

# شبيه سازى عددى پديده انجماد آب در لوله‌ها

(staggered)

## A Numerical Simulation of Two-Phase Solidification Problem in a pipe

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### **ABSTRACT**

*In this research, a fixed-grid finite volume numerical approach is developed and used to simulate physical details of convection-dominated solidification problems for the pipe flow. This approach is based on the enthalpy-porosity method which is used to track the motion of the liquid-solid front and to obtain the freezing length and time of the solidification. The Navier-Stokes equations are solved on a staggered mesh by pressure-based implicit procedure. Results of the solidification are then validated against experimental data. Findings show a remarkable quality of the simulation of solidification problems.*

**Keywords:** Heat transfer, Solidification, Enthalpy-Porosity, Time of Solidification

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(Hwang)  
[ ] (Keary) ( )

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(Akyurt)

[ ]  
(Conda) (Mushy Zone)  
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(Variable grid) (Fixed grid)  
(Apparent capacity)  
Source ) (Effective capacity)  
Stream function ) (based  
(Primitive variable) (vorticity  
- [ ] (Enthalpy)

[ ] (Eyres)

[ ](Oleinik)

[ ](Atthey)

[ ] (Voller)

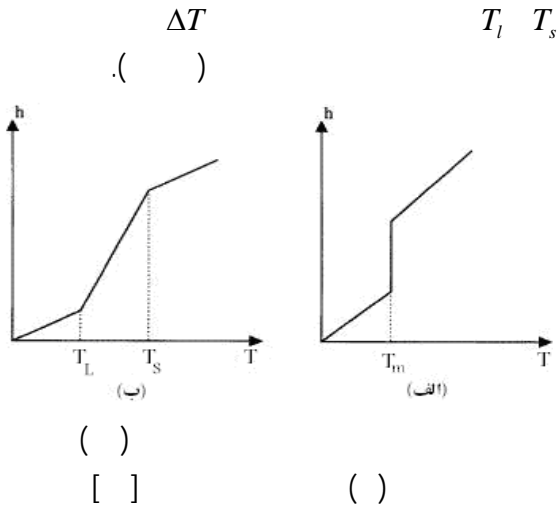
(Cao)

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$$f = \begin{cases} 1 & T > T_l \\ (T - T_s) / \Delta T & T_s \leq T \leq T_l \\ 0 & T < T_s \end{cases} \quad (1)$$

$T_s \quad T_l \quad \Delta T = T_l - T_s$



$$\frac{\partial}{\partial t} (\rho c T + \rho f L) + \nabla \cdot (\rho c T U) = \nabla \cdot (k \nabla T) + \rho g U \quad (2)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0 \quad (3)$$

$$\frac{\partial (\rho U)}{\partial t} + \nabla \cdot (\rho U U) = -\nabla p + \nabla \cdot (\mu \nabla U) + \rho g \quad (4)$$

$$\frac{\partial (\rho h)}{\partial t} + \nabla \cdot (\rho U h) = \nabla \cdot (k \nabla T) + \rho g U \quad (5)$$

$$h = h_{ref} + \int_{T_{ref}}^T c dT \quad (6)$$

$$h = fL + cT \quad (7)$$

(Mushy Zone)

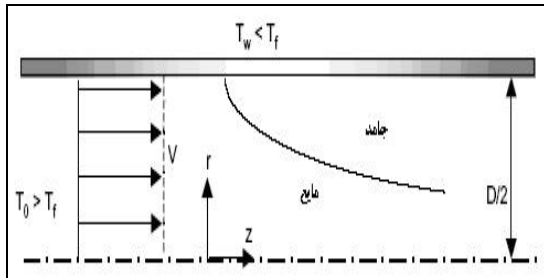
ε |

( )  
 $T_w$   
 $T_0$

(SIMPLE)

(Second Order Upwind)

(Three Diagonal Matrix) TDM



$$\rho_l = 999.840281 + 0.067326T - 0.008944T^2 + 0.000087T^3 - 0.000000666T^4 \quad ( )$$

$$c_l = 8.95866 - 0.040534 T + 0.0001123 T^2 - 0.0000001013 T^3 \quad [kJ / kg \cdot K] \quad ( )$$

$$k_l = 0.812 \times \exp(-0.0005 T) - 0.247 \times \exp(-0.0106 T) \quad [W / m \cdot K] \quad ( )$$

$$\mu = 0.00179 \times \exp\left[6.18 \times 10^7 \left(\frac{1}{T^3} - \frac{1}{(273.15)^3}\right)\right] \quad [N \cdot s / m^2] \quad ( )$$

( )

$$\rho_s = 949.948 \times \exp(-0.000125 T) - 1.86 \times 10^{-13} \times \exp(0.109 T) \quad ( )$$

$$c_s = 7.07 \times T^{1.016} - 0.122 \quad [kJ / kg \cdot K] \quad ( )$$

$$k_s = 6.99949 \cdot 948 \times \exp(-0.00408 \times T) \quad [W / m \cdot K] \quad ( )$$

(Mushy Zone)

$$\lambda = \lambda_s + f(\lambda_l - \lambda_s) \quad ( )$$

λ

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o |

$$: (t^*)$$

$$t^* = \frac{4(T_s - T_w)k_s}{\rho L D^2} t = f(Z^*, W_l, Re, W_s)$$

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$$W_l \quad Z^* = \frac{Z}{D}$$

$$(L) \quad (\Gamma)$$

$$(\Gamma)$$

$$(\gamma)$$

$$\gamma_{water} = 0.696$$

( )

$$W_l = \frac{\Gamma}{L} = \frac{L + \gamma c_l (T_0 - T_f)}{L} \quad ( )$$

$$W_s$$

$$W_s = L / c_s (T_f - T_w) \quad ( )$$

( )

$$Re = \frac{\rho V D}{\mu} \quad ( )$$

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$$t^* = C Z^{*a} W_l^b Re^n W_s^m \quad ( )$$

( )

$$t^* = 0.094595 Z^{*1.371606} W_l^{-3.254362} \quad ( )$$

$$Re^{0.354471} W_s^{-0.783680}$$

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*Re = 2000*

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(a)

(b)

/

(e) (d) (c)

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(f)

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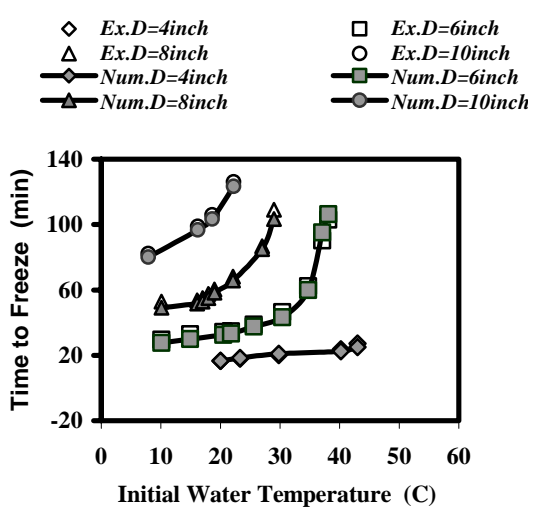
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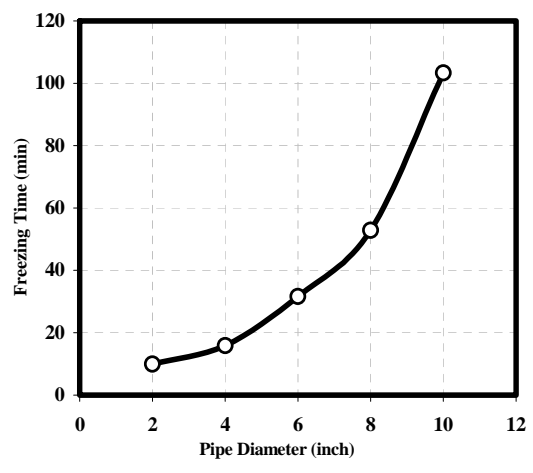
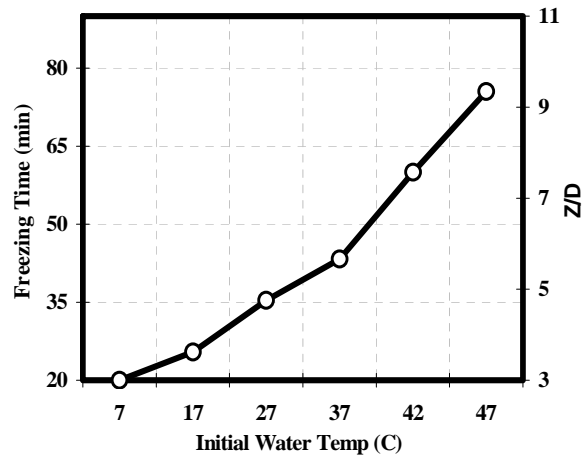
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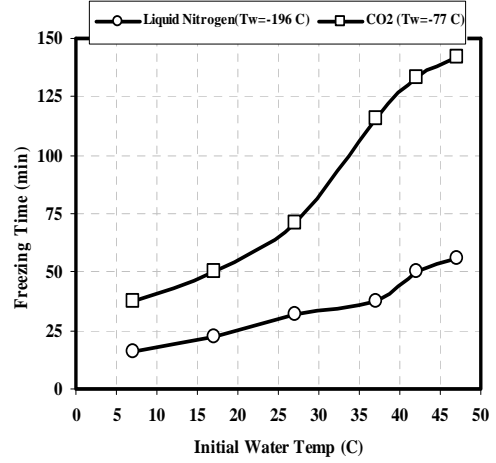
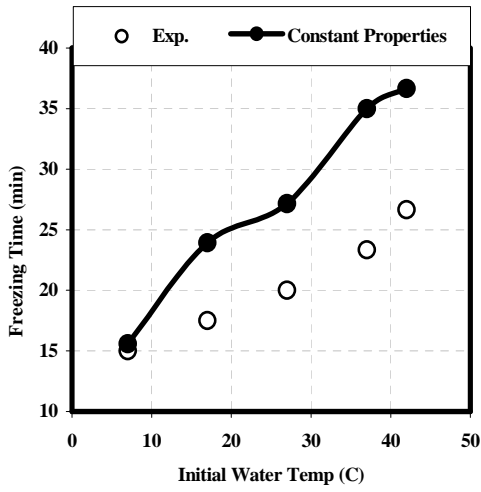
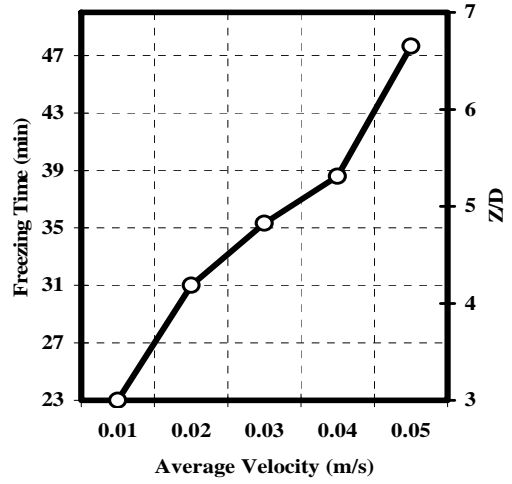
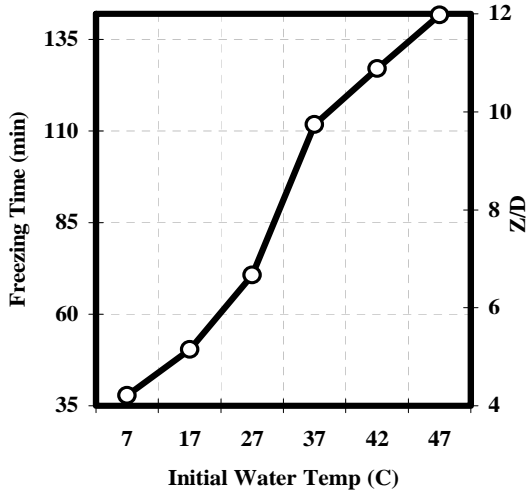
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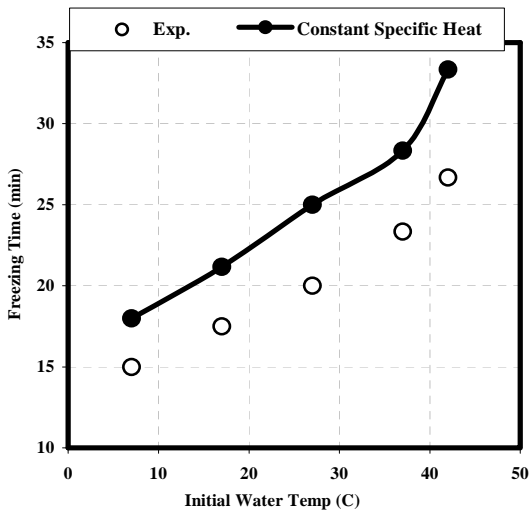
		$\rho(kg / m^3)$
		$C_p(j / kg.K^o)$
/	/	$k(W / m.K^o)$
		$L(kj / kg)$
		$T_m(^oK)$
/		$\mu(kg / m.s)$



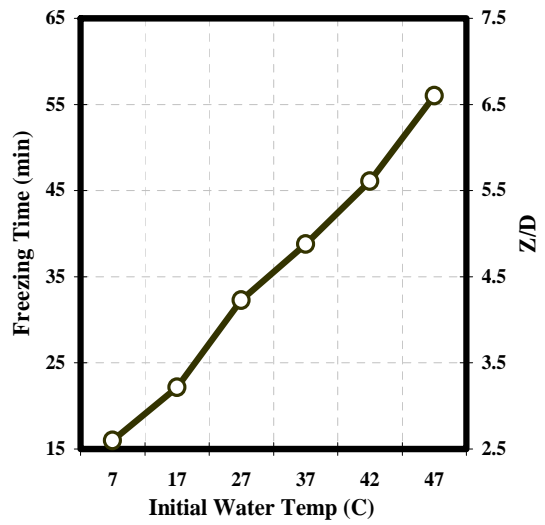


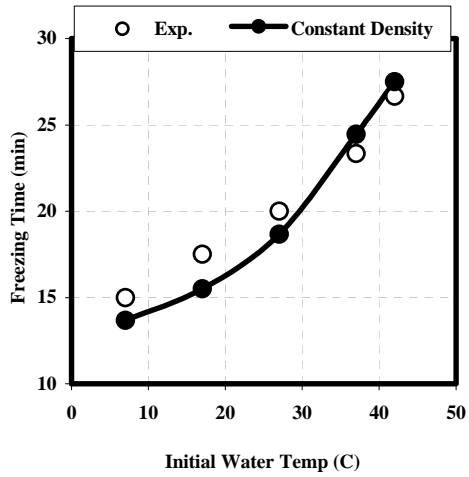


(a)

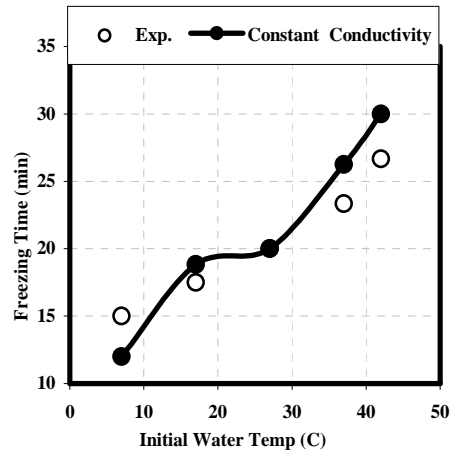


(b)

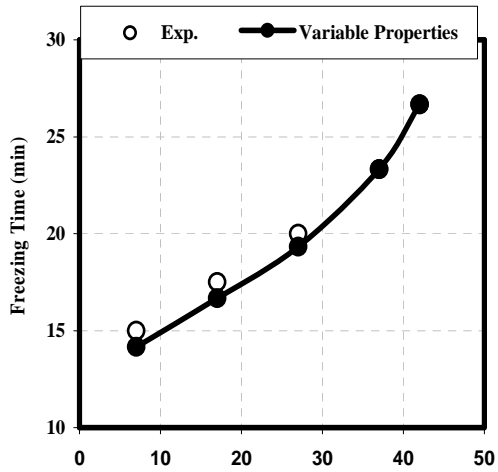




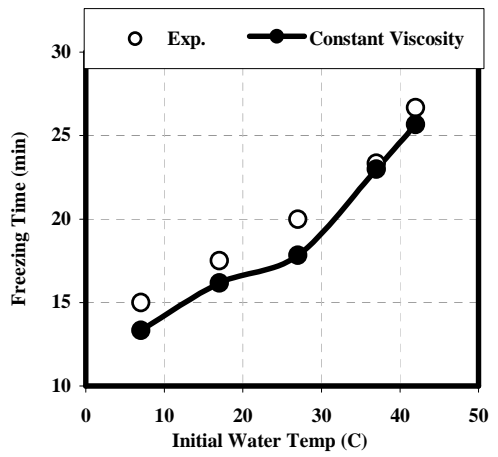
(e)



(c)



(f)



(d)