation in a two-ion-temperature plasma containing isothermal ions and warm adiabatic dusty plasmas [15]. Dusty plasmas containing electrons and ions with space dependent densities have been studied by Zhang and Xue. They have shown that both shock waves and solitary waves can be propagated in such this medium [16]. In all

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above cases the charge of the dust particles is constant. Modified KdV equation for propagation of nonlinear dust acoustic waves has been derived in inhomogeneous dusty plasmas consisting of electrons, ions, and charged dust particles [17]. The charging process of dust particles is an important effect which has been investigated in [8,9,18,19]. This phenomenon was also studied by using semi-inverse method, applied to ion-acoustic plasma waves in [10]. Properties of small amplitude dust-ion-acoustic (DIA) solitary waves in warm plasmas containing two temperature electrons with external oblique magnetic field are studied by Shalaby et.al [20].

In the presented paper, the dusty plasma with variable dust charge and two temperature ions has been considered. One can obtain the KP equation using the reductive perturbation method (RPM) on two dimensional unmagnetized case of this system. Balancing between nonlinear and dispersion effects cause to formation of symmetrically solitary waves. The KP equation has been obtained for dust acoustic waves in hot dusty plasmas and also in dust-ion-acoustic dusty plasmas [21,22]. In Section 2, the basic set of equations is introduced and in Section 3, the KP equation has been derived. Section 4 contains discussion on solitonic solution and its stability conditions. The energy of the soliton will be calculated in Section 5. The linear dispersion relation and effects of variable dust charge on this relation has been discussed in this section too. In Section 6 the modified KP equation is derived at the critical values of the plasma parameters. Conclusions are given in Section 7.

2. Basic equations

We consider the propagation of dust acoustic waves in collisionless, unmagnetized dusty plasma consisting of electrons, two temperature ions and high negatively charged dust grains. Total charge neutrality at equilibrium requires that

$$n_{0e} + n_{0d}Z_{0d} = n_{0il} + n_{0ih} \tag{1}$$

where n_{0e}, n_{0d}, n_{0il} and $n_{0ih}$are the equilibrium values of electrons, dust, lower temperature ions and higher temperature ions number densities respectively. Z_{0d} is the unperturbed number of charges on the dust particles. The following set of normalized two dimensional equations of continuity, motion for the dust and Poisson, describe dynamics of dust acoustic wave

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}(n_d u_d) + \frac{\partial}{\partial y}(n_d v_d) = 0 \tag{2}$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial x} + v_d\frac{\partial u_d}{\partial y} = Z_d\frac{\partial \phi}{\partial x} \tag{3}$$

$$\frac{\partial v_d}{\partial t} + u_d\frac{\partial v_d}{\partial x} + v_d\frac{\partial v_d}{\partial y} = Z_d\frac{\partial \phi}{\partial y} \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = Z_d n_d + n_e - n_{il} - n_{ih} \tag{5}$$

where u_d and v_d are velocity components of the dust particles in x and y-directions. n_d, ϕ and Z_d are dust number density, electrostatic potential and variable charge number of dust grains, respectively. Note that all of the above variables have been normalized by $n_{\circ d}$. T_{eff} is effective temperature and it is given by:

$$\frac{1}{T_{eff}} = \frac{Z_{\circ d}n_{\circ d}}{\left(\frac{n_{0e}}{T_e} + \frac{n_{0il}}{T_{il}} + \frac{n_{0ih}}{T_{ih}}\right)} \tag{6}$$

Also dust acoustic speed, Debye length and inverse of dust plasma frequency are defined by $C_d = \left(\frac{Z_{0d}T_{eff}}{m_d}\right)^{\frac{1}{2}}$, $\lambda_d = \left(\frac{T_{eff}}{4\pi Z_{0d}^2 n_{0d} e^2}\right)^{\frac{1}{2}}$ and $\omega_{pd}^{-1} = \left(\frac{m_d}{4\pi n_{0d} Z_{0d} e^2}\right)^{\frac{1}{2}}$ respectively.

Electrons and ions are assumed to be distributed with Maxwell–Boltzmann distribution functions. So related dimensionless number densities for electrons (n_e), low temperature ions (n_{il}) and high temperature ions (n_{ih}) are:

$$n_e = \frac{n_{0e}}{n_{0d}Z_d}\exp(\beta_1 s\phi) \tag{7}$$

$$n_{il} = \frac{n_{0il}}{n_{0d}Z_d}\exp(-s\phi) \tag{8}$$

$$n_{ih} = \frac{n_{0ih}}{n_{0d}Z_d}\exp(-\beta s\phi) \tag{9}$$

where

$$\beta_1 = \frac{T_{il}}{T_e}, \quad \beta_2 = \frac{T_{ih}}{T_e}, \quad \beta = \frac{\beta_1}{\beta_2} = \frac{T_{il}}{T_{ih}}, \quad s = \frac{T_{eff}}{T_{il}}, \quad \delta_1 = \frac{n_{0il}}{n_{0e}}, \quad \delta_2 = \frac{n_{0ih}}{n_{0e}} \tag{10}$$

And from (1) it follows

$$\delta_1 + \delta_2 - 1 \geqslant 0 \tag{11}$$

The dust charge variable Q_d is obtained from the charge-current balance equation [23]

$$\left(\frac{\partial}{\partial t}+\vec{V}.\vec{\nabla}\right)Q_d = I_e + I_{il} + I_{ih} \tag{12}$$

where and I_e, I_{il} and I_{ih} are the electron and ions (low and high temperature) currents. We further suppose that the streaming velocities of electrons and ions are much smaller than the thermal velocities. Thus $\frac{dQ_d}{dt} \ll I_e, I_{il}, I_{ih}$ and charge-current balance Eq. (12) reads $I_e + I_{il} + I_{ih} \approx 0$. The electron and ions currents are [24]

$$I_e = -e\pi r^2\left(\frac{8T_e}{\pi m_e}\right)^{\frac{1}{2}} n_e \exp\left(\frac{e\Phi}{T_e}\right) \tag{13}$$

$$I_{il} = e\pi r^2\left(\frac{8T_{il}}{\pi m_i}\right)^{\frac{1}{2}} n_{il}\left(1-\frac{e\Phi}{T_{il}}\right) \tag{14}$$

$$I_{ih} = e\pi r^2\left(\frac{8T_{ih}}{\pi m_i}\right)^{\frac{1}{2}} n_{ih}\left(1-\frac{e\Phi}{T_{ih}}\right) \tag{15}$$

Where Φ denotes the dust grain surface potential relative to the plasma potential ϕ [25].

The normalized dust charge, Z_d is obtained from

$$Z_d = \frac{\psi}{\psi_0}$$

where $\psi = \frac{e\Phi}{T_{eff}}$ and $\psi_0 = \psi$ ($\phi = 0$). By expanding Z_d with respect to ϕ we have [11]

$$Z_d = 1 + \gamma_1\phi + \gamma_2\phi^2 + \gamma_3\phi^3 + \cdots \tag{16}$$

where $\gamma_1 = \frac{1}{\psi_0}\frac{d\psi(\phi)}{d\phi}\Big|_{\phi=0}$ and $\gamma_2 = \frac{1}{2\psi_0}\frac{d^2\psi(\phi)}{d\phi^2}\Big|_{\phi=0}$, $\quad \gamma_3 = \frac{1}{6\psi_0}\frac{d^3\psi(\phi)}{d\phi^3}\Big|_{\phi=0}$.

3. The derivation of KP equation

According to the general method of reductive perturbation theory, we choose the independent variables as

$$\xi = \varepsilon(x - v_\circ t), \quad \tau = \varepsilon^3 t, \quad \eta = \varepsilon^2 y \tag{17}$$

where ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity in the system and v_0 is the phase velocity of the wave along the x direction. We can expand physical quantities which have been appeared in (2)–(5) in term of the expansion parameter ε as

$$n_d = 1 + \varepsilon^2 n_{1d} + \varepsilon^4 n_{2d} + \cdots \tag{18}$$

$$u_d = \varepsilon^2 u_{1d} + \varepsilon^4 u_{2d} + \cdots \tag{19}$$

$$v_d = \varepsilon^3 v_{1d} + \varepsilon^5 v_{2d} + \cdots \tag{20}$$

$$\phi = \varepsilon^2\phi_1 + \varepsilon^4\phi_2 + \cdots \tag{21}$$

$$Z_d = 1 + \varepsilon^2 Z_{1d} + \varepsilon^4 Z_{2d} \tag{22}$$

Also with using (10) one can find

$$s = \frac{T_{eff}}{T_{il}} = \frac{\delta_1+\delta_2-1}{\delta_1+\delta_2\beta+\beta_1} \tag{23}$$

Substituting (17)–(22) into Eqs. 2,3,5 and collecting terms with same powers ofε, from the coefficients of lowest order we have:

$$n_{1d} = -\frac{\phi_1}{v_0^2}, \quad u_{1d} = -\frac{\phi_1}{v_0}, \quad v_0 = \frac{1}{\sqrt{1+\gamma_1}} \tag{24}$$

$$\frac{\partial v_{1d}}{\partial \xi} = -\frac{1}{v_0}\frac{\partial \phi_1}{\partial \eta} \tag{25}$$

And for the higher orders of ε

$$-v_\circ\frac{\partial n_{2d}}{\partial \xi} + \frac{\partial n_{1d}}{\partial \tau} + \frac{\partial(u_{2d}+n_{1d}u_{1d})}{\partial \xi} + \frac{\partial v_{1d}}{\partial \eta} = 0 \tag{26}$$

$$-v_\circ\frac{\partial u_{2d}}{\partial \xi} + \frac{\partial u_{1d}}{\partial \tau} + u_{1d}\frac{\partial u_{1d}}{\partial \xi} = Z_{1d}\frac{\partial \phi_1}{\partial \xi} + \frac{\partial \phi_2}{\partial \xi} \tag{27}$$

$$\frac{\partial^2\phi_1}{\partial \xi^2} = Z_{2d} + Z_{1d}n_{1d} + n_{2d} + \phi_2 - \frac{1}{2}(\delta_1 + \delta_2\beta^2 - \beta_1^2)\frac{s^2}{(\delta_1+\delta_2-1)}\phi_1^2 \tag{28}$$

The KP equation is derived from the above equations

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+a\phi_1\frac{\partial\phi_1}{\partial\xi}+b\frac{\partial^3\phi_1}{\partial\xi^3}\right]+c\frac{\partial^2\phi_1}{\partial\eta^2}=0 \tag{29}$$

where

$$a=\frac{v_\circ^3}{2}\left[(\delta_1+\delta_2\beta^2-\beta_1^2)\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)^2}-2\gamma_2\right]+\frac{3}{2}\gamma_1 v_0-\frac{3}{2v_0},\quad b=\frac{v_\circ^3}{2},\quad c=\frac{v_\circ}{2} \tag{30}$$

Eq. (30) with $\gamma_1 = \gamma_2 = 0$, reduces to the results of [14] for warm plasmas with one ion. The effects of dust charge variation and nonthermal ions on dust acoustic solitary wave structure in magnetized dusty plasmas has been studied using the KdV equation in [19]. Notice that the derived parameter "a" is different from what has been reported in [11]. Our calculation shows that what has been appeared in [11] can not be correct.

From (10) one can find that $\beta_1,\beta < 1$. Now let us examine sign of "a" which has been defined in (30). Parameter "a" reaches its maximum where the first term becomes maxima and the second term attains its minimum value. The first term is maximum when$\gamma_2 = 0$. Thus for $\gamma_2 = 0$ and $\gamma_1 \neq 0$ "a" is maximal. We choose $\gamma_1 = \gamma_2 = 0$ and in this case "a" is

$$a=\frac{1}{2}\left[(\delta_1+\delta_2\beta^2-\beta_1^2)\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)^2}-3\right]$$

Obviously $(\delta_1+\delta_2\beta^2-\beta_1^2)$ is always less than $(\delta_1 + \delta_2\beta + \beta_1)$, but for term $\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)}$ we have

$$\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)}=\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2-1)+1+\beta_1-(1-\beta)\delta_2}$$

It is clear that above term is less than 1 if $\delta_2<\frac{1+\beta_1}{1-\beta}$ and in this case "a" is always negative and rarefactive solitons always exist. Also above mentioned term is more than 1 if $\delta_2>\frac{1+\beta_1}{1-\beta}$ and in this case "a" can get positive or negative values and in these cases both compressive and rarefactive solitary waves can be propagated. Figs. (4)1–3 show the variation of "a" with respect to different values of β, β_1, δ_1 and δ_2.

In Fig. 1 "a" is plotted as a function of β andβ_1 when$\delta_1 = 1$, $\delta_2 = 4$ and $v_0 = 1$.

Fig. 2 presents "a" as a function of β and δ_2 when$\delta_1 = 1.1$, $\beta_1 = 0.01$ and$v_0 = 1$. Fig. 3 demonstrates "a" respect to β_1 and δ_1 when $\delta_2 = 1.1$ and β=0.01 with$v_0 = 1$.

We can see that with a fixed value for δ_1 and δ_2, "a" reaches its maximum when β and β_1 have their minimum values. Fig. 4 presents "a" as a function of δ_1 and δ_2, with $\beta = 0.01$, $\beta_1 = 0.5$ and $v_0 = 1$.

All of the figures show that "a" is negative for most of the acceptable values of the parameters and it is positive only in small region of parameters. Exact solutions of the KP equation have been derived in [26]. The existence and stability of one-soliton solution of (29) is discussed in the next section.

4. Discussion

We introduce the variable

$$\chi = l\xi + m\eta - u\tau \tag{31}$$

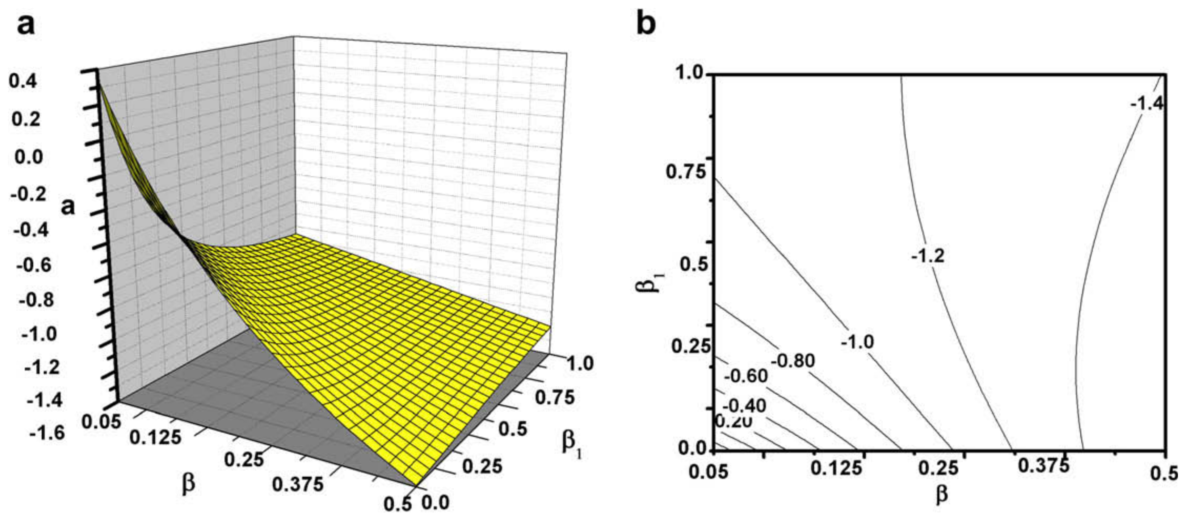

Fig. 1. The parameter "a" as a function of β and β_1 with $\delta_1 = 1$, $\delta_2 = 4$ and $v_0 = 1$. Fig. 1b is the contour plot of Fig. 1a.

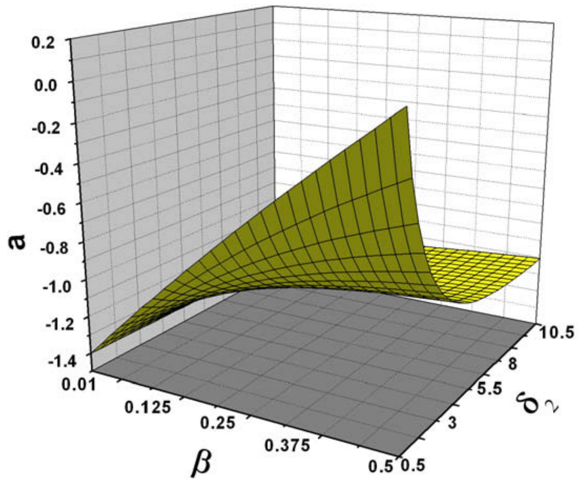

Fig. 2. The parameter "a" as a function of β and δ_2 with $\delta_1 = 1.1$, $\beta_1 = 0.01$ and $v_0 = 1$.

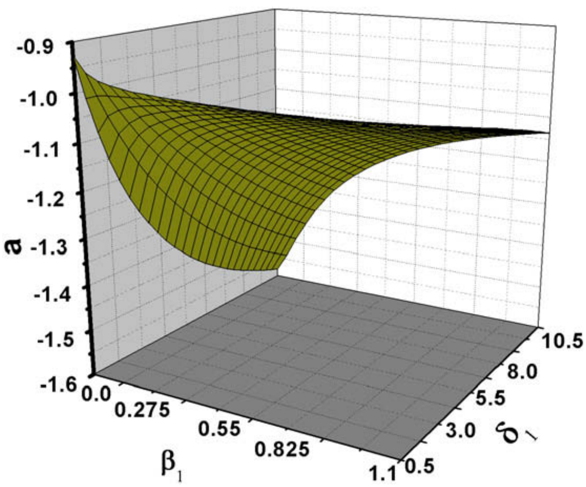

Fig. 3. "a" as a function of β_1 and δ_1 with $\delta_2 = 1.1$, $\beta = 0.01$ and $v_0 = 1$.

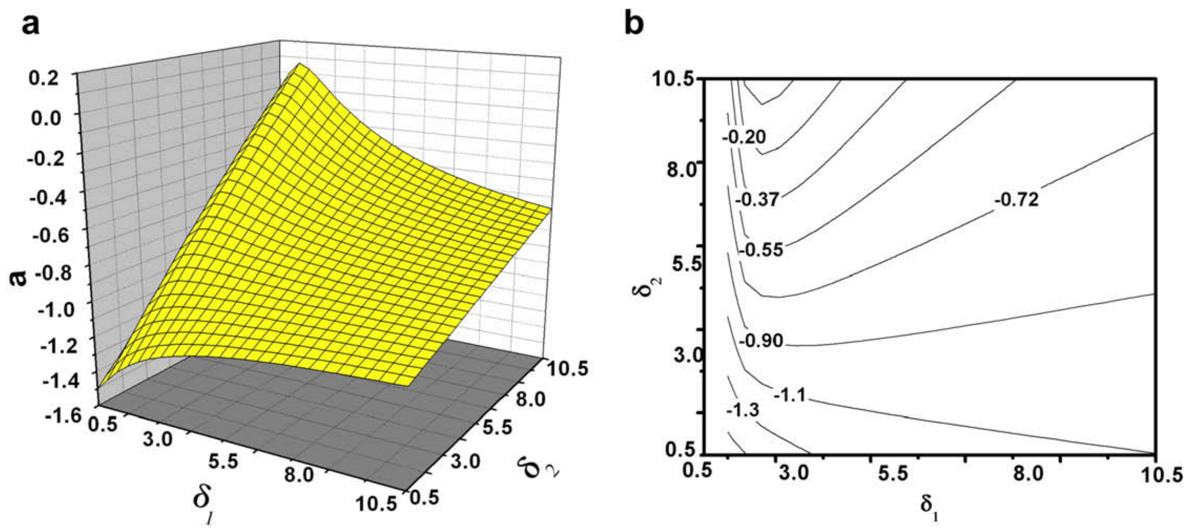

Fig. 4. The parameter "a" as a function of δ_1 and δ_2 when $\beta = 0.01$, $\beta_1 = 0.5$ and $v_0 = 1$.

where χ is the transformed coordinate relative to a frame which moves with the velocity u. "l" and "m" are the directional cosines of the wave vector "k" along the ξ and η respectively, in the way that $l^2 + m^2 = 1$.

By integrating (29) respect to the variable χ and using the vanishing boundary condition for ϕ_1 and its derivatives up to the second-order for $|\chi| \to \infty$, we have

$$\frac{d^2\phi_1}{d\chi^2} = \frac{h}{l^4 b}\phi_1 - \frac{a}{2l^2 b}\phi_1^2 \tag{32}$$

where

$$h = ul - m^2 c \tag{33}$$

Eq. (32) has solitonic solutions and one-soliton solution for this equation is given by

$$\phi_1 = \phi_\circ \sec h^2\left[\frac{\chi}{W}\right] \tag{34}$$

where $\phi_\circ = \frac{3h}{l^2 a}$ is the amplitude while $W = 2\sqrt{\frac{l^4 b}{h}}$ is the width of the soliton.

For investigating the stability conditions of this solution, we use a method based on the energy considerations [27]. Thus we are going to find the Sagdeev potential for this situation. Eq. (32) can be written as

$$\frac{d^2\phi_1}{d\chi^2} = \frac{h}{l^4 b}\phi_1 - \frac{a}{2l^2 b}\phi_1^2 = -\frac{dV(\phi_1)}{d\phi_1} \tag{35}$$

In order to obtain the Sagdeev potential, Eq. (35) is integrated to yield the nonlinear equation of motion as

$$\frac{1}{2}\left[\frac{d\phi_1}{d\chi}\right]^2 + V(\phi_1) = 0 \tag{36}$$

where

$$V(\phi_1) = \frac{a}{6l^2 b}\phi_1^3 - \frac{h}{2l^4 b}\phi_1^2 \tag{37}$$

It is clear that $V(\phi_1) = 0$ and $\frac{dV(\phi_1)}{d\phi_1} = 0$ at $\phi_1 = 0$. A stable solitonic solution must satisfy the following conditions [28,29]

(I) $\left[\frac{d^2V}{d\phi_1^2}\right]_{\phi_1=0} < 0$

(II) There must exists a nonzero crossing point $\phi_1 = \phi_\circ$ that $V(\phi_1 = \phi_0) = 0$.

(III) There must exists a ϕ_1 between $\phi_1 = 0$ and $\phi_1 = \phi_\circ$ to make $V(\phi_1) < 0$.

Thus, from (36) and (37) we have

$$\left.\frac{d^2V(\phi_1)}{d\phi_1}\right|_{\phi_1=0} = -\frac{h}{l^4 b} < 0 \tag{38}$$

The parameters, l and b are positive, Therefore $h > 0$ or

$$ul - m^2 c > 0 \tag{39}$$

It is clear that the width (W) of a stable solitary wave is real.

We found that h > 0 and also for most of the cases the parameter "a" is negative. By these conditions the term $\phi_\circ = \frac{3h}{l^2 a}$ is negative. Therefore the solution is a rarefactive soliton in most of the cases.

Now let us find the stability conditions for the above solution. From the (39) we have

$$u > \frac{m^2}{l}c$$

or

$$u > \left(\frac{1-l^2}{l}\right)c \tag{40}$$

If $\frac{1-l^2}{l} > 1$ then u > c and when $\frac{1-l^2}{l} < 1$ we have u < c. Thus the soliton is stable if

$$\begin{cases} u \geqslant c & \text{when} \quad 0 < l \leqslant 0.62 \\ 0 < u < c & \text{when} \quad 0.62 < l < 1 \end{cases} \tag{41}$$

Fig. 5 shows the soliton amplitude ($\phi_\circ$) as a function of velocity "u" and Fig. 6 presents the soliton width respect to the velocity "u".

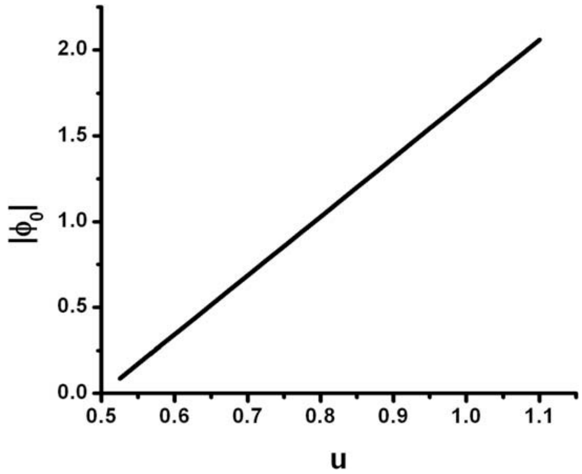

Fig. 5. Soliton amplitude as a function of "u". The figure was plotted with $\beta = 0.01$, $v_\circ = 1$ and $l = 0.6$.

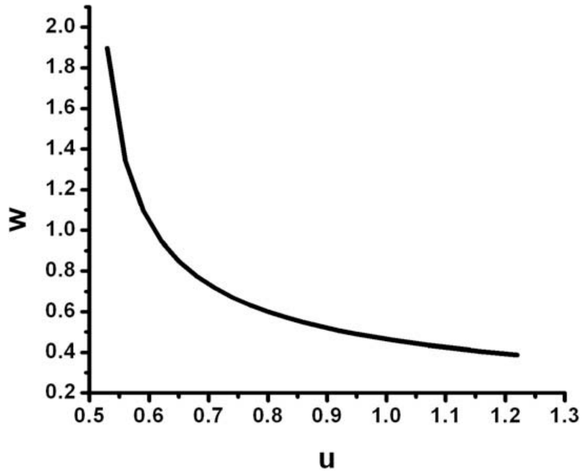

Fig. 6. Soliton width as a function of velocity "u" with $\beta = 0.01$, $v_\circ = 1$ and $l = 0.6$.

We can see that the amplitude of the soliton ($\phi_\circ$) increases when "u" is increased, while its width decreases with an increasing velocity "u". On the other hand, from the definition of the soliton amplitude and its width, one can find that the amplitude (width) decreases (increases) with an increasing value for the parameter "l". This means that the parameters "u" and "l" have important roles in the stability of soliton. Thus a soliton is stable when the effects of these two phenomena cancel out each other.

Finally for the case $\frac{m^2}{l} = 1$ and u > c, we have

$$W = 2\sqrt{\frac{l^3 b}{u-c}}, \quad \phi_\circ = \frac{3(u-c)}{la}, \quad \phi_1 = \phi_\circ \sec h^2\left(\frac{\chi}{W}\right) \tag{42}$$

And the potential is

$$V(\phi_1) = \frac{a}{6l^2 b}\phi_1^3 - \frac{(u-c)}{2l^3 b}\phi_1^2 \tag{43}$$

5. Energy of soliton and linear dispersion relation

The soliton energy can be obtained using the following equation [30]

$$E = \int_{-\infty}^{+\infty} u_{1d}^2 d\chi \tag{44}$$

After the integration, we have [30]

$$E = \frac{4}{3}u_m^2 W = \frac{24(u-c)^2}{a^2}(1+\gamma_1)\sqrt{\frac{b}{u-c}} \tag{45}$$

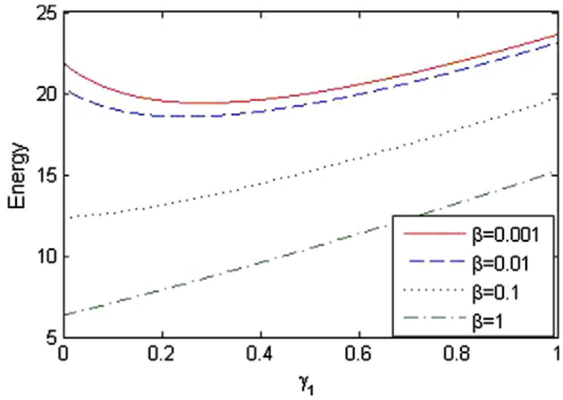

Fig. 7. Energy of the soliton as a function of γ_1 for $\gamma_2 = 0$, $\delta_1 = 2$, $\delta_2 = 3$, $\beta_1 = 0.1$, and u=1.1.

Fig. 7 indicates that the energy of soliton increases with an increasing. This figure also shows that, the soliton energy decreases when the temperature of cold ion approaches the temperature of warm ion.

The above calculated energy comes from the motion of the dust particles so this is a kinetic energy. We can add the electrostatic potential energy into this quantity. The electrostatic potential energy is

$$E_p = \frac{1}{2}\int_{-\infty}^{+\infty}\left(-\frac{d\phi_1}{d\chi}\right)^2 d\chi \tag{46}$$

where $\left(-\frac{d\phi_1}{d\chi}\right)$ is the electrostatic filed. Using (42) we have

$$E_p = \frac{48}{5}\frac{(u-c)^2}{l^2a^2}\sqrt{\frac{u-c}{bl^3}} \tag{47}$$

Linear dispersion relation can be obtained as follows. According to the standard normal-mode analysis, by linearization of dependent variables n_d, ϕ and Z_d in terms of their equilibrium and perturbed parts [31,32], we have

$$n_d = 1 + n_{1d}, \quad \phi = \phi_1, \quad u_d = u_{1d}, \quad Z_d = 1 + Z_{1d} = 1 + \gamma_1\phi_1 \tag{48}$$

We assume that all the perturbed quantities are proportional to $e^{i(kx-\omega t)}$ where 'K' is the wave propagation constant in the direction of x-axis. Therefore we have $\frac{\partial}{\partial t} = -i\omega$, $\frac{\partial}{\partial x} = ik$. Substituting (48) into (10)–(12), (14) and (15) and using their linear terms one obtains linear dispersion relation as

$$\omega^2 = \frac{k^2}{k^2 + 1 + \gamma_1} \tag{49}$$

Fig. 8 shows the angular frequency (ω) as a function of k for $\gamma_1 = 0$ and $\gamma_1 = 0.2$.

Fig. 8 indicates that increasing k (γ_1) leads to increasing (decreasing) values for theω. For real values of ω, all perturbation variables oscillate harmonically and if any or all of the ω's have positive imaginary parts, then the system is unstable since those normal-modes will grow in time [32,33].

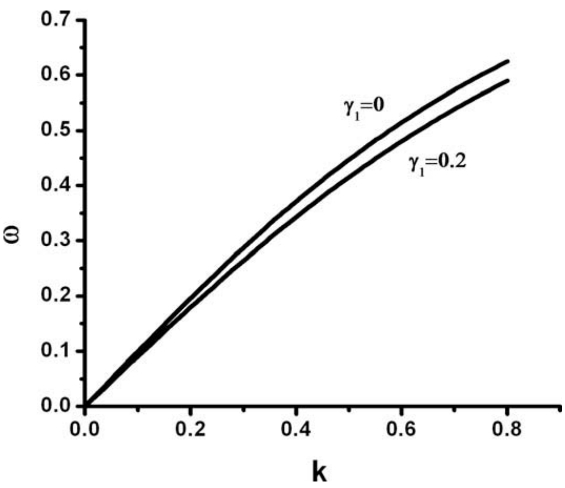

Fig. 8. The angular frequency respect to k for $\gamma_1 = 0$ and $\gamma_1 = 0.2$.

6. Modified KP equation

The strength of the nonlinear term in KP equation depends on the value of the parameter "a" which is a function of β_1,β, δ_1,δ_2, σ_i, γ_1 and γ_2. The dependency of "a" was studied by plotting this quantity as a function of other parameters. We saw that the parameter "a" can be positive or negative, so by taking a specific value for the plasma parameters (which is called the critical parameters) it is possible that "a" becomes zero and thus ϕ_m increases to infinity. For example with the $\gamma_1 = \gamma_2 = 0$ 'a' becomes zero if

$$(\delta_1 + \delta_2\beta^2 - \beta_1^2)\frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)^2} = 3 \tag{50}$$

In this case the stretching coordinate transformation is not valid. But we can save the equations by using a new set of parameters as follows

$$\begin{aligned}
n_d &= 1 + \varepsilon n_{1d} + \varepsilon^2 n_{2d} + \varepsilon^3 n_{3d} + \cdots \\
u_d &= \varepsilon u_{1d} + \varepsilon^2 u_{2d} + \varepsilon^3 u_{3d} + \cdots \\
v_d &= \varepsilon^2 v_{1d} + \varepsilon^3 v_{2d} + \varepsilon^4 v_{3d} + \cdots \\
\phi &= \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \cdots \\
Z_d &= 1 + \varepsilon^1\gamma_1\phi_1 + \varepsilon^2(\gamma_1\phi_2 + \gamma_2\phi_1) + \varepsilon^3(\gamma_1\phi_3 + 2\gamma_2\phi_1\phi_2 + \gamma_3\phi_1^3)
\end{aligned} \tag{51}$$

Again by using (51) in the main Eqs. (2)–(5) and collecting terms with the same powers of expanding parameter ε we have (24) and (25) again, for the lowest order. But for higher orders of ε we will find

$$\begin{aligned}
& n_{2d} = \frac{1}{2v_\circ^2}\left(\frac{3}{v_\circ^2} - \gamma_1\right)\phi_1^2, \quad u_{2d} = \frac{1}{2v_\circ}\left(\frac{1}{v_\circ^2} - \gamma_1\right)\phi_1^2 - \frac{\phi_2}{v_\circ}, \quad v_0 = \frac{1}{\sqrt{1+\gamma_1}} \\
& \frac{\partial n_{1d}}{\partial\tau} - v_\circ\frac{\partial n_{3d}}{\partial\xi} + \frac{\partial}{\partial\xi}(n_{1d}u_{2d} + n_{2d}u_{1d} + u_{3d}) + \frac{\partial v_{1d}}{\partial\eta} = 0 \\
& \frac{\partial u_{1d}}{\partial\tau} + \frac{\partial}{\partial\xi}(u_{1d}u_{2d}) - v_\circ\frac{\partial u_{3d}}{\partial\xi} = \frac{\partial\phi_3}{\partial\xi} + Z_1\frac{\partial\phi_2}{\partial\xi} + Z_2\frac{\partial\phi_1}{\partial\xi} \\
& \frac{\partial^2\phi_1}{\partial\xi^2} = n_1 Z_2 + n_2 Z_1 + n_3 + Z_3 + \phi_3 + \frac{1}{6}\left(\frac{\delta_1 + \delta_2 - 1}{\delta_1 + \delta_2\beta + \beta_1}\right)^3(1 + \beta_1^3 + \beta^3)\phi_1^3 + \left(\frac{\delta_1 + \delta_2 - 1}{\delta_1 + \delta_2\beta + \beta_1}\right)^2(1 + \beta^2 + \beta_1^2)(\phi_1\phi_2)
\end{aligned} \tag{52}$$

and $-v_\circ\frac{\partial v_{1d}}{\partial\xi} = \frac{\partial\phi_1}{\partial\eta}$

Finally we have the following equation

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau} + A\phi_1^2\frac{\partial\phi_1}{\partial\xi} + E\frac{\partial}{\partial\xi}(\phi_1\phi_2) + B\frac{\partial^3\phi_1}{\partial\xi^3}\right] + C\frac{\partial^2\phi_1}{\partial\eta^2} = 0 \tag{53}$$

Where A, E, B and C are

$$\begin{aligned}
A &= \frac{v_\circ^3}{4}\left(\frac{4}{3}\gamma_2 + \frac{\gamma_1^2}{2}\right)\left[(\delta_1 + \delta_2\beta^2 - \beta_1^2)\frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)^2} - 2\gamma_2 + \gamma_3\right] + \frac{15}{4}\left[\frac{3}{2}\gamma_1 v_0 + (\delta_1 + \delta_2\beta^2 - \beta_1^2)\frac{\delta_1 + \delta_2 - 1}{(\delta_1 + \delta_2\beta + \beta_1)^2}\right] - \frac{1}{v_0}\gamma_1(\delta_1 + \delta_2 - 1 + 2\gamma_1) \\
B &= \frac{v_\circ^3}{2}, \quad C = \frac{v_\circ}{2} \\
E &= \frac{v_\circ^3}{2}\left[(\delta_1 + \delta_2\beta^2 - \beta_1^2)\frac{(\delta_1 + \delta_2 - 1)}{(\delta_1 + \delta_2\beta + \beta_1)^2} - 2\gamma_2\right] + \frac{3}{2}\gamma_1 v_0 - \frac{3}{2v_0}
\end{aligned} \tag{54}$$

It is clear that $a = E$, so for critical parameters "E" becomes zero and in this situation (53) reduces into the modified KP equation

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau} + A\phi_1^2\frac{\partial\phi_1}{\partial\xi} + B\frac{\partial^3\phi_1}{\partial\xi^3}\right] + C\frac{\partial^2\phi_1}{\partial\eta^2} = 0 \tag{55}$$

This equation has solitonic solutions. One-soliton solution for this equation is [34,35]

$$\phi_1 = \pm\phi_m \operatorname{sec}h[(\xi + \eta - u\tau)/W] \tag{56}$$

where u, $\phi_m = \sqrt{6(u-C)/A}$ and $W = \sqrt{B/(u-C)}$ are velocity, amplitude and width of the solitary wave respectively. The above results for one-dimensional propagation with$\gamma_1 = \gamma_2 = \gamma_3 = 0$ can be compared with results of [36]. It is clear that "A" is always positive.

7. Conclusion

The KP equation was obtained in unmagnetized dusty plasma with variable dust charge and two temperature ions. For the KP equation (29), parameters "b" and "c" are always positive. But parameter "a" can be positive or negative; however it is negative for most of the cases. This means that generally a rarefactive soliton is appeared in the medium. Consequently amplitude of the solitary waves is smaller as compared to the one-dimensional case [25].

The Sagdeev potential was derived and stability conditions were investigated. One can find that for a stable soliton the velocity "u" has some limitations (see (4)). This means that the solitons are stable only if the effects of dust and ions motion cancel out each other. Analytically, the coefficients of the dispersive terms, "b" and "c" depend on the parameter γ_1. Indeed dispersion decreases when γ_1 is increased. The parameter "a" is coefficient of nonlinear term. It is function of relative densities, relative temperatures, γ_1 and γ_2. Therefore, it is possible that the competition between the nonlinear term and dispersion terms, lead to the formation of a soliton. The energy of soliton and linear dispersion relation have been derived and discussed too.

Since the parameter "a" can be positive or negative it can be zero too. But a solitonic solution can not be established when "a' is zero. This means that "a" has critical values. In this situation we have derived modified KP equation and solitonic solution of this equation which has finite amplitude.

References

[1] Shukla PK, Mamun AA. Introduction to dusty plasma Physics. London, UK: IOP; 2002.
[2] Rao NN, Shukla PK, Yu My. Dust-acoustic waves in dusty plasmas. Planet Space Sci 1990;38:543–6.
[3] Mendis DA, Rosenberg M. Cosmic dusty plasma. Annu Rev Astron Astrophys 1994;32:419.
[4] Goertz CK. Dusty plasmas in the solar. Rev Geophys 1989;27:271.
[5] Whipple EC, Northrop TG, Mendis DA. The electrostatics of a dusty plasma. J Geophys Res 1985;90:7405.
[6] Duan WS, Lu KP, Zhao JB. Hot dust acoustic solitary waves in dust plasma with variable dust charge chin. Phys Lett 2001;18:1088.
[7] Shukla PK, Mamun AA. Introduction to dusty plasma. Bristol, UK: IOP, Publishing; 2002.
[8] Bharuthram R, Shukla PK. Large amplitude ion-acoustic solitons in a dusty plasma. Planet Space Sci 1992;40:973.
[9] Nakamura Y, Sarma A. Observation of ion-acoustic solitary waves in a dusty plasma. Phys Plasmas 2001;8:3921.
[10] He JH. Variational principles for some nonlinear partial differential equations with variable coefficients. Chaos, Soliton and Fractals 2004;19:84.
[11] Gill TS. Nareshpal Singh Saini, Harvinder Kaur,The Kadomstev–Petviashvili equation in dusty plasma with variable dust charge and two temperature ions. Chaos, Solitons and Fractals 2006;28(4):1106–11.
[12] Lin MM, Duan WS. The Kadomtsev–Petvishcili (KP), MKP, and coupled KP equations for two- ion-teperature dusty plasmas. Chaos, Solitons and Fractals 2005;23:929.
[13] Lin MM, Duan WS. Dust acoustic solitary waves in dusty plasmas with nonthermal ions. Chaos, Solitons and Fractalse 2007;33(4):1189–96.
[14] Duan WS. The KP equation of dust acoustic waves for hot dust plasmas. Chaos, Solitons and Fractals 2002;14(3):503–6.
[15] Duan WS. Acoustic mode in a two-ion-temperature warm dusty plasma under the weakly transverse perturbations. Chaos, Solitons and Fractals 2002;14:1315–8.
[16] Xue JK, Zhang LP. Non-linear waves in complex plasma. Chaos, Solitons and Fractals 2007;32:592.
[17] Xue J. Electrostatic waves in inhomogeneous dusty plasmas, with grain charge fluctuation. Chaos, Solitons and Fractals 2004;19:971–5.
[18] Duan WS, Xue-ren S, Jian-an S. Envelop solitons in dusty plasmas for warm dust. Chaos, Solitons and Fractals 2003;16(5):767–77.
[19] Zhang LP, Xue J. Effects of the dust charge variation and non-thermal ions on multi-dimensional dust acoustic solitary structures in magnetized dusty plasmas. Chaos, Solitons and Fractals 2005;23(2):543–50.
[20] Shalaby M, El-Labany SK, El-Shamy EF, El-Sherif LS. Dust-ion-acoustic solitary waves in a two-temperature electrons with charge fluctuations in a dusty plasma. Chaos, Solitons and Fractals 2009;41:1208–17.
[21] Nejoh YN. The dust charging effect on electrostatic ion waves in a dusty plasma with trapped electrons. Phys Plasmas 1997;4:2813.
[22] Waleed MM. Dust ion acoustic solitons and shocks in dusty plasmas. Chaos, Solitons and Fractals 2006;28(4):994–9.
[23] Melandso F, Askalsen FT, Havnes O. A new damping effect for the dust-acoustic wave. Planet Space Sci 1993;41:321.
[24] Duan WS. The Kadomtsev–Petviashvili (KP) equation of dust acoustic waves for hot dust plasmas. Chaos, Solitons and Fractals 2002;14:503.
[25] Xie BS, He KF, Huang ZQ. Dust-acoustic solitary waves and double layers in dusty plasma with variable dust charge and two-temperature ions. Phys Plasmas 1999;6:3808.
[26] Zhang S. Further improved F-expansion method and new exact solutions of Kadomstev–Petviashvili equation. Chaos, Solitons and Fractals 2007;32(4):1375–83.
[27] Krall N, Trivelpiece AW. Principles of plasma physics. McGraw-Hill, Inc.; 1973.
[28] Moslem WM. Dust-ion-acoustic solitons in a strong magnetic field. Phys Lett A 2006;351:290.
[29] Gill Ts, Kaur H, Saini NS. Dust-acoustic solitary waves in a finite temperature dusty plasma with variable dust charge and two temperature ions. J Plasma Phys 2004;70:481.
[30] Singh S, Honzawa T. Kadomtsev–Petviashivili equation for an ion-acoustic soliton in a collisionless weakly relativistic plasma with finite ion temperature. Phys Fluids B 1993;5:2093.
[31] Verheest F, Lakhina S. Oblique solitary Alfvén modes in relativistic electron–positron plasmas. Astrophys Space Sci 1996;240:215.
[32] Samanta S, Misra AP, Chowdhury AR. Effects of obliqueness and external magnetic field on the large amplitude solitary waves in a dusty plasma. Planet Space Sci 2007;55:1380.
[33] Krall NA, Trivelpiece AW. Principles of physics. San Francisco, CA: San Francisco Press; 1986.
[34] Wang ZX, Wang X, Ren LW, Liu J-Y, Yue Liu. Effect of negative ions on dust-acoustic soliton in a dusty plasma. Phys Lett A 2005;339:96.
[35] Wadati M. The modified Korteweg-de Vries equation. J Phys Soc Jpn 1973;34:1289.
[36] Lin MM, Duan WS. Dust acoustic solitary waves in a dusty plasmas with nonthermal ions. Chaos, Solitons and Fractals 2007;33:1189.