Close form Solutions for Inverse and Direct Position Analysis of a Special 3-PSP Parallel Manipulator

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Abstract

This paper presents the direct and inverse kinematics solutions for a 3-DOF special 3-PSP parallel manipulator. The 3-PSP mechanism consists of two rigid bodies, a movable platform (formed like a star) and a fixed base that are connected to each other by means of three PSP legs. For the direct kinematics, unlike traditional methods which use constraint equations and numerical methods, a novel approach is used to formulate the direct kinematics problem. The approach uses relatively simpler geometric relations and results in a closed form solution with unique answer. This enables implementation of control methods requiring fast computation times. Manipulator's structural properties also lead us to formulate nine coupled trigonometric constraint equations that are utilized in inverse kinematics analysis. Additionally, two relevant inverse kinematics formulations are investigated. The first formulation uses xyz coordinate of tool that leads to an exact solution with multiple answers. The second formulation uses orientation of the platform as well as its z coordinate which also leads to an exact solution with unique answer.

Keywords: Inverse kinematics, Direct kinematics, 3-PSP Parallel Manipulator, Closed form solution

Introduction

In recent years, parallel robots have become an active research area due to their merits in terms of high accuracy, high stiffness and high load carrying capacity over their serial counterparts. Their principal drawbacks are limited workspace and complex forward position kinematics problem. A parallel manipulator typically consists of a moving platform that is connected to a fixed base by several limbs or legs in parallel. A definite advantage of parallel manipulators is the fact that, in most cases, actuators can be placed on or near the truss, thus imposing a limited weight on the moving parts. This makes possible for parallel manipulators to achieve high speed. An exhaustive enumeration of parallel robot's mechanical architectures and their versatile applications are described in [1].

Parallel mechanical architectures were first introduced in tire testing by Gough and Whitehall [2] and later used by Stewart [3] as motion simulators with Six degrees of freedom. Parallel manipulators have many advantages, as mentioned above. However, 6-DOF mechanisms are not always required for many applications in practice. Recently, parallel manipulators with less than 6-DOF have attracted attention of various researchers. The use of lower degrees of freedom is always recommended when the application makes it possible [4-11]. 3-DOF mechanisms constitute an important subset of 6-DOF mechanisms, since their translational and/or orientational motion can be augmented with additional mechanisms to provide the required DOF. Many 3-DOF parallel manipulators have been designed and investigated for relevant applications. Gregorio analyzed the kinematics of a 3-URC parallel manipulator [4]. Tsai and Joshi worked on Kinematics and optimization of a spatial 3-UPU parallel manipulator [5].

This paper studies inverse and direct position analysis of a special 3-PSP parallel mechanism. In general, 3-PSP parallel manipulator is not considered sufficiently in literatures. Gregorio and Parenti-Castelli studied a general form of 3-PSP parallel manipulator and derived the direct and inverse kinematics problem in analytical form [4]. However, because of the general form of the structure, they were not able to obtain close form solution and thus had to use numerical approach to solve the problem.

By simplifying manipulator's structure, through introducing star platform, eliminating link length and link offset between passive spherical and passive prismatic joints we arrived at a mechanism design that enabled us to solve both inverse and direct position problem in a closed form. However eliminating the linkage between two passive joints in each leg may cause the mechanism more simple also will make it special considering the PSP series. The specification of the designed mechanism is that the three passive prismatic joints at the end of each leg will always meet each other in the plane of moving platform.

Depending on the problem formulation, we obtained either unique or multiple answers. Each leg of the proposed mechanism consists of a prismatic actuated joint and a spherical joint which is paired with a passive prismatic joint. The solution for both direct and inverse kinematics applies to general class of the special 3-PSP. This means that the fixed base of the manipulator can form any triangular shape.

The problem of determining actuated joint coordinates for a given pose of the end-effector is called the inverse kinematics. The solution to the inverse kinematics problem for parallel robot is usually simple to obtain [8]. The other important and challenging problem is to determine the pose of the parallel robot end-effector given its actuated joint coordinates. This is called direct kinematics problem. This relation has a clear practical interest for the control of the pose of the manipulator. It is shown that, in general, the solution for this problem is neither simple nor unique [1-11].

This mechanism offers three degrees-of-freedom (DOF) and can be used as a base in machine tool applications requiring two angular and one linear DOF.

The significant property of this parallel manipulator is that manipulator's moving platform has two angular and one linear DOF. This specific property results in two suggested kinds of problems for solving the inverse kinematics. The first formulation of inverse kinematics, uses the xyz coordinate of the moving star, manipulator's tip, and finds the actuated prismatic joints position. This results in a closed form solution with multiple answers. The second formulation uses the desired orientation of the moving star and its z-height coordinate to arrive at the actuated prismatic joints position. This also results in a closed form solution with a unique answer.

As mentioned before, one of the more complicated problems of parallel manipulators is finding the exact solution to the direct kinematics. Most parallel manipulators have non-exact multiple solutions and require the use of numerical methods. This will cause errors in dynamics, control and simulation. However we simplified 3-PSP mechanism and were able to obtain closed form equations with unique solution.

Manipulator's structure

The manipulator consists of three legs. Each leg is made of one actuated prismatic joint and a passive spherical joint which itself is paired with a passive prismatic joint. It includes three closed kinematics loops where two of them are independent and the other is dependent. The kinematic chain of the general mechanism is shown in “Figure1 ”.

In this paper a special kind of 3-PSP parallel manipulator is studied. An industrial model of this manipulator type is shown in “Figure 2”. To calculate the degrees of freedom of the system we must find the number of the one-DOF joints and the number of movable rigid bodies [11]. Each spherical joint is equivalent to three 1-DOF revolute joints with two zero weighted movable rigid bodies with negligible sizes [11]. Each prismatic joint is considered as one-DOF joint. Therefore, the structure has fifteen one-degree-of-freedom joints (m) and thirteen movable rigid bodies (r). For the 3-PSP mechanisms we can write,

$$n = 6\times r - 5\times m$$
$$= 6\times 13 - 5\times 15 = 3 \quad (1)$$

$$l = m - r$$
$$= 15 - 13 = 2 \quad (2)$$

This results in three degrees-of-freedom (n) and two independent kinematic loops (l).

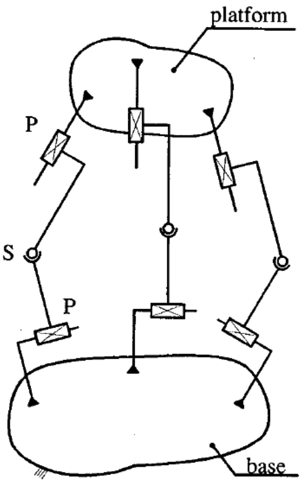

Figure 1: General 3-PSP mechanism [10]

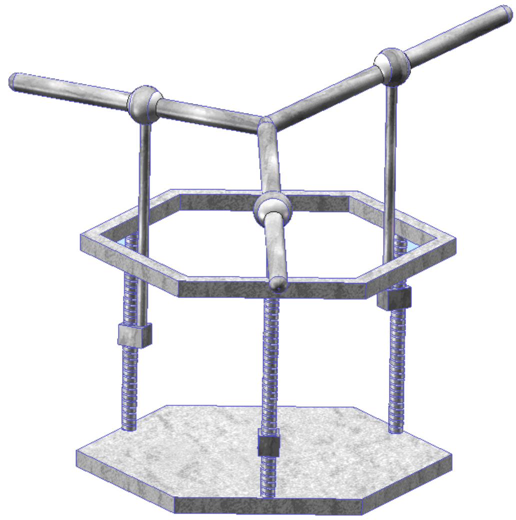
Figure 2: The suggested model of the 3-PSP

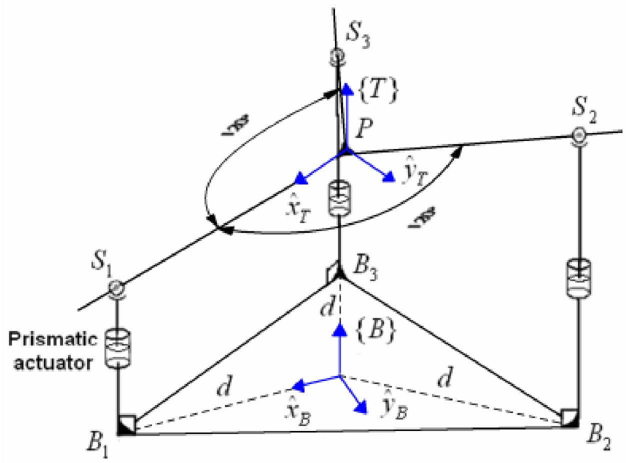

Figure 3: structure of the special 3-PSP parallel manipulator

As shown in “Figure 3”, there is one actuated prismatic joint for each link and one passive spherical joint which is paired with a passive prismatic joint. The movable platform is illustrated as a welded three bar in the form of a planar star. Legs of the star platform make 120 degrees with each other. By extending or descending each actuated prismatic joint we could define position and orientation of a frame, {T}, attached to the center of the star platform with respect to fixed frame, {B}, attached to fixed base. The base platform is chosen to be an equilateral triangle. This creates a rather symmetrical shape for the 3-PSP robot. This symmetrical structure helps arrive at constraint equations that are more algebraically similar. Therefore, it will be easier to solve them manually. However, it is completely acceptable if the structure has no symmetry. Our solution method stated in this paper, can still be applied.

Because the prismatic actuators are welded to the fixed base, the manipulator could not have any rotation about z-axis of the base frame. The three degrees-of-freedom of the manipulator are manifested by changes in z-height of point P and changes in orientation of the moving star about x and y axes of frame {B}.

Inverse Kinematics

Obtaining the position of the prismatic actuators given the pose of the end-effector (moving star) is our goal. As mentioned earlier, we can formulate the inverse kinematics in two different ways,

- What are the actuated prismatic joints values given the x, y, z coordinate of frame {T}, placed on the center of the moving platform, with respect to frame {B}?
- What are the actuated prismatic joints values given the z coordinate as well as orientation of frame {T} about x and y axis of frame {B}?

Formulating the problem in these two ways is meaningful since the workspace of the robot has a spatial shape. Therefore, depending on the application, one may reach any x,y,z position within this space or alternatively one may reach any z position with some specific orientation.

1. First formulation

In this circumstance, we have an exact position for the center of the moving star, P, defined in {B}. Utilizing the manipulator's structural properties we could obtain main constraint equations.

As shown in "Figure 4", Frame {B} is located at the center of the fixed equilateral triangular base. Frame {T} is located at the center of the moving star. The unit vectors u_i start at the base of frame {B} and point towards the corners of the fixed base. The unit vectors v_i start at the base frame {T} and are along the legs of the moving star to indicate the moving platforms orientation. Considering the geometry of the structure, unit vectors v_i and u_i are each 120 degrees apart from each other. For simplicity and without loss of generality, the x axes of {B} and {T} frames are each along u_1 and v_1 unit vectors, respectively. The transformation matrix ${}^B_T T$ will define the position and orientation of frame {T} with respect to {B} frame. This transformation may be defined using a translation and three Euler angle rotations. Utilizing the represented parameters in "Figure 3" and "Figure 4" the main constraint equations are derived as follows;

$$\vec{s}_i = d\vec{u}_i \qquad i = \{1,2,3\} \tag{3}$$

Where:

$$\vec{u}_1 = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix},\ \vec{u}_2 = \begin{Bmatrix} -d/2 \\ \sqrt{3}d/2 \\ 0 \end{Bmatrix},\ \vec{u}_3 = \begin{Bmatrix} -d/2 \\ -\sqrt{3}d/2 \\ 0 \end{Bmatrix} \tag{4}$$

Where d is the distance between the base of each actuated prismatic joint and the base frame.

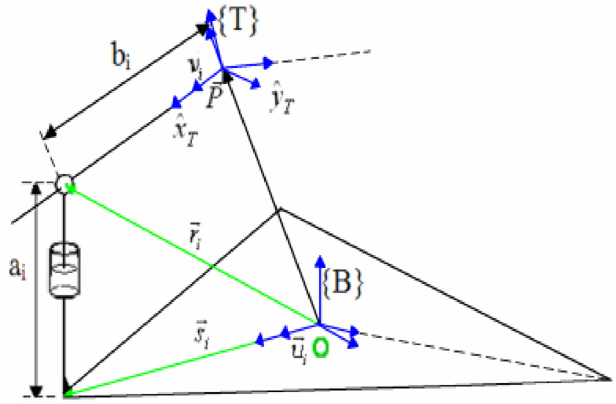

Figure 4: Disassembled leg of 3-PSP structure

$$\vec{v}_i = R_{3\times3} \times \vec{u}_i \tag{5}$$

$${}^B_T T = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}_{4\times4} \tag{6}$$

$$R = R(z,\lambda) \times R(y,\varphi) \times R(x,\theta) \tag{7}$$

Where T is the transformation matrix in which R is the 3×3 rotation matrix consisting of three Euler angles λ, φ and θ about the unit vectors of the frame {B}. This relates orientation of the moving star with respect to the base frame and t is the translation vector between frames {B} and {T}.

$$t = \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8}$$

$${}^B_T R = R(z,\lambda) \times R(y,\varphi) \times R(x,\theta) \tag{9}$$

Where:

$$R(z,\lambda) = \begin{bmatrix} \cos\lambda & -\sin\lambda & 0 \\ \sin\lambda & \cos\lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

$$R(y,\varphi)=\begin{bmatrix}\cos\varphi & 0 & \sin\varphi\\ 0 & 1 & 0\\ -\sin\varphi & 0 & \cos\varphi\end{bmatrix} \tag{11}$$

$$R(x,\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix} \tag{12}$$

Considering "Figure 4", the nine constraint equations can now be written as,

$$\vec{P}+b_i\vec{v}_i=\vec{r}_i=\vec{s}_i+\begin{Bmatrix}0\\0\\a_i\end{Bmatrix}\quad i=\{1,2,3\} \tag{13}$$

$$\vec{P}+b_iR\vec{u}_i-\vec{s}_i-\begin{Bmatrix}0\\0\\a_i\end{Bmatrix}=0\quad i=\{1,2,3\} \tag{14}$$

By expanding derived equations in (14), the main constraint equations can be derived as,

$$\begin{cases}x+b_1\cos\lambda\cos\varphi-d=0\\ y+b_1\sin\lambda\cos\varphi=0\\ z-b_1\sin\varphi-a_1=0\\ x+b_2\left\{-1/2\cos\lambda\cos\varphi+\sqrt{3}/2(-\sin\lambda\cos\theta+\cos\lambda\sin\varphi\sin\theta)\right\}+d/2=0\\ y+b_2\left\{-1/2\sin\lambda\cos\varphi+\sqrt{3}/2(\cos\lambda\cos\theta+\sin\lambda\sin\varphi\sin\theta)\right\}-\sqrt{3}d/2=0\\ z+b_2(1/2\sin\varphi+\sqrt{3}/2\cos\varphi\sin\theta)-a_2=0\\ x+b_3\left\{-1/2\cos\lambda\cos\varphi-\sqrt{3}/2(-\sin\lambda\cos\theta+\cos\lambda\sin\varphi\sin\theta)\right\}+d/2=0\\ y+b_3\left\{-1/2\sin\lambda\cos\varphi-\sqrt{3}/2(\cos\lambda\cos\theta+\sin\lambda\sin\varphi\sin\theta)\right\}+\sqrt{3}d/2=0\\ z+b_3(1/2\sin\varphi-\sqrt{3}/2\cos\varphi\sin\theta)-a_3=0\end{cases} \tag{15}$$

For the first problem of inverse kinematics when we have the coordinate of the moving star as the input data, the 9 nonlinear equations consist of 9 unknowns, a_1, a_2, a_3, b_1, b_2, b_3, θ, φ and λ. The unknown λ can easily be calculated from the first two set of equations,

$$x+b_1\cos\lambda\cos\varphi-d=0\quad\Rightarrow\cos\lambda=\frac{d-x}{b_1\cos\varphi}$$

$$y+b_1\sin\lambda\cos\varphi=0\quad\Rightarrow\sin\lambda=\frac{-y}{b_1\cos\varphi} \tag{16}$$

$$\Rightarrow\lambda=a\tan 2\frac{-y}{d-x}$$

Therefore, there remain 8 equations and 8 unknowns. We used Maple software to solve this set of equations. Because of the length of the solution we decided not to present them. Instead, in this paper, we present a numerical example.

2. Second formulation

In this formulation we have the manipulator's tip z-position and two orientations (θ,φ) as input data. Placing these known variables into (14) will result in 9 nonlinear constraint equations with a_1, a_2, a_3, b_1, b_2, b_3, x, y, λ as unknowns. Utilizing same procedure as before we could compute λ in the terms of x and y from (16). By eliminating one of the first two equations used in obtaining λ and placing λ in the terms of x and y into the remaining eight equations we will achieve a system of eight linear equations and eight unknowns. Hence, by linear algebra, we can easily conclude that we will have one exact real solution for this formulation of inverse kinematics. We used Matlab software to solve this set of linear equations. As before, because of the length of the solution we decided not to present them. Instead in this paper, we present a numerical example.

Direct Kinematics

Determining the pose of the end-effector of a parallel robot for the given actuated joint coordinates is called direct position analysis. In direct position analysis, values of actuators displacement a_1, a_2, a_3 are given and we wish to find the orientation of moving platform and its centeral position P.

As shown in "Figure 5", frame {W} is attached to corner of the moving platform, S_1, its x-y axes in plane $S_1S_2S_3$ (plane-II) and its x-axis directed along line S_1S_2. We will also define station frame {S} with its origin at point B_1, its x-y axes in base plane and its x-axis directed along B_1B_2 .

We will show that given the actuator's displacements (a_1, a_2, a_3) we can express rotation of platform's frame {W} with respect to base frame {B} by two relative Euler angles about a-a and b-b axes. This transformation can be obtained by one translation and two rotations about a-a and b-b respectively.

The translation vector is as following,

$$^{S}_{W}t=\begin{Bmatrix}0\\0\\a_1\end{Bmatrix} \tag{17}$$

The rotation matrix $^{S}_{W}R$ defining frame {S} with respect to {W} is defined as follows,

$$^{S}_{W}R=R(y,\beta)\times R(x,\gamma) \tag{18}$$

$$^{S}_{W}R=\begin{bmatrix}\cos\beta & 0 & \sin\beta\\ 0 & 1 & 0\\ -\sin\beta & 0 & \cos\beta\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\gamma & -\sin\gamma\\ 0 & \sin\gamma & \cos\gamma\end{bmatrix} \tag{19}$$

In (19), β and γ represent the relative Euler angles which are shown in "Figure 5". The relation between {S} frame and {B} frame is defined using the transformation matrix,

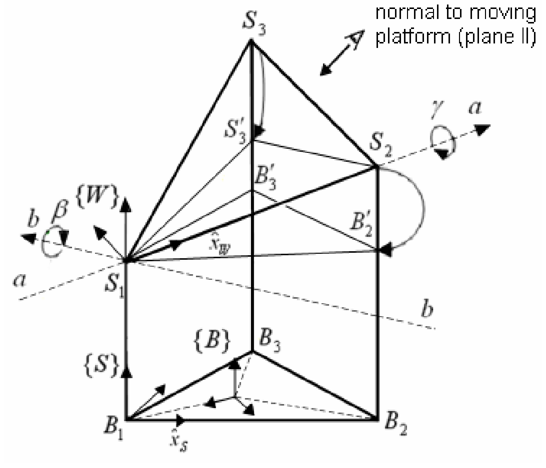

Figure 5: Base, Station and platform frame

$$ {}^B_S T = \begin{bmatrix} {}^B_S R & {}^B_S t \\ 0 & 1 \end{bmatrix}_{4\times4} \tag{20}$$

Where,

$$ {}^B_S R = \begin{bmatrix} \cos(-7\pi/6) & \cos(-7\pi/6) & 0 \\ \cos(-7\pi/6) & \cos(-7\pi/6) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{21}$$

And,

$$ {}^B_S t = \begin{Bmatrix} d \\ 0 \\ 0 \end{Bmatrix} \tag{22}$$

The transformation matrix between the platform frame, {W}, and the base frame, {B}, can now be defined as,

$$ {}^B_W T = {}^B_S T \times {}^S_W T \tag{23}$$

This transformation will later be used to illustrate the direct kinematics solution in the base frame.

Next, we will prove that there is a unique answer for the direct kinematics problem which satisfies the geometrical bounds. Considering the actuator's displacements, a_1, a_2 and a_3 we can obtain S_1, S_2 and S_3 coordinates. These points make up a plane called plane II. Star platform is considered to be made of 3 welded bars in one plane. Therefore, this star should also be placed in plane II. Because the three legs of the star make a 120° with each other, the angles $S_1\hat{P}S_2$, $S_2\hat{P}S_3$ and $S_1\hat{P}S_3$ must also be equal to 120°. Point P could be defined by the intersection of two special circles in plane-II. The first circle crosses points S_1 and S_2 while the second circle crosses points S_1 and S_3. The center of these two circles, C_1 and C_2 are located so that the angles $S_1C_1S_2$ and $S_1C_2S_3$ become 120°. See "Figure 6". The coordinates of point P can now be defined in frame {W}. These coordinates are shown by primed letters. Therefore, we will have the following equations for the intersection point,

$$\begin{cases} (x'-x'_1)^2 + (y'-y'_1)^2 = r_1^2 \\ (x'-x'_2)^2 + (y'-y'_2)^2 = r_2^2 \end{cases} \tag{24}$$

There are two real solutions for this set of equations. One answer is always at $(0,0)$ and the second answer, which is, acceptable defines point P in frame {W}.

Next, we need to define point P in base frame {B}. The position of point P according to frame {B} is,

$$ {}^B P = {}^B_S T \times {}^S_W T \times {}^W P \tag{25}$$

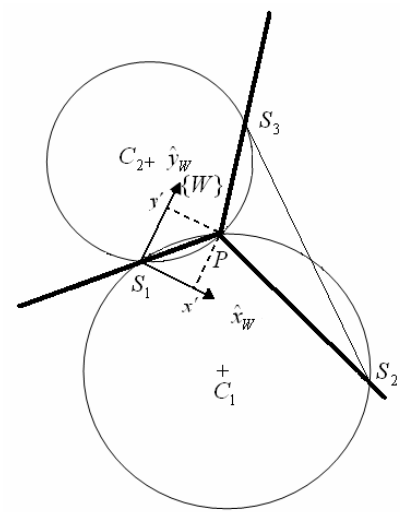

Figure 6: proving the uniqueness of the Direct position analysis

While we have:

$$ {}^B P = \begin{Bmatrix} x \\ y \\ z \\ 1 \end{Bmatrix}, \quad {}^W P = \begin{Bmatrix} x' \\ y' \\ 0 \\ 1 \end{Bmatrix} \tag{26}$$

Numerical examples

Here, we present three examples to illustrate the correctness of the solutions.

1. First formulation of inverse position analysis:

The sample input parameter values are:

$d = 1(m), \quad x = -0.3168, y = -0.4174, z = 4(m)$

The first formulation led up to 8 possible solutions. However, the given input variables will have four real solutions and are shown in "Table 1". These solutions are shown in "Figure 7" using Matlab program. It should be noted that the 1st and 3rd solutions as well as the 2nd and 4th solutions have equal prismatic joint positions (equal a_1, a_2 and a_3). As proved earlier, the answer to direct kinematics is unique. Therefore, solution 1 and 3 as well as 2 and 4 are the same.

Table 1: Parameters of the first relevant problem of inverse

	Parameter (unit)	Parameter value			
		1st solution	2nd solution	3rd solution	4th solution
Input	x (m)	-0.316	-0.316	-0.316	-0.316
	y (m)	-0.417	-0.417	-0.417	-0.417
	z (m)	4	4	4	4
	d (m)	1	1	1	1
Output	a1 (m)	1.606	6.393	1.606	6.393
	a2 (m)	5.107	2.892	5.107	2.892
	a3 (m)	4.107	3.892	4.107	3.892
	b1 (m)	2.763	2.763	2.763	2.763
	B2 (m)	1.705	1.705	1.705	1.705
	B3 (m)	0.496	0.496	0.496	0.496
	θ (deg)	-150.004	150.004	29.995	-29.995
	φ (deg)	119.992	-119.992	60.007	-60.007
	λ (deg)	-162.412	-162.412	17.587	17.587

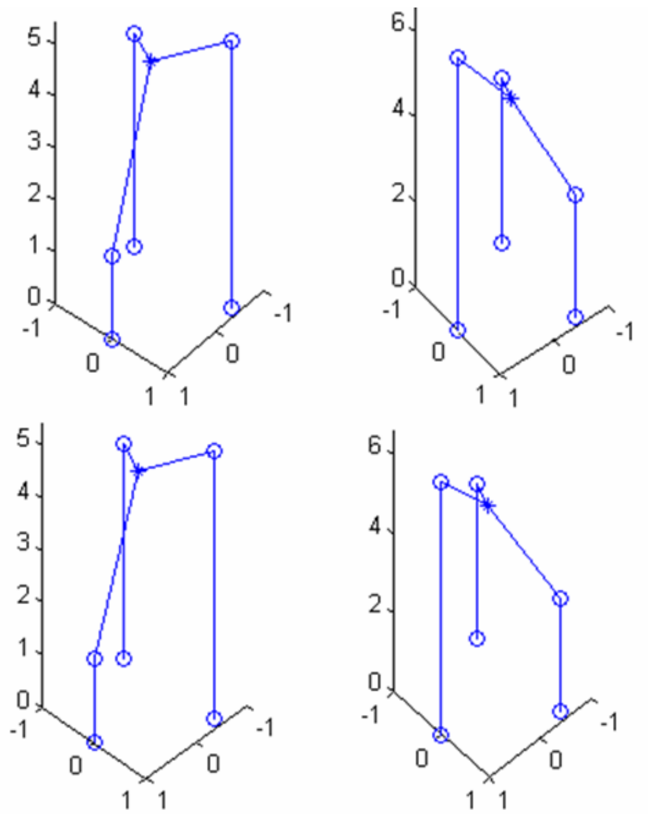

Figure 7: Solutions to the first formulation of inverse Position

2. Second formulation of inverse position analysis:

The input values for this example are chosen based on the first example (third answer),

$$d = 1(m), \quad \theta = 30^\circ, \varphi = 60^\circ, \quad z = 4(m)$$

As stated before, the solution to this formulation of inverse kinematics is unique and is shown in “Table 2”. This solution is depicted in “Figure 8” using Matlab program.

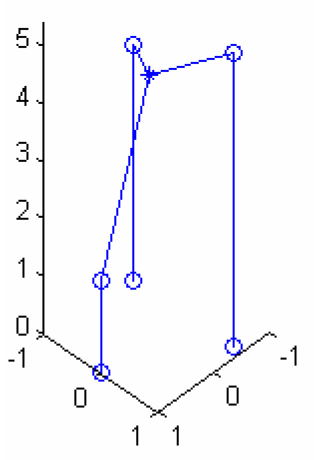

Figure 8: Solutions to the second formulation of inverse Position

Table 2: Parameters of the second relevant problem of inverse position solution

	Parameter(unit)	Parameter’s value
Input data	θ (deg)	30
	φ (deg)	60
	z(m)	4
	d(m)	1
Output data	x(m)	-0.316
	y(m)	-0.417
	a1(m)	1.607
	a2(m)	5.107
	a3(m)	4.101
	b1(m)	2.762
	b2(m)	1.705
	b3(m)	0.496
	λ (deg)	17.587

It should be noted that the solution of example #2 matches with the third solution of solution #1. This confirms the correctness of the solution.

3. Direct position analysis

In order to double check results obtained in previous two examples, we will use the same outputs obtained in the inverse kinematic as inputs for the direct kinematics analysis,

a_1=1.6075(m), a_2=5.1075(m), a_3=4.1017(m)

As stated before, the solution to this formulation of direct kinematic is unique and is shown in “Table 3”. This solution is depicted using Matlab program and shown in “Figure 9”.

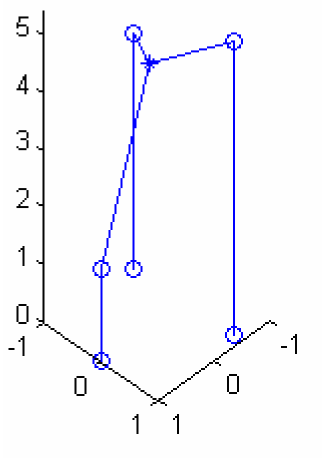

Figure 9: Direct Position solution

Table 3:Parameters of the direct Position solution

	Parameter(unit)	Parameter's value
Input data	a1(m)	1.607
	a2(m)	5.107
	a3(m)	4.101
	d(m)	1
Output data	x(m)	-0.316
	y(m)	-0.417
	z(m)	4
	b1(m)	2.762
	b2(m)	1.705
	b3(m)	0.496
	θ (deg)	30
	φ (deg)	60
	λ (deg)	17.587

As it is clear the results of direct position analysis match results obtained in the inverse kinematics analysis.

Conclusion

In this paper we presented the direct and inverse position solutions for a special 3-PSP parallel manipulator. Manipulator's structural properties also lead us to formulate nine coupled trigonometric constraint equations that are utilized in inverse kinematics analysis. Based on desired input types, two formulations for inverse kinematics were considered. The first formulation leads us to closed form solution with multiple answers. The second formulation also resulted in a closed form solution with a unique answer. A novel approach is used to formulate the direct kinematics problem. The approach uses relatively simpler geometric relations and results in a closed form solution with unique answer. Three examples were given. The answers of the three examples confirmed the correctness of the solution.

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