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GENERAL THREE-DIMENSIONAL STAGNATION-POINT FLOW AND HEAT TRANSFER ON A FLAT PLATE

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Abstract:
The existing solutions of Navier-Stokes and energy equations in the literature regarding the three-dimensional problem of stagnation-point flow either on a flat plate or on a cylinder with or without transpiration are only for the case of axisymmetric formulation. In this study the non-axisymmetric three-dimensional steady viscous stagnation-point flow and heat transfer in the vicinity of a flat plate are investigated when suction and blowing are also considered in the model. A similarity solution of the Navier-Stokes equations and energy equation is presented in this problem. A reduction of these equations is obtained by use of appropriate similarity transformations. Velocity profiles and surface stress-tensors and temperature profiles along with pressure profile are presented for different values of velocity ratios, and Prandtl number for sample cases of transpiration.

1- Introduction

There are many exact solutions for Navier-Stocks and energy equations regarding the problem of stagnation-point flow and heat transfer in the vicinity of a flat plate or a cylinder but in all the three-dimensional cases, only axisymmetric formulation of the problem has been considered. Fundamental studies are including by: Stokes [1851], Hiemenz [1911], Karman [1921], Griffith and Meredith [1936], Homann [1936], Wang [1974], Howarth [1951], Stuart [1956], Glauert [1956], Stuart [1959], Kelly [1965], Gorla [1976], Wang [1973], Cunning et al. [1998], Jung et al. [1992], Wang [1989], Weidman et al. [1997], Saleh and Rahimi [2004], Rahimi and Saleh [2007], Rahimi and Saleh [2008].

In this study the nonaxisymmetric three-dimensional steady viscous stagnation-point flow and heat transfer in the vicinity of a flat plate are investigated in the presence of suction and blowing. The external fluid, along z-direction, with strain rate a impinges on this flat plate and produces a two-dimensional flow with different components of velocity on the plate. A similarity solution of the Navier-Stokes equations and energy equation is derived in this problem. A reduction of these equations is obtained by use of these appropriate similarity transformations. The obtained coupled ordinary differential equations are solved using numerical techniques. Velocity profiles and surface stress-tensors along with temperature profiles are presented for different values of impinging fluid strain rate, different forms of jet arrangements, Prandtl number, and sample values of suction and blowing parameter.

2- Problem Formulation

Flow is considered in Cartesian coordinates (x, y, z) with corresponding velocity components (u, v, w). We consider the laminar steady incompressible flow and heat transfer of a viscous fluid in the neighborhood of stagnation-point on a flat plate located in the plane $z = 0$. An external fluid, along z-direction, with strain rate a impinges on this flat plate and produces a two-dimensional flow with different components of velocity on the plate. The governing equations are the steady Navier-Stokes and energy equations in Cartesian coordinates.

3- Self-Similar Solution

3-1 Fluid Flow Solution

An inviscid solution of the governing equations, valid far above the plane, is given by:

$$U = a\lambda x \ , \quad 0 < \lambda \le 1 \tag{1}$$

$$V = ay \tag{2}$$

$$w = -a(\lambda+1)z - W_0 \tag{3}$$

$$P_\infty = P_0 - \frac{1}{2}\rho a^2 \left[\lambda^2 x^2 + y^2 + (\lambda+1)^2 z^2 + W_0^2 - 2aW_0(\lambda+1)z\right] \tag{4}$$

where ρ is density, p_0 is stagnation pressure and λ is the coefficient which indicates the difference between the velocity components in x and y directions. The velocity components in these directions are the same if $\lambda = 1$, indicating that the each two adjacent single jets are far enough from each other and therefore there is no interactions between them. W_o is suction or blowing rate in z direction.

A reduction of the Navier-Stokes equations is sought by the following coordinate separation in which the solution of the viscous problem inside the boundary layer is obtained by composing the inviscid and viscous parts of the velocity components as the following:

$$u = a\lambda x f'(\eta) \ , \quad 0 < \lambda \le 1 \ , \quad v = ay[f'(\eta) + g'(\eta)]$$
$$w = -\sqrt{a\nu}\,[g(\eta) + (\lambda+1) f(\eta)] - W_0 \ , \qquad \eta = \sqrt{a/\nu}\, z$$

in which the terms involving $f(\eta)$ and $g(\eta)$ comprise the Cartesian similarity form for steady stagnation-point flow and prime denotes differentiation with respect to η. Note, boundary layer is defined here as the edge of the points where their velocity is 99% of their corresponding potential velocity. These transformations satisfy continuity automatically and their insertion into momentum equations yields a coupled system of ordinary differential equations in terms of $f(\eta)$ and $g(\eta)$ and an expression for the pressure:

$$f''' + [(\lambda+1) f + g - S] f'' + \lambda[1 - (f')^2] = 0 \tag{5}$$

$$g''' + [(\lambda+1) f + g - S] g'' - [g' + 2f'] g' - (1-\lambda)[(f')^2 - 1] = 0 \tag{6}$$

$$\begin{aligned} p(x,y,z) = \ & P_0 - \frac{\rho a^2}{2}\left[\lambda^2 x^2 + y^2\right] + \frac{1}{2}\rho a^3 S\left[S\nu - 2\sqrt{a\nu}(\lambda+1)z\right] \\ & - \rho a \nu\left[\frac{1}{2}[(\lambda+1)f + g]^2 + (f' + g') + \lambda f' - (\lambda+1)\right] + \rho a\nu[\eta(\lambda+1)(\gamma - aS)] \end{aligned} \tag{7}$$

in which $\gamma = \lim_{\eta\to\infty} g(\eta) = Const.$, and : $S = \dfrac{W_0}{\sqrt{a\upsilon}}$, S>0: Suction & S<0: Blowing.

The boundary conditions are:

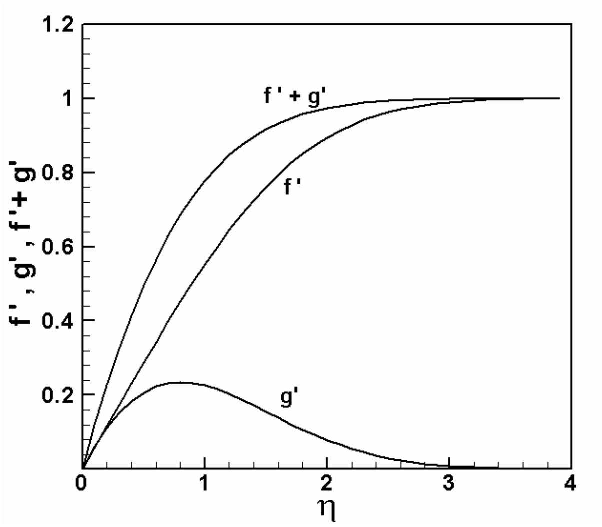

Fig. 3. Typical u and v velocity components for $\lambda = 0.01$.

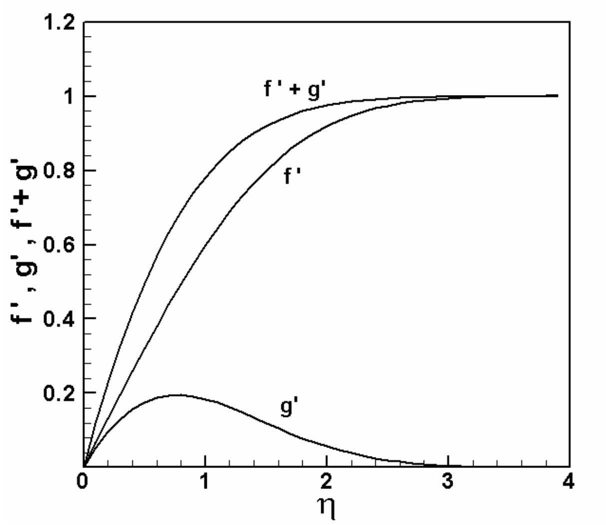

Fig. 4. typical u and v velocity components for $\lambda = 0.1$.

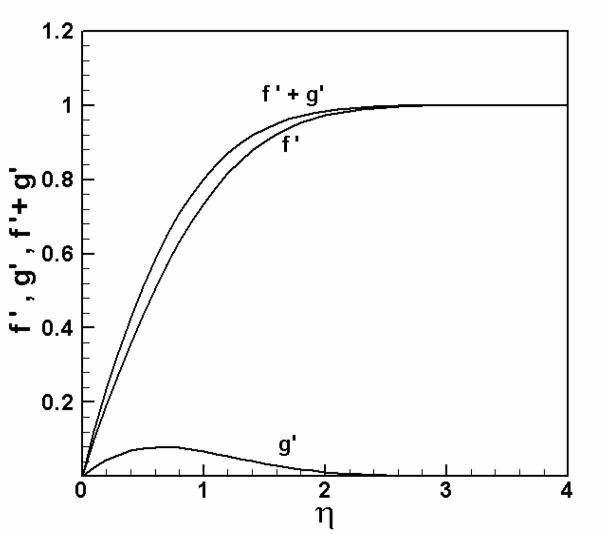

Fig. 5. Typical u and v velocity components for $\lambda = 0.50$.

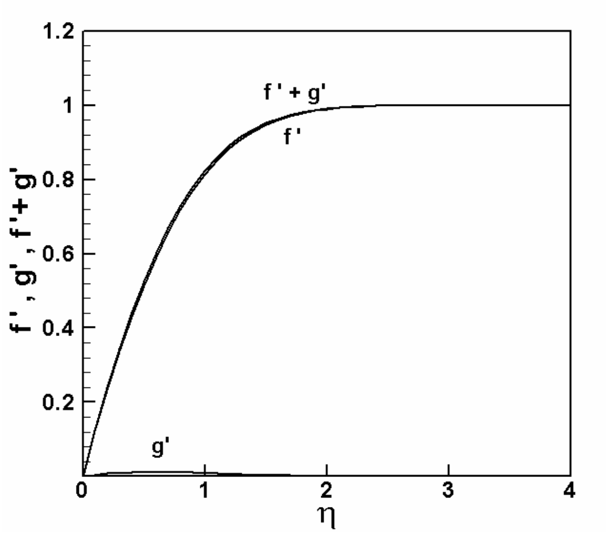

Fig. 6. Typical u and v velocity components for $\lambda = 0.95$.

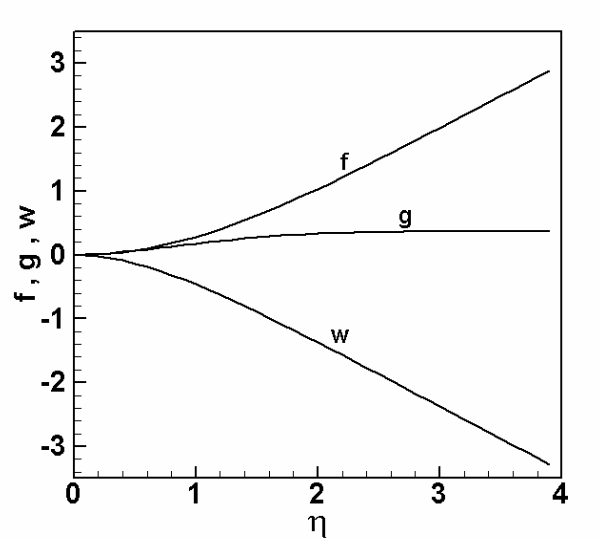

Fig. 7. Typical w- component of velocity for $\lambda = 0.01$.

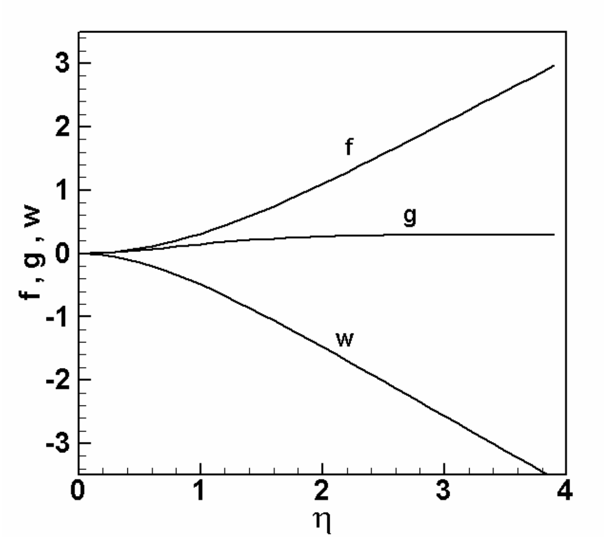

Fig. 8. Typical w-component of velocity for $\lambda = 0.10$.

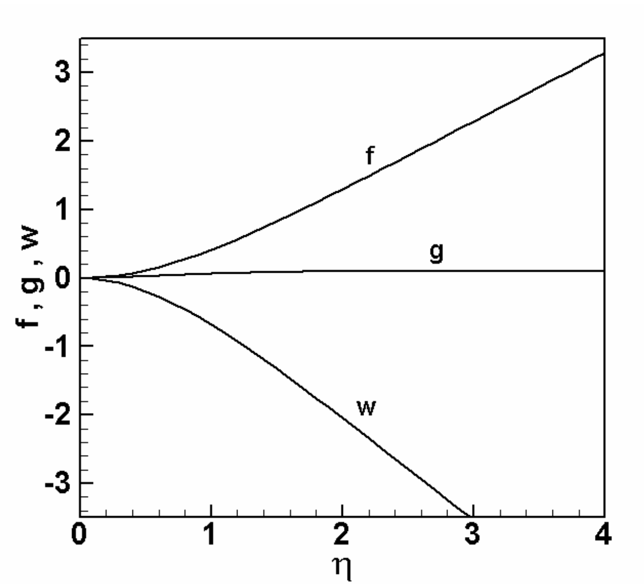

Fig. 9. Typical w-component of velocity for $\lambda = 0.50$.

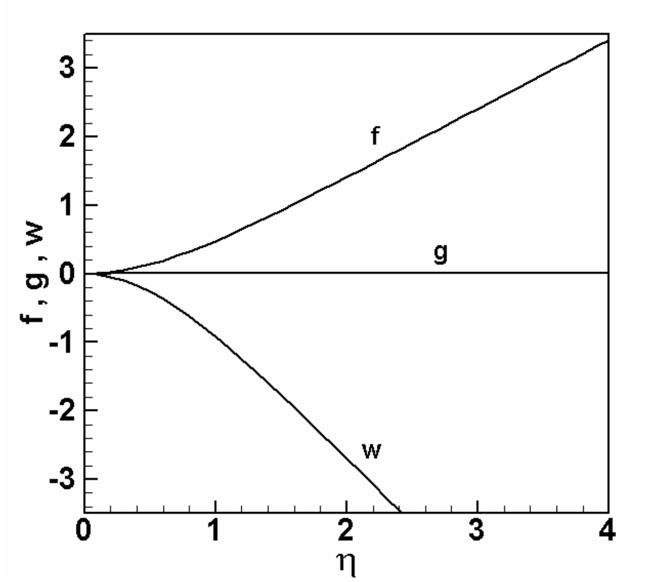

Fig. 10. Typical w-component of velocity for $\lambda = 0.95$.

$$\eta = 0: \quad f = 0, f' = 0, g = 0, g' = 0 \tag{8}$$

$$\eta \to \infty: f' = 1, \quad g' = 0. \tag{9}$$

Note that, when $\lambda = 1$ the case of axisymmetric three-dimensional results are obtained, Homman [5]. When $\lambda = 0$, the results are the same as two-dimensional problem.

3.2 Heat Transfer Solution:

To transform the energy equation into a non-dimensional form for the case of defined wall temperature, we introduce:

$$\theta = \frac{T(\eta) - T_\infty}{T_w - T_\infty} \tag{10}$$

Making use of similarity transformations, the energy equation may be written as:

$$\theta'' + \Pr .\theta'(g + (\lambda + 1)f - S) = 0 \tag{11}$$

with the boundary conditions as:

$$\eta = 0: \qquad \theta = 1 \;, \qquad \eta \to \infty: \qquad \theta = 0 \tag{12}$$

Where $\Pr = \nu / \alpha$, is Prandtl number and prime indicates differentiation with respect to η. The Equations (5), (6), and (11) are solved numerically using a shooting method trial and error and based on the Runge-Kutta algorithm and the results are presented for selected values of λ and $\Pr$ in following sections.

4- Shear-Stress

The shear-stress at the wall surface is calculated from:

$$\tau = \mu(\frac{\partial u}{\partial z}\vec{e}_x + \frac{\partial v}{\partial z}\vec{e}_y)_{z=0} \tag{13}$$

where μ is the fluid viscosity. Using the similarity transformations, the shear-stress at the flat plate surface becomes:

$$\tau = \rho \nu^{\frac{1}{2}} a^{\frac{3}{2}} (\lambda^2 x^2 f''^2 + y^2 (f'' + g'')^2)^{\frac{1}{2}} \tag{14}$$

This quantity is presented for different values of λ in later sections.

5- Presentation of Results

The solution of the self-similar Equations (5), (6), (7) and (11) along with the surface shear-stresses for different values of velocity ratios and Prandtl numbers are presented.

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