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USE OF ELASTIC FOLLOW-UP IN INTEGRITY ASSESSMENT OF STRUCTURES

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ABSTRACT

This paper reviews the concepts and definitions related to elastic follow-up, Z, together with its potential use in stress classification. Based on the principles governing benchmark multiple bar structures elastic follow-up (EFU) is quantified. Local nonlinearities arising within a structure influence elastic follow-up. These include variations in the geometry of structure, its material properties, effects of plasticity and creep, structural discontinuities and boundary conditions. Elastic follow-up is shown to be simple to evaluate, is physically meaningful (as it relates strain accumulation in the structure to its cause) and is useful in design practice. In this generalised definition Z=1 indicates no follow-up and represents a fully displacement controlled situation. In contrast Z=∞ represents the extreme case of fully load controlled situation. Presence of mixed boundary conditions is interpreted as 1<Z<∞.

A methodology that overcomes the singularity problem of cracked structure to determine Z is then proposed. The distinctive characteristic of the proposed approach is that it takes account of situations where the structure contains defects.

NOMENCLATURE

A; area
E; Young Modulus
F; load
K; stiffness (AE/L)
L; length
l_0; initial length
n; number
P; applied load
t; time
α; relative stiffness (stiffness ratio)
δ_0; misfit displacement
δ; displacement
Δ; applied displacement
ε; strain
$\dot{\varepsilon}$; strain rate
λ, λ_E, Z; elastic follow-up factor
σ; stress
c; creep
el; elastic
eq; equivalent
inc; increment
pl; plastic
n; exponent
yp; yield point
max; maximum

INTRODUCTION

The concept of elastic follow-up originated from a model for creep deformation of a piping system (Robinson[1]) where the possibility of "self springing" being accompanied by excessive creep in localised regions of high stress was noted. Robinson illustrated elastic follow-up through several examples of creep relaxation using a basic creep power law and provided a clear description of the concept by analysis of Coffin's spring loaded creep furnace (referenced from: " The Effect of temperature on Materials Required in Turbine Design" by S H Weaver, General Electric Review, 1930). The essence of elastic follow-up in the context of creep stress relaxation is the argument that "the elastic action of the least strained component of a structure prevents the stress in the most highly strained component from relaxing quickly enough".

The term elastic follow-up was probably first used by Goldhoff et al[2] who simulated flange-bolt interaction in a model bolt test incorporating elastic follow-up. They noticed that the residual stress variation in the model bolts tests were quite similar to those obtained in the conventional creep relaxation test.

In general a structure is subjected to combined (primary and secondary) loading and boundary conditions and exhibits nonlinear behaviour due to material nonlinearities such as plasticity and or creep as well as geometrical nonlinearities such as changes in geometry or presence of defects (cracks). In a practical structure the stress state therefore corresponds to situations that are neither fully load-controlled, nor fully displacement-controlled. This may be referred to as a "mixed boundary-value" problem. Laboratory scale test pieces however are often tested either under load or displacement control but rarely under mixed boundary conditions. The test piece may be viewed as an extracted region from the practical structure and if the extracted region does see a mixed boundary condition (such as combined residual and applied stresses) how is this mimicked in the laboratory condition? The purpose of this study is to explore and establish a methodology that helps to translate the findings from experiments carried out on test pieces to the practical structure.

The features highlight some important issues:

1. How to classify stresses (including residual stresses) into primary and secondary for integrity assessment procedures?

2. How to understand the response of a practical structure from the test results obtained from laboratory scale specimens?

Exploring and quantifying elastic follow up could provide the key information required to answer the above questions. The paper first reviews earlier work and then explores two benchmark models that quantify elastic follow-up. The outcomes are then discussed in relation to the behaviour of a cracked structure.

EARLIER ANALYSES

It is often stated in the literature that the classification of residual stresses for the purpose of integrity assessment depends on the "high" or "low" level of associated elastic follow-up. However there is very little that provides a practical procedure for categorisation of those components. Roche[3, 4, 5] suggested a procedure which would allow part of the secondary stress to be retained for the primary stresses.

A procedure for stress classification at structural discontinuities was suggested by Dhalla[6, 7] with the objective to provide practical design guidelines along with reducing conservatism in the ASME code. Dhalla used the concept of a reduced elastic (secant) modulus to quantify elastic follow-up and outlined a simplified stress classification procedure to help quantify the discontinuity stresses into primary and secondary components in the design of piping systems and pressure vessels.

A basic example problem often used for definition of elastic follow-up is a two-bar in series model subjected to fixed displacement experiencing creep stress relaxation as shown in Figure 1 (Boyle and Nakamura[8]).

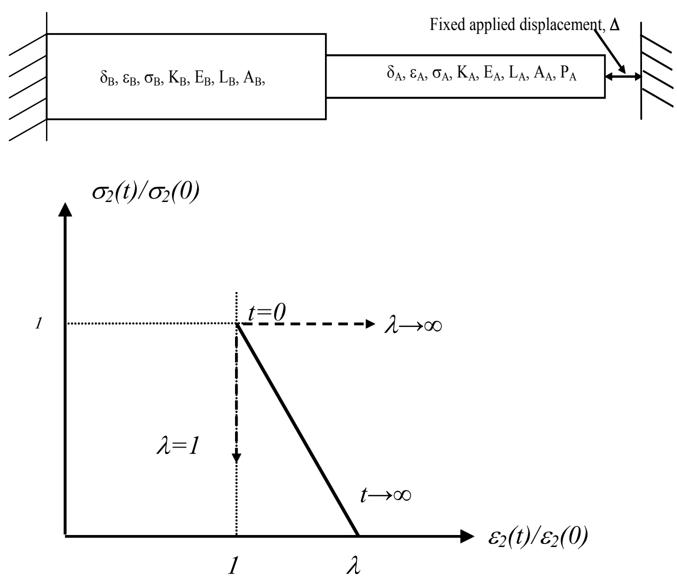

Figure 1: Basic two-bar model subjected to creep (Reproduced from Boyle and Nakamura[8])

Assuming a time-hardening rule for creep given by

$$\dot{\varepsilon}_c = g(t)\sigma^n$$

Boyle and Nakamura[8] show that

$$\frac{\varepsilon_2(t)}{\varepsilon_2(0)} = 1 + (\lambda - 1)\left(1 - \frac{\sigma_2(t)}{\sigma_2(0)}\right) \quad \text{where} \quad \lambda = \frac{L_2/L_1 + A_2/A_1}{L_2/L_1 + (A_2/A_1)^n}$$

where λ is the additional strain accumulated during elastic follow-up.

Boyle and Nakamura[8] also presented a description of simplified methods for inelastic flexibility analysis. These include the Severud / Dhalla method. Since elastic follow up is primarily an elastic effect, the resulting inelastic accumulated strain (in their case study a pipe bend) could be estimated by repeatedly lowering the stiffness and performing a sequence of elastic piping flexibility analyses. Assuming that bar 1 in Figure 1 is purely elastic and does not exhibit creep, the coefficient λ_E can be used as λ in the stress ratio and strain ratio equations to obtain an exact solution, independent from the creep constitutive relation and only depending on geometry. Thus λ_E can be obtained by performing a sequence of elastic calculations varying the stiffness of bar No. 2 as E/f with f>1. The strain ratio is now given by

$$\frac{\varepsilon_2(t)}{\varepsilon_2(0)} = 1 + (\lambda_E - 1)\left(1 - \frac{\sigma_2(t)}{\sigma_2(0)}\right) \quad \text{where} \quad \lambda_E = \frac{L_2/L_1 + A_2/A_1}{L_2/L_1}$$

Boyle and Spence[9] suggested a procedure for classification of initial elastic stress into primary and secondary that is in principle an alternative interpretation for the approach suggested by Roche[5] that leads to higher degree of conservatism although it is simpler to evaluate.

Whereas the majority of work on elastic follow-up deals with the high temperature plastic creep response of complex piping systems and it is less used in general structural problems. There is also limited evidence of studies aimed in simplifying the analysis of structural discontinuities using elastic follow-up. Gamboni et al[10] proposed a procedure for quantifying elastic follow up due to a generic structural discontinuity resulting in inelastic strain concentration in the presence of a deformation controlled loading. An essential requirement for elastic follow-up to occur is that the system has to be loaded in-elastically. They proposed two simplified methods, a “simplified inelastic method based upon creep response” and an “equivalent elastic modulus procedure” to estimate elastic follow-up.

In the simplified inelastic method based on creep response two different finite element models were employed to evaluate the un-relaxed secondary stresses in high temperature applications. It was suggested that the ratio of stress relaxations obtained from the two analyses may be used to give the primary stress portion of thermal expansion stress. The alternative simplified method, i.e. the equivalent elastic modulus procedure also suggested that estimation of EFU from simple elastic analysis often provided adequate results if all necessary approximations were correctly defined. A graphical presentation of this approach is shown in Figure 2.

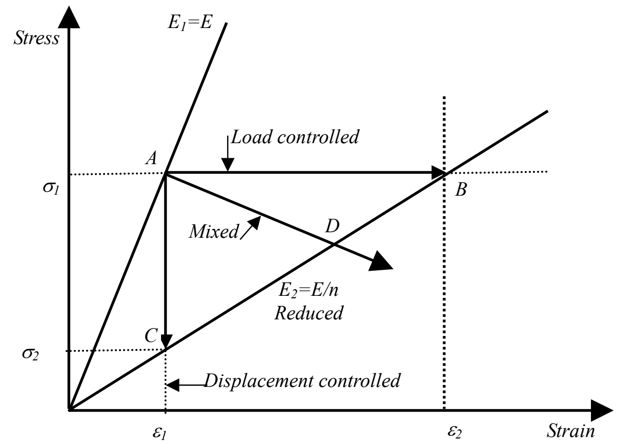

Figure 2. Graphical presentation of equivalent elastic modulus procedure (reproduced from Gamboni et al[10])

Treatment of stresses in integrity assessment depends on whether the stresses are classified as being of primary or secondary importance. The concept of elastic follow up is closely related to the proximity of the structure to a load controlled or displacement controlled situation (e.g. Green and Knowles[11]). For a load controlled situation elastic follow up is very high and the corresponding stresses have a primary effect on failure of structure, hence should be classified as primary. In contrast for a displacement controlled situation elastic follow up is insignificant and the corresponding stresses are classified as secondary.

Kasahara et al[12] proposed combined elastic follow-up models for specific structures using a power law that incorporates plasticity and creep effects as well as the structural discontinuities. In their models elastic follow-up parameters differ among structures and need to be estimated for each specific configuration. They evaluated the creep-fatigue response of a fast breeder reactor component using finite element analysis. Further work was carried out by Kasahara[13] to predict strain concentrations at structural discontinuities using a “compliance change” concept. Recent research related to nuclear codes and standards that provide a fracture toughness estimation method based on elastic follow-up was reported by Azada[14].

The difficulty of treating of elastic follow-up shared by most design codes has been noted in R5[15]. In R5 elastic follow-up is described as “a consequence of the change in kinematics between fully elastic and inelastic behaviour” it is due to disproportionately large strains that can be generated in highly stressed regions (high temperature regions in case of creep) during inelastic deformation. The surrounding lower stressed elastic regions then prevent the stress relaxation in the weak region with little increase in net strains. This means larger increase in strain compared with pure relaxation. Elastic follow-up represents this effect and depends on the local constraint of the structure.

No unified approach is currently available for incorporating the effects of elastic follow-up in integrity assessment procedures for cracked structures. The high temperature integrity assessment procedure R5 incorporates an elastic follow-up factor, Z in the assessment procedure. The definition of Z is consistent with the concept introduced for elastic follow-up associated with creep for un-cracked bodies[16]. The influence of EFU on creep fatigue crack growth is also considered in R5 and an alternative Z factor to that of un-cracked creep problem has been introduced[16]. The methods for determination of Z in R5 are based on judgment and experience. Extension of these procedures to cracked structures and cases where significant localised plasticity and crack growth introduce elastic follow-up remains to be addressed. Based on analyses for elastic follow-up R5 suggests that increasing the Z factor calculated for an un-cracked body by unity is adequate for the case of cracked structures and thereby allow for the additional follow-up due to presence of crack.

BENCHMARK MODELS

Boyle and Nakamura[4] used two bars in series subjected to a fixed displacement to introduce elastic follow up due to creep stress relaxation as shown in Figure 1. This model may be used to represent a structure in which a region (or component) exhibiting a softening behaviour is linked in a series with the rest of the structure. Similar to creep, the softening behaviour may also represent plasticity or presence of crack that is acting as the source of nonlinearity.

Alternatively the softening region of the structure is linked in parallel with the remaining structure. The general case however, may be considered as a combination of series and parallel components in which a region within the structure is experiencing one or more sources of nonlinear behaviour.

In the following both series and parallel multi-bar structures are used to explore EFU due to plasticity. It is assumed that the material exhibits elastic perfectly plastic behaviour in the soft region. Idealizing a structure as multiple bars allows simple estimates of EFU to be provided.

The elastic follow-up factor Z in the high temperature integrity assessment procedure R5 is defined such that it simultaneously accommodates the associated stress and strain variations. The R5[15] definition of the elastic follow-up factor, Z, for creep that incorporates strain accumulation and stress relaxation is:

$$\frac{d\bar{\varepsilon}_c}{dt} + \frac{Z}{E}\frac{d\bar{\sigma}}{dt} = 0 \quad \text{or} \quad Z = \frac{\Delta\bar{\varepsilon}_{inc} + \Delta\bar{\sigma}'/\bar{E}}{\Delta\bar{\sigma}'/\bar{E}} = \frac{\Delta\bar{\varepsilon}_c}{\Delta\bar{\varepsilon}_{el}} \quad (1)$$

It can also be shown that equation 1 can be re-written for a general case as:

$$Z = \frac{(\varepsilon)_{final} - \left(\varepsilon_{eq}^{el}\right)_{final}}{(\varepsilon)_{initial} - \left(\varepsilon_{eq}^{el}\right)_{final}} \quad \text{where} \quad \left(\varepsilon_{eq}^{el}\right)_{final} = (\sigma)_{final}/E \quad (2)$$

Equation 2 is adopted as a generalized description for quantifying elastic follow-up. Difficulties arise when discontinuities are present and for a structure containing a crack the above description results in values of elastic follow-up that depend on location in the structure. In R5 this is overcome by quantifying Z based on reference stresses and strains. Alternatively, in this paper a new method based on defining a "crack affected zone" is developed and this is described later.

a. Series bar structure

Consider a two bar structure subjected to a fixed end displacement as shown in Figure 1.

For the purely elastic response of the structure the strains in the bars are:

$$\varepsilon_B = \frac{\Delta}{L_B}\left(\frac{1}{1+\alpha}\right), \ \varepsilon_A = \frac{\Delta}{L_A}\left(\frac{\alpha}{1+\alpha}\right) \quad (3)$$

The stiffness ratio of the two bars is

$$\alpha = \frac{K_B}{K_A} \quad (4)$$

As the input displacement increases beyond the elastic limit, bar A continues to extend at no additional load. Thus the load remains unchanged and ε_A increases with increasing Δ.. The strains in bars are given by:

$$\varepsilon_B = \left(\frac{\sigma_A^{yp}}{E_A}\right)\left(\frac{L_A}{L_B}\right)\left(\frac{1}{\alpha}\right), \qquad \varepsilon_A = \frac{\Delta}{L_A} - \left(\frac{\sigma_A^{yp}}{E_A}\right)\left(\frac{1}{\alpha}\right) \quad (5)$$

where σ_A^{yp} is the yield stress for material A. The strains given by Eqn 5 correspond to the external displacement given by:

$$\Delta \geq \left(\frac{\sigma_A^{yp}}{E_A}\right)L_A\left(\frac{\alpha+1}{\alpha}\right) \quad (6)$$

Using Eqn 2 and noting that

$$(\varepsilon)_{final} = \varepsilon_A^{ep} = \frac{\Delta}{L_A} - \left(\frac{\sigma_A^{yp}}{E_A}\right)\left(\frac{1}{\alpha}\right)$$

$$(\varepsilon)_{initnal} = \varepsilon_A^{e} = \frac{\Delta}{L_A}\left(\frac{\alpha}{1+\alpha}\right) \tag{7}$$

It can be shown that

$$Z = \frac{\alpha+1}{\alpha} \tag{8}$$

b. Parallel bar structure

In this analysis an initial residual stress field is introduced into the structure by imposing an initial misfit, δ as shown in Figure 3. The structure is then loaded through a rigid block such that subsequent displacements in bars are essentially identical. It is assumed that the applied load is sufficient to cause plasticity in the centre bar A but the surrounding structure represented by bars B remain elastic. The system is then unloaded.

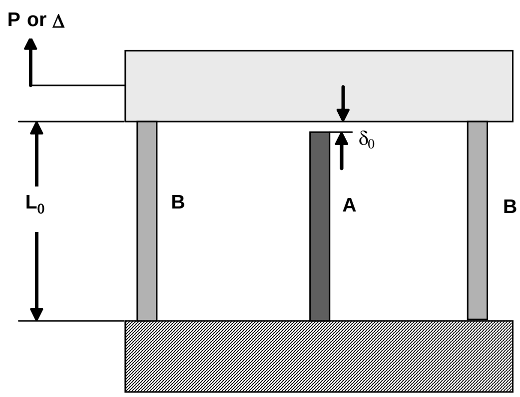

Figure 3. Parallel bars with an initial misfit subjected to load, P

The following expressions provide strains in bars A and B at three states; state 1 corresponding to the initial misfit, state 2 subsequent loading and finally state 3 is when the structure is unloaded.

For state 1 the strains due to misfit are:

$$\varepsilon_{B1} = \frac{\delta_0}{(\alpha+1)} \Big/ l_0 \text{ and } \varepsilon_{A1} = \frac{\alpha\delta_0}{(\alpha+1)} \Big/ l_0 \tag{9}$$

Here the stiffness ratio is given by

$$\alpha = nK_B / K_A \tag{10}$$

where n is the number of bars B

In state 2 after loading the strains are:

$$\varepsilon_{B2} = \left(\Delta - \frac{\delta_0}{(\alpha+1)}\right) \Big/ l_0 \text{ and } \varepsilon_{A2} = \left(\Delta + \frac{\alpha\delta_0}{(\alpha+1)}\right) \Big/ l_0 \tag{11}$$

Assuming that bar A is plastic whereas bars B remain elastic the corresponding load to the displacement Δ is:

$$P = \left(\sigma_A^{yp} A_A + nE_B \varepsilon_B A_B\right) = \sigma_A^{yp} A_A + \alpha K_A \left(\Delta - \frac{\delta_0}{(\alpha+1)}\right) \tag{12}$$

Following unloading, at state 3 the strains are:

$$: \varepsilon_{B3} = \left(\Delta - \frac{\delta_0}{(\alpha+1)} - \frac{l_0 \sigma_A^{yp}}{E_A}\right)\left(\frac{1}{\alpha+1}\right) \Big/ l_0$$

$$\text{and } \varepsilon_{A3} = \left(\Delta - \frac{\delta_0}{(\alpha+1)} - \frac{l_0 \sigma_A^{yp}}{E_A}\right)\left(\frac{\alpha}{\alpha+1}\right) \Big/ l_0 \tag{13}$$

To ensure the system does not collapse the applied external load is less than P_{max} where

:

$$P < P_{\max} = \left(\sigma_A^{yp} A_A + n\sigma_B^{yp} A_B\right) \tag{14}$$

To obtain a generalised solution for the parallel bar structure a simple Fortran routine was developed that provides stress strain data for incremental steps of displacement for various combinations of misfit, yield strength, and relative stiffness of the structural components. Within the code the external load corresponding to the applied displacement for the case of elastic-plastic response of the structure is calculated and then applied to the structure assuming purely elastic response to obtain the corresponding strains.

The information obtained from elastic plastic and elastic analyses was then be used to calculate the strains required to determine Z (equation 2), thus:

$$(\varepsilon)_{final} = \varepsilon_{A2}^{ep} = \left(\Delta + \frac{\alpha\delta_0}{(\alpha+1)} \right) \Big/ l_0$$

$$(\varepsilon)_{initial} = \varepsilon_{A2}^{el} = \delta_{A2}/l_0 = \frac{P/K_A + \delta_0\alpha}{(\alpha+1)l_0} \quad (15)$$

Assuming that bar A is plastic and bars B remain elastic, the corresponding load to the displacement Δ will be:

$$P = \left(\sigma_A^{yp} A_A + nE_B\varepsilon_B A_B\right) = \sigma_A^{yp} A_A + \alpha K_A \left(\Delta - \frac{\delta_0}{(\alpha+1)} \right) \quad (16)$$

Finally, the elastic equivalent strain in bar A corresponding to the final state is:

$$\left(\varepsilon_{eq}^{el}\right)_{final} = (\sigma)_{final}/E = \sigma_A^{yp}/E_A \quad (17)$$

Introducing equations 15 and 17 into equation 2 gives

$$Z = \frac{\alpha+1}{\alpha} \quad (18)$$

This result is based on the assumption that the applied load to the structure is fixed. Equation 18 is identical to equation 8 for the series bar structure in which the applied displacement was fixed. This is an important aspect of the external loading to the structure that will be discussed later.

EFU FOR BENCHMARK MODELS

Both structures described in previous sections were examined in detail and results are presented and discussed in this section. In both structures equations 8 and 18 show that as

$$\alpha \to 0 \text{ then } Z \to \infty \text{ and as } \alpha \to \infty \text{ then } Z \to 1$$

a. Series bar structure

The closed form description provided for elastic follow-up due to plasticity in the series bar structure was based on the assumption of perfect plasticity and on the application of a fixed displacement to the structure. In a series structure the load transfers through the bars so that the same load is applied to both bars A and B. It is the applied displacement that is distributed depending on the load level. It was shown that on this basis elastic follow-up only depends on the relative stiffness of the bars. Elastic follow-up for this case is independent of the applied displacement as long as it is sufficient to introduce plasticity into bar A. For a structure where the surrounding structure is stiffer than the "weaker region" A, i.e. $\alpha \geq 1$, a lower bound value of Z is 2. For relatively large stiffness ratios Z tends to 1 implying no associated EFU and the boundary conditions essentially represent displacement controlled conditions.

Assuming that the surrounding structure has a very low stiffness (e.g. long bar B compared to A with same material and cross section properties as bar A) the relative stiffness would be small (α<<1) resulting in high estimates of Z.

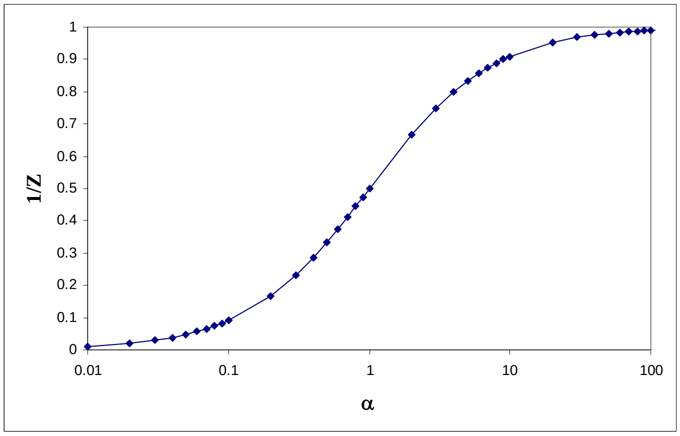

Figure 4. Variation of the inverse of Z (elastic follow-up) with stiffness ratio in a series bar structure

The variation of the inverse of Z as a function of the stiffness ratio α is shown in Figure 4. The extreme case of high Z, equivalent to the load control, $(1/z \to 0)$ can also be considered by assuming the application of the same load rather that displacement to the model. Once plasticity is reached in bar A the load will remain constant for perfect plasticity.

The applied load is:

$$P = A_A \sigma_A^{yp} \quad (19)$$

Applied to the elastic plastic structure gives the following strains

$$(\varepsilon)_{final} = \frac{\sigma_A^{yp}}{E_A} + \varepsilon_A^{pl}, (\varepsilon)_{initial} = \frac{\sigma_A^{yp}}{E_A}, \left(\varepsilon_{eq}^{el}\right)_{ifinall} = \frac{\sigma_A^{yp}}{E_A} \quad (20)$$

from which:

$$Z = \frac{(\varepsilon)_{final} - \left(\varepsilon_{eq}^{el}\right)_{final}}{(\varepsilon)_{initial} - \left(\varepsilon_{eq}^{el}\right)_{final}} = \frac{\varepsilon_A^{pl}}{0} = \infty \qquad (21)$$

b. Parallel bar structure

Based on the analysis presented in the previous section stress strain distributions for the elastic plastic structure are shown for relative stiffness ratios of α=1.0 and α=10 in Figure 5. Also are the distributions assuming purely elastic response.

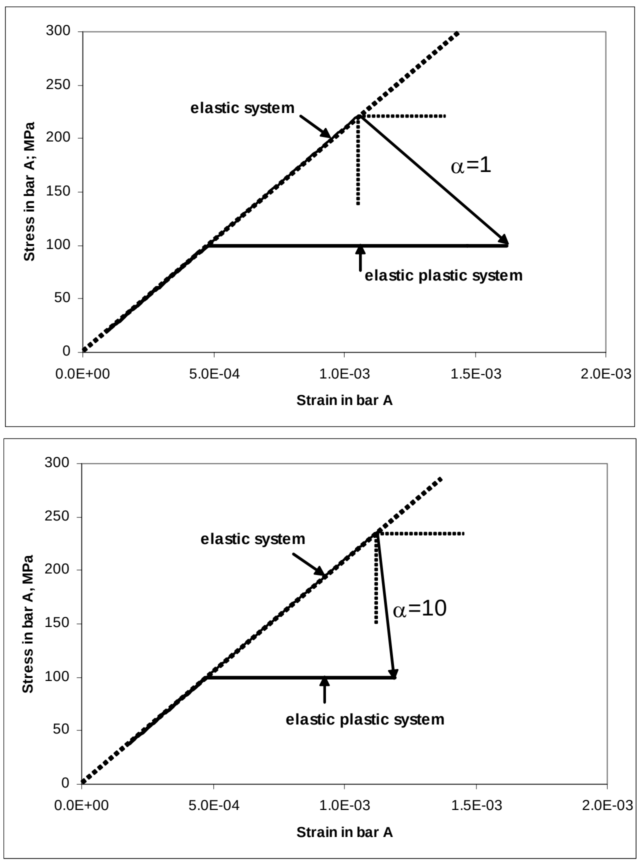

Figure 5. Stress strain response of bar A in parallel bar structure for α=1 and α=10

The solutions presented earlier suggest that elastic follow-up is only a geometrical feature of the structure and the initial (residual) global stress field introduced through the initial misfit provides no contribution to Z. If the initial residual stress field due to misfit is an elastic field then on subsequent loading of the structure the residual stress remains unchanged as long as no plasticity is introduced. That is to say if the structure is unloaded following this applied load the initial residual stress remains the same.

In contrast, when plasticity occurs in bar A some of the original misfit is accommodated. This will continue until the extent of plasticity equals the initial misfit. At this stage the initial residual stress is fully relaxed. The introduction of further plastic deformation in bar A will create a new (reversed) residual stress since misfit is now introduced in the direction opposite to the initial misfit. This is demonstrated for the two relative stiffness values in Figure 6. It is important to note that for a high relative stiffness a higher level of residual stress may be relaxed for the same level of plastic strain i.e. for the same initial misfit.

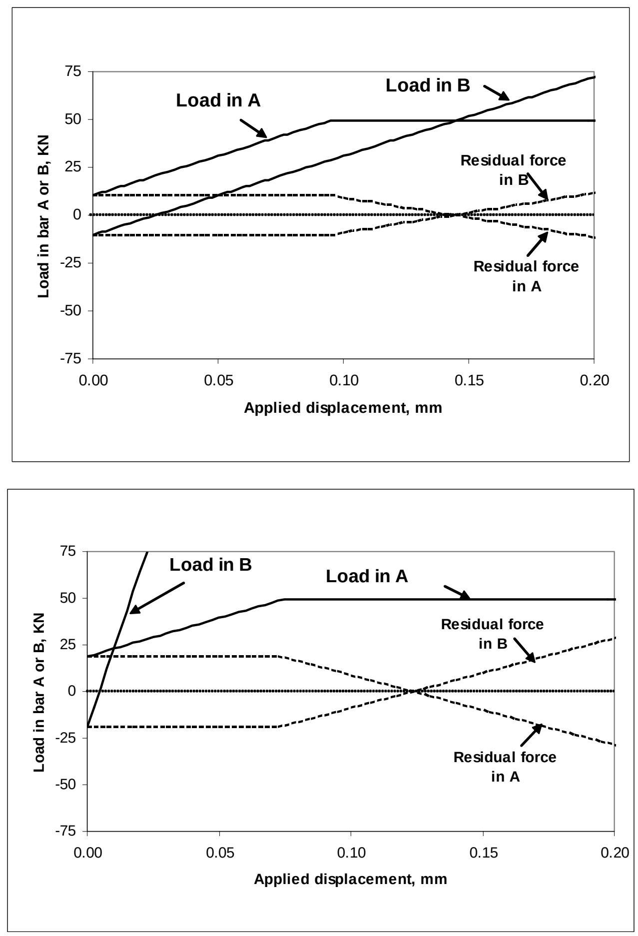

Figure 6. Relaxation of residual stresses (forces) introduced by misfit due to plastic straining in bar "A" in parallel formation

Values for Z for the selected relative stiffness ratios of 1 and 10 are plotted as a function of the applied load in Figure 7.

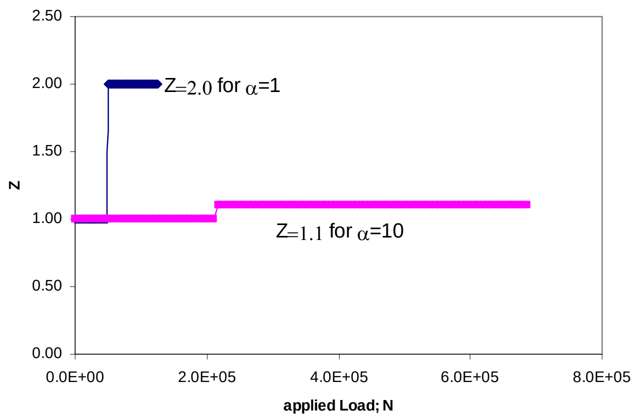

Figure 7. Elastic follow-up as a function of load (the assumed behaviour of bar A is elastic perfectly plastic)

DISCUSSION

The study of the benchmark models provides important insight into the source of the elastic follow-up in structures. There are various mechanisms, such as creep, plasticity and geometric features, which can aid the introduction of elastic follow-up into the structure. In both the series and parallel structures the essence of the model is that a "region" within the structure follows a combined boundary condition from that followed by the rest of the structure. Provided the effect of the mechanism on the response of the structure is the same the elastic follow up is the same. Elastic follow-up is a consequence of a presence of a region of differing stiffness relative to the remainder of the structure.

Based on the results of the benchmark models a methodology for quantifying elastic follow-up in a cracked structure is suggested. Similar to the cases of creep and plasticity, a crack in a structure also introduces nonlinearity. The added complication in case of crack presence is the presence of a stress and strain singularity. To overcome this, a "crack affected zone" (CAZ) in a structure may be defined as a representative of a softening component of the structure whereas the remaining of the structure, the surrounding continuum remains essentially elastic. This is shown schematically in Figure 8. The reference (initial) state is the un-cracked structure. Softening is due to the appearance of crack in the structure. For a fixed displacement applied to the structure, as the crack grows the far field stress decreases leading to an increase in strain in the crack affected zone.

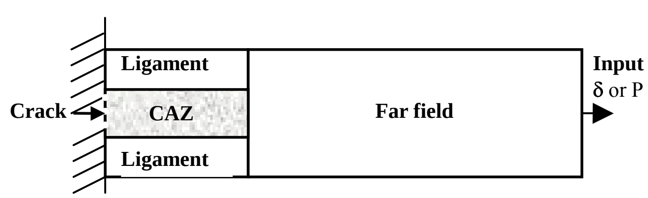

Figure 8. Crack affected zone (CAZ), accommodating local stress strain distributions due to presence of crack

CONCLUDING REMARKS

- The concept of elastic follow-up originally used as a means of simplifying creep and creep fatigue analysis methods, has been examined for the cases where nonlinear response of structure is due to localised plasticity within the structure
- Benchmark models in form of series and parallel bar structures with idealised material behaviour have been used to obtain closed form solutions for the stress strain response of the plastically deforming softening component of the structure.
- Based on the findings from the analysis of the benchmark models the concept of elastic follow-up has been extended to the characterisation of localised plasticity.
- A pragmatic approach that tackles the singularity problem associated with cracked structure analysis has been outlined. This has been achieved by introduction of a "crack affected zone" approach (CAZ) that is analogous with the concept of the "softening region".

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