Applications of Workspace Categorization for Parallel Manipulators In Identification of Desired Direct Kinematics Solution

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Abstract:
Direct kinematics of parallel manipulators usually is a complicated problem which in general does not have a close form solution. It is well known that parallel manipulators admit generally several direct kinematic solutions for a given set of input joint values. Direct kinematic computation is an essential part in control and simulation. Bezout's elimination method is a way to obtain all solutions. Using this method direct kinematics solutions will be obtained as roots of a polynomial. However, obtaining the one desired solution has been a challenging problem. It is shown that workspace of a parallel manipulator could be categorized to special regions named basic regions. Each region contains one of the polynomial's solutions. Using workspace categorization, a method is proposed to determine the desired solution from the polynomial set solutions. The study is illustrated all along the paper with a 3-RRR planar parallel manipulator. Finally some future works are suggested where workspace categorization may be combined with available methods in solving direct kinematics.

Keywords: Direct kinematics, 3-RRR parallel manipulator, Solution categorizing

Introduction

Parallel manipulators are defined as ''a closed-loop kinematic chain mechanisms which end-effector is linked to the base by several independent kinematic chains '' [1]. Due to their high stiffness, high speed and large load carrying capacity, parallel mechanisms have become very popular in the past decade. Direct kinematics of parallel manipulator has been studied by few researchers, [1-3]. In many field of robotics such as control and simulation direct kinematics is an essential part. It is shown that usually there exist multiple solutions for the direct kinematics problem [2]. In past decades, different approaches such as numerical methods [4], Bezout's eliminating method [5] and artificial neural networks modeling (ANN) [6] have been implemented in order to find the direct kinematics solutions. Each of these methods presents its own challenges. Numerical methods are used to find one of the solutions which may not be the desired solution. Bezout's elimination method is traditionally used to obtain all solutions. However, obtaining the one desired solution has been a challenging problem. Some researchers have introduced methods to select the desired solution [4].

Most of these methods need information on prior position in order to find the desired solution. An earlier algorithm assumed that given initial assembly mode for the robot, only the direct kinematics solution, among all possible solutions, that can be reaches from this initial assembly with a trajectory that is singularity-free may be valid solution for the current pose. But this condition is not sufficient, [4]. It was shown by Innocenti that in a planar robot, two different direct kinematics solutions may be connected through a singularity-free trajectory [7]. Therefore, designing an algorithm for a complete direct kinematic verification is difficult and proving that it will be lead to a unique solution is still an open problem. The notion of *aspect* was introduced by Borrel for parallel manipulators which are singularity free domains in workspace [8]. As mentioned before, it is possible to link several solutions of the forward kinematics problem without meeting a singularity. This means that aspects are not unique domains and may contain several solutions to the forward kinematics problem. Wenger and Chablat defined *characteristic surfaces*, which divide the workspace of parallel manipulators into *basic regions* and yield unique domains. These regions are domains in which contain only one direct kinematics solution [9].

In this work, as a case study, direct kinematics of a 3-RRR planar parallel manipulator is studied. Singular points are specified and Bezout's elimination method is used to find all possible solutions of direct kinematics. Secondly, *aspects* and *basic regions* are illustrated for the manipulator. A method is purposed to find the proper solution among all real solutions. Note that if we distinguish the determined solution number for each region it will consequence the next position's region and it's specific solution number. Therefore, our goal is to identify the region of the next position. The algorithm measures the distance from current position to borders of the current region. If the distance is far enough then the current region most likely contains the next position. Otherwise, additional methods should be used in the selection process. Finally some applications of this method in direct kinematics modeling of parallel manipulators are suggested.

Manipulator's structure

A 3-RRR planar three-degree-of-freedom parallel manipulator is studied in this paper. Structure of this manipulator is shown in "Figure 1". Each leg of manipulator consists of one active and two passive revolute joints. The three motors M_1, M_2 and M_3 are fixed and placed on the vertices of an equilateral triangle. Triangle ABC is the moving platform of manipulator. This manipulator consists of a kinematics chain with three closed loops, namely M_1DABEM_2, M_2EBCFM_3, and M_3FCADM_1. It should be noted that only two of the aforementioned loops are kinematically independent [10].

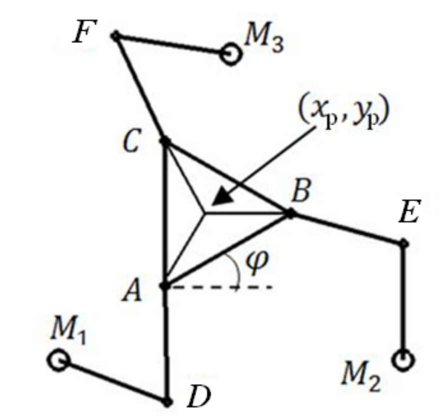

Figure 1: planar parallel 3-RRR manipulator [2]

Inverse Kinematics

The inverse kinematics consists of establishing values of the joint coordinates given the end-effector position and orientation. Obtaining inverse kinematics of manipulator is essential for trajectory generation and manipulator simulation. Inverse kinematics problem typically leads to a close form solution with multiple answers. The correct answer could be easily selected with respect to robot assembly mode. The assembly mode is defined when a robot is first assembled and will not change while robot traces a singularity free path. A complete inverse kinematics analysis for 3-RRR parallel manipulator is suggested [2].

Direct Kinematics

The degree of difficulty involved in finding a solution to the direct kinematics problem of parallel manipulators is higher than corresponding serial manipulators. In many field of robotics such as control and simulation, direct kinematics is an essential part. It is shown that there exist multiple solutions for the direct kinematics problem. Usually the closed form solution to the direct kinematics is impossible to obtain for most parallel manipulators including planar 3-RRR type. Therefore, the solution for the 3-RRR manipulator requires utilization of a numerical method which in general may not be the desired solution. The problem for 3-RRR manipulator leads to a maximum of 6 real solutions [2]. Referring to "Figure 1", if the three input angles are specified, the position of points *D, E* and *F* are readily computed. Moreover, the chain DABE could be considered as a four-bar linkage as shown in "Figure 2". Point C is a prominent point of the coupler link generating a coupler curve. A solution for the closure of the whole kinematics chain (manipulator) is obtained whenever the coupler curve described by the motion of point *C* intersects the circle defined by the rotation of link *F C* around point *F.* The forgoing principle is now used to derive the equations that will lead to the two following coupled trigonometric equations:

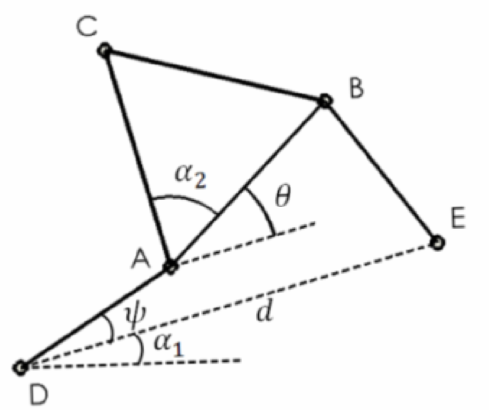

Figure 2: equivalent four bar mechanism [2]

$$x_C = x_D + l_2\cos(\alpha_1+\psi) + \sqrt{3}l_3\cos(\alpha_1+\alpha_2+\theta) \tag{1}$$

$$y_C = y_D + l_2\sin(\alpha_1+\psi) + \sqrt{3}l_3\sin(\alpha_1+\alpha_2+\theta) \tag{2}$$

Where:

$$\alpha_2 = \pi/3 \tag{3}$$

$$\alpha_1 = \tan^{-1}\left[\frac{y_E - y_D}{x_E - x_D}\right] \tag{4}$$

$$\theta_{1,2} = 2\tan^{-1}\left[\frac{b \pm \sqrt{b^2 - ac}}{a}\right] \tag{5}$$

With A, B and C that summarized in the following form:

$$a = m_1 - m_2 + (1+m_3)\cos\psi \tag{6}$$

$$b = \sin(\psi) \tag{7}$$

$$c = m_1 + m_2 + (m_3 - 1)\cos(\psi) \tag{8}$$

Where the capsulated forms are:

$$m_1 = \frac{-d^2 - 3l_3^2}{2\sqrt{3}l_2l_3},\ m_2 = \frac{d}{l_2},\ m_3 = \frac{d}{\sqrt{3}l_3} \tag{9-11}$$

$$d = \sqrt{(x_E - x_D)^2 + (y_E - y_D)^2} \tag{12}$$

The coupler curve intersects the circle defined by the rotation of link *F C* around point *F.* Therefore, following equation could be obtained:

$$(x_C - x_F)^2 + (y_C - y_F)^2 = l_2 \tag{13}$$

Now, the nonlinear relations that must be solved are equations (5) and (13).

By substitution variables θ & ψ in equations (5) and (13), a set of non-linear equations in terms of variables θ & ψ, will be obtained. These two equations could be solved numerically for angles θ and ψ. Derivative methods such as Secant method are usually suggested to solve these problems [11]. A disadvantage of these methods is that the computation is time consuming. Additionally, these methods provide only one of the solutions which itself depends on the initial guess. As pointed out earlier, the direct kinematics problem has multiple solutions. However, considering the requirement of the trajectory following, only one solution is possible which may not be the one provided by the derivative methods. Therefore another method should be utilized.

Singularity analysis

Singular configurations are particular poses of the end-effector, for which parallel robots lose their inherent infinite rigidity, and in which the end-effector will have uncontrollable degrees of freedom. Therefore, such poses should be generally avoided. In order to find singular poses first we should drive the relation between actuators angular velocity and moving platform velocities. In the 3-RRR parallel manipulator this relation, named velocity inversion, could be written as following:

$$Jt + K\dot{\theta} = 0 \tag{14}$$

Where t is Cartesian vector, $t = [\dot{x}, \dot{y}, \dot{\varphi}]^T$, and $\dot{\theta}$ is the vector of actuated joint rates, $\dot{\theta} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T$. J and K are 3 by 3 matrices represented in [2]. For this manipulator there are three types of singularities. First type of singularities occurs when determinant of J matrix vanishes. Second type of singularities occurs when determinant of K matrix vanishes. The third type which also is structural dependent occurs when determinant of both K and J matrices become zero. The contour diagrams of determinant values of both J and K matrices for our 3-RRR manipulator are shown in Fig. 3 and Fig. 4 respectively. As shown, there are poses where determinant of K and/or J become zero.

The notion of aspect was introduced by [8] to cope with the existence of multiple inverse kinematics solutions in serial manipulators. An equivalent definition was used in [12] for a special case of parallel manipulators, but no formal, more general definition has been set. The aspects are redefined formally by [9]. Aspect is defined as the maximal singularity-free connected regions *in the workspace*. The moving platform of 3-RRR parallel manipulator has three degrees of freedom resulting in a 3-dimensional x, y and φ workspace. In this space, *aspects* will also be 3-dimensional regions. Fig. 5 illustrates a cross section of the aspects of 3-RRR manipulator for $\varphi = 0^o$.

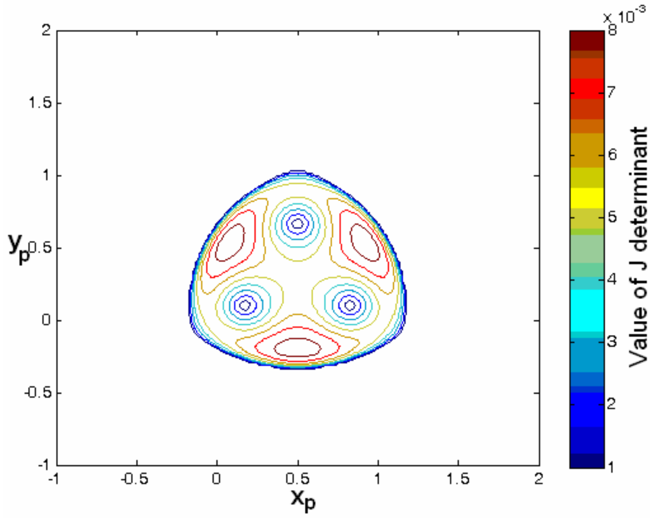

Figure 3: Contour of J determinant in manipulator workspace for $\varphi = 0^o$

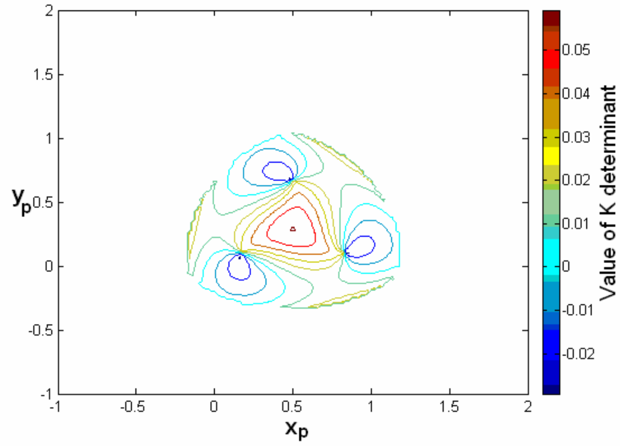

Figure 4: Contour of K determinant in manipulator workspace for $\varphi = 0^o$

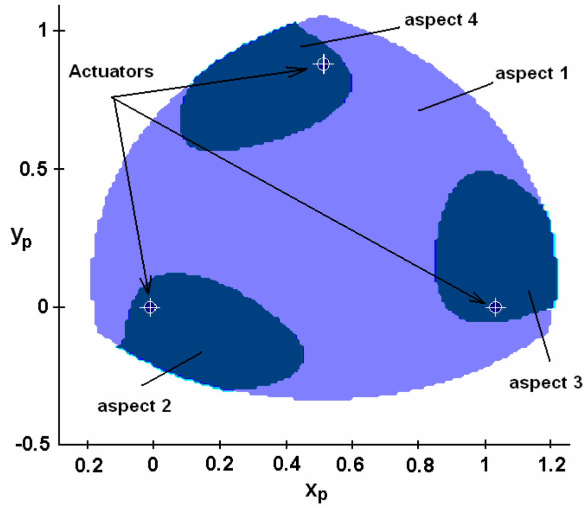

Figure 5: A cross section of aspect of 3-RRR robot for $\varphi = 0^o$

Using the Bezout's elimination method to obtain all solutions

The Bezout's elimination method is traditionally used for reducing a set of polynomials of multiple variables into a polynomial of only one variable [4]. For solving the nonlinear Eqs. (5) and (13) by Bezout's method, this trigonometric equations must be transformed into a set of polynomials. This transformation can be achieved by using the following trigonometric identities:

$$\tan(\frac{\theta}{2}) = z_1,\ \tan(\frac{\psi}{2}) = z_2 \qquad (15, 16)$$

$$\cos(\theta) = \frac{1-z_1^2}{1+z_1^2},\ \sin(\psi) = \frac{2z_2}{1+z_2^2} \qquad (17, 18)$$

$$\sin(\theta) = \frac{2z_1}{1+z_1^2},\ \cos(\psi) = \frac{1-z_2^2}{1+z_2^2} \qquad (19, 20)$$

Substituting the above expression into (5) and (13), and applying some simplifications, one can obtain the following polynomials:

$$\sum_{i=1}^{3}(\sum_{j=1}^{3}(a_{ij}z_1^{i-1}z_2^{j-1})) = 0 \qquad (21)$$

$$\sum_{i=1}^{3}(\sum_{j=1}^{3}(b_{ij}z_1^{i-1}z_2^{j-1})) = 0 \qquad (22)$$

Where a_{ij} and b_{ij} are the coefficient of polynomial equations (21) and (22) which are collected by the powers of z_1 and z_2.

With the Bezout's method, variable z_1 could be eliminated in the equations (21) and (22) and the resulting equation is given as follows:

$$\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix} = 0 \qquad (23)$$

Where F_{ij} will define in following equations:

$$F_{11} = \begin{vmatrix} a_{33}z_2^2 + a_{32}z_2 + a_{31} & a_{13}z_2^2 + a_{12}z_2 + a_{11} \\ b_{33}z_2^2 + b_{32}z_2 + b_{31} & b_{13}z_2^2 + b_{12}z_2 + b_{11} \end{vmatrix} \qquad (24)$$

$$F_{12} = \begin{vmatrix} b_{23}z_2^2 + b_{22}z_2 + a_{21} & a_{23}z_2^2 + a_{22}z_2 + a_{21} \\ b_{33}z_2^2 + b_{32}z_2 + b_{31} & a_{33}z_2^2 + a_{32}z_2 + a_{31} \end{vmatrix} \qquad (25)$$

$$F_{21} = \begin{vmatrix} a_{23}z_2^2 + a_{22}z_2 + a_{21} & a_{13}z_2^2 + a_{12}z_2 + a_{11} \\ b_{23}z_2^2 + b_{22}z_2 + a_{21} & b_{13}z_2^2 + b_{12}z_2 + b_{11} \end{vmatrix} \qquad (26)$$

$$F_{22} = \begin{vmatrix} a_{33}z_2^2 + a_{32}z_2 + a_{31} & a_{13}z_2^2 + a_{12}z_2 + a_{11} \\ b_{33}z_2^2 + b_{32}z_2 + b_{31} & b_{13}z_2^2 + b_{12}z_2 + b_{11} \end{vmatrix} \qquad (27)$$

Where $|*|$ denotes the determinant of a matrix. After expanding and simplification, Eq. (17) becomes:

$$L_9z_2^8 + L_8z_2^7 + L_7z_2^6 + L_6z_2^5 + L_5z_2^4 + L_4z_2^3 + L_3z_2^2 + L_2z_2^1 + L_1 = 0 \qquad (28)$$

Where L_i are the functions of a_{ij} and b_{ij} only. Detailed expressions of L_i could be developed by expanding Eq. (23). The solutions of z_2 could be found numerically by solving equation (28). Substituting each solutions of z_2 in equation (21) or (22) achieve two solutions for z_1. Therefore, finally 16 solutions will be obtained. As it was stated earlier, a maximum of 6 solutions for direct kinematics exists. Therefore other solutions are imaginary and are not acceptable. θ and ψ can now be calculated.

$$\theta = 2a\tan(z_1)\ \psi = 2a\tan(z_2) \qquad (29, 30)$$

Then platform position and orientation could be obtained as following:

$$x_p = x_D + l_2\cos(\alpha_1 + \psi) + l_3\cos(\alpha_1 + \frac{\alpha_2}{2} + \theta) \qquad (31)$$

$$y_p = y_D + l_2\sin(\alpha_1 + \psi) + l_3\sin(\alpha_1 + \frac{\alpha_2}{2} + \theta) \qquad (32)$$

$$\varphi = \psi + \theta \qquad (33)$$

An example is performed for the manipulator with properties shown in Table 1. Direct kinematics results are shown in Table 2.

Table 1. Manipulator properties

	First leg	Second leg	Third leg
l1 (m)	0.5	0.5	0.2
l2 (m)	0.5	0.5	0.2
l3 (m)	0.5	0.5	0.2
Actuator's position (m)	$\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$	$\begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$	$\begin{Bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{Bmatrix}$
Actuator's angle (degrees)	-89.825	-6.778	36.986

Table 2. The positions obtained by using Bezout's elimination method

Solution Number	x_p (m)	y_p (m)	φ_p (degrees)
1	-	-	-
2	-	-	-
3	-	-	-
4	-	-	-
5	-	-	-
6	-	-	-
7	0.8000	0.7000	30.00
8	-	-	-
9	-	-	-

10	-	-	-
11	-	-	-
12	-	-	-
13	1.0739	-0.0665	8.0157
14	-	-	-
15	-	-	-
16	0.9278	0.3581	-69.6659

Categorizing the workspace

In this section a method is applied to categorize workspace of the manipulator into regions according to multiple solutions of direct kinematics problem. The following algorithm is used for this purpose:

1. First we select a desired assembly mode.

2. The three Actuator's value, θ_1, θ_2 and θ_3 are each varied from 0 to 2π with small steps. All combinations of these values are used as input to direct kinematics problem. For each set of input values, Bezout's elimination method is used to find all 16 solutions.

3. The sixteen solutions obtained for each set is numbered from one to sixteen (1^{st}, 2^{nd}, 3^{rd}, …, 16^{th}). Imaginary answers are ignored. Real solutions are identified and their original numbering scheme is maintained.

4. Inverse kinematic algorithm will be utilized to verify if each real solution (direct kinematics) satisfies the assembly mode. Non acceptable solutions will be ignored. Acceptable (direct kinematics) solutions will maintain their original number defined in step 3.

5. Workspace is mapped by identifying all acceptable direct kinematics solutions x, y and φ.

6. The workspace is separated into regions which are formed by associating each direct kinematics solution (x, y and φ) with its original solution number

When solving direct kinematics we arrive at multiple solutions. If through some algorithm we could identify that the answer (desired x, y *and* φ of the moving platform) is in a specific region then from Fig. 3 we can conclude that the desired answer is the solution number related to this region.

It is difficult to graphically show all regions, therefore, for simplification cross sectional views representing constant values of φ are shown, Fig. 7 and Fig. 8 for $\varphi = 0^o$ and $\varphi = 30^o$ respectively. In these figures regions related to each solution number is filled with a specific color. This categorization will help us to select the desired solution in direct kinematics analysis. In Fig. 8 the singular points and workspace categorization are depicted together. This figure shows that in singular points always there is a change in workspace direct kinematics solutions regions (*basic regions*). But regions also may change where there is not a singularity.

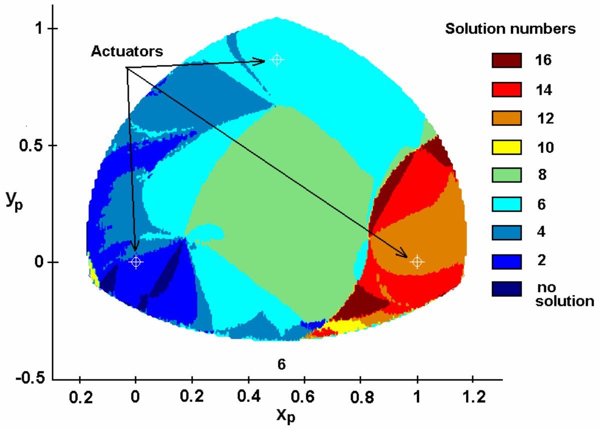

Figure 6: Cross section of categorized workspace for $\varphi = 0^o$

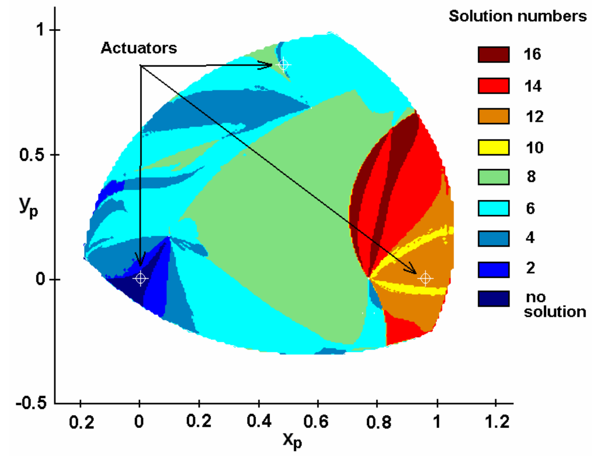

Figure 7: Cross section of categorized workspace for $\varphi = 30^o$

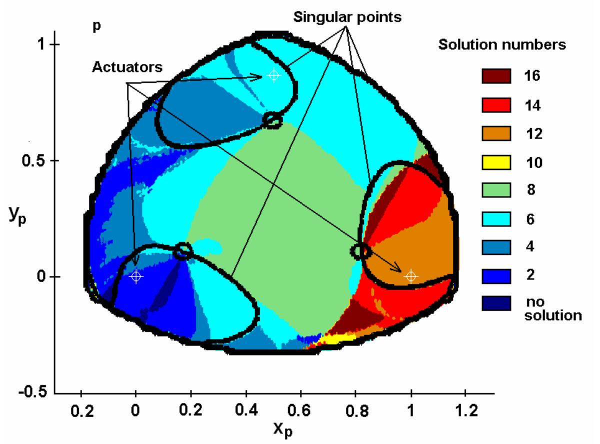

Figure 8: Cross section of categorized workspace for $\varphi = 0^o$ and singular points together

Future works

In past decades, different approaches are implemented to find direct kinematics solutions such as numerical methods [4] and artificial neural networks modeling (ANN) [6]. None of these methods presents a reliable method to find correct and desired direct kinematics solution. Bezout's elimination method is traditionally used to obtain all solutions [5]. However, obtaining the one desired solution has been a challenging problem. The algorithm presented in this paper may be implemented in combination with other methods to overcome their shortages. Some cases are appending below:

1. Numerical approaches, which consequently obtain one solution according to initial guess:
These approaches use an initial guess, usually prior position, and a gradient base method to become closer to one of solutions which usually is the nearest solution to the initial guess. Our proposed categorization method could be combined with these approaches to verify if the found solution is the desired solution or not.

2. Methods which find all possible solutions such as Bezout's elimination method:
It is preferred to use Bezout's elimination method to find all possible solutions when the computational time is not important for direct kinematics analysis, for example in robot simulation for off line controller tuning. However, as mentioned before, selecting the desired solution is a challenging problem. Our proposed method introduced in this paper will aid to select the one desired solution easily.

3. Using neural network to model direct kinematics of manipulator:
Artificial neural networks could be used to estimate a function for a set of input data and target data. Therefore it could be useful in direct kinematics solution. In many literature ANNs are used to model direct kinematics of parallel manipulators. In these methods they obtain sets of actuator positions related to platform position by inverse kinematics analysis and then utilizing actuator positions as inputs and platform position as targets for training the ANN. A parallel robot has many assembly modes. Each assembly mode will have its own inverse kinematics solution. Therefore these methods use separate ANNs for each assembly mode. For direct kinematics problem, given a specific assembly mode, we may have multiple solutions representing different platform positions. However, ANNs provides only one output, platform position, for each input, actuator's position. Therefore, methods which used ANN may not be reliable. We propose combining ANN with our workspace classification. As was shown earlier, distinguishing the region of moving platform's current position, will proof the existence of only one solution for the direct kinematics. Therefore, we suggested, using a separate ANN for each region. For example, assuming 8 possible assembly modes and 16 possible direct kinematics solutions, we will need up to 128 (8×16) ANNs to overcome this problem.

Conclusion

Direct kinematics of parallel manipulators is usually a complicated problem which generally does not have a close form solution. Direct kinematics problem usually leads to multiple solutions. A new approach is presented to select the desired solution to direct kinematics in parallel manipulators while robot's tool follows a trajectory. The approach used the concept of *basic regions* for defining the domains containing unique solution for direct kinematics. Bezout's elimination method is used to obtain all possible solutions. It is noted that if the region of manipulator is known we can easily select the desired solution among them. Finally considering the suggested future works, workspace categorization (basic regions concept) may be combined with available methods for solving direct kinematics to obtaining precise and reliable results.

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